# Estimated probability of the partial coverage of QSOs by intervening $H_2$ -clouds at formation of QSO absorption-line spectra

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Abstract. We have estimated the probability of a partial coverage of QSO broad-line regions by intervening H<sub>2</sub>-clouds. This effect has been revealed by an analysis of H<sub>2</sub> absorption systems in QSO spectra [1, 2]. Accounting of the effect may change significantly physical parameters of interstellar clouds derived from the spectral analysis [2].

We show that the probability of incomplete coverage turns out to be not lower than about 8%. Actually, a frequency of the effect revealing may occur essentially higher than this quantity, encouraging us in further systematic searches of the incomplete coverage in the  $H_2$  absorption QSO spectra.

## 1. Introduction

It was widely accepted until recently that intervening H<sub>2</sub> absorption clouds fully cover cosmologically remote quasars (QSOs) usually treated as point-like powerful sources of radiation. The interstellar clouds in remote galaxies situated on lines of sight between QSOs and an observer imprint a set of absorption lines into initial QSO spectra which consist of a smooth continuum and broad emission lines. The latter are formed within a broad-line region (BLR) in a wider vicinity of a central QSO machine.

The partial coverage of a BLR by an intervening absorption cloud was firstly reported in [1] and investigated in detail by [2]. It was demonstrated that the H<sub>2</sub>-bearing cloud covers the QSO 1232+082 ( $z_{em} = 2.57$ ) intrinsic continuum source completely but only a part of the BLR. The details of this unique effect and different alternative interpretations were discussed in the paper by [2].

Recently another H<sub>2</sub> absorption system with partial coverage of the appropriate QSO has been revealed in Q0528-250 spectrum [3]. Since taking into account of the partial coverage leads to serious changes of the results of QSO spectra analysis (e.g. column densities of absorbers), it is important to investigate possibility of the partial coverage effect in other QSO H<sub>2</sub> absorption systems. Following [2], one can introduce noncoverage factor f as the ratio of a light flux passing by the cloud (i.e. came to the detector without absorption) to a flux which would be detected in absence of the cloud. It is useful to calculate a distribution function of f, i.e. a probability to reveal  $f > f_0$  ( $f_0$  is an assigned value) for an arbitrary absorption system.



Figure 1. Schematic illustration of an absorption cloud with a transverse size  $l_c$  (grey circle) situated between the observer and QSO broad-line region (BLR) with a transverse size  $l_q$  (red ellipse);  $\theta_q$  is an angular size of BLR,  $\Delta \theta$  is an angle between the line-of-sights from the observer to the centres of QSO and cloud.  $\Omega_q$  is a solid angle (light cone) of the whole BLR radiation flux,  $\Omega_{uncov}$  is a solid angle filled by non-absorbed radiation. Radiation from the QSO without H<sub>2</sub>-absorption systems is shown as yellow area of the light cone and radiation containing the absorption systems – as a dimly coloured area. Note that the light cone is curved due to the expansion of the Universe.

Note that there is an important reason to search the partial coverage phenomenon for molecular clouds in contrast with HI clouds (mostly forming Ly $\alpha$  forest) which is likely to have transverse sizes well exceeding the probable sizes of QSO BLRs (e.g. [4]).

In the present paper, we build the probability function of noncoverage factor f for QSO BLRs with  $z_{em} = z_q$  (QSOs) by absorption systems with  $z_{abs} = z_c$  (clouds). We consider an arbitrary angular distance between line-of-sights to the centres of the cloud and corresponding BLR, as well as an arbitrary ratio  $\kappa$  of the cloud to BLR transverse sizes. In Section 2 we introduce noncoverage factor f and consider the main parameters determinative f. In Section 3 we calculate distribution of noncoverage factor for certain distributions of physical parameters. In Section 4 we discuss obtained results and their relation to observations.

#### 2. Geometry of partial coverage

A sketch of partial coverage is represented on Fig.1. Light from the QSO propagates through the Universe and it is registered by an observer. The observer detects a light cone within an angle  $\theta_q$  or solid angle  $\Omega_q$ . Some part of light passes by the cloud and comes to the observer without formation of an absorption-line system in a spectrum. This part is comprised within a solid angle  $\Omega_{uncov}$ . The rest light is partly screened by the cloud in such a way that absorption of molecules in the cloud imprints a set of absorption lines in the initial spectrum. In result the observer detects a complex radiation from QSO with spectrum integrated over angles. A screened (covered) flux of radiation may be approximately estimated as a flux comprised by a solid angle  $\Omega_{cov} = \Omega_q - \Omega_{uncov}$ .

Noncoverage factor f is a ratio of the flux which goes by the cloud without absorption to the flux of radiation which would come to the observer if there was no cloud on its path. Let us assume roughly that both the screened and unscreened fluxes are uniformly distributed within their solid angles. Thus f may be estimated as a ratio of the solid angle  $\Omega_{uncov}$  to the whole solid angle  $\Omega_q$  of BLR observations:

$$f = \frac{\Omega_{uncov}}{\Omega_q},\tag{1}$$

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where at  $\theta_q \ll 1$  one can write  $\Omega_q = \pi \theta_q^2$ .

Using the well-known definition of the angular size distance,  $D_A(z)$ , for cosmologically distant objects with a proper transverse size l at a cosmological redshift z, one can determine the angular size [5]  $\theta = l/D_A(z)$ , where  $D_A = cd_A(z)/H_0$  is the angular size distance,  $H_0$  is the Hubble constant at present, c is the speed of light and the dimensionless value  $d_A(z)$  in the standard  $\Lambda$ CDM cosmological model is:

$$d_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}}.$$
 (2)

Here  $\Omega_m$  is the dimensionless cold matter density parameter and  $\Omega_{\Lambda} = \Lambda c^2/(8\pi G)$  is determined by the cosmological constant  $\Lambda$ , G – gravitational constant. According to Fig. 1 one can set  $\theta_q = \theta/2$  and employ the equation  $\theta_q = lH_0/(2cd_A(z))$ . Then using this relation we obtain the final equation for  $\Omega_q$ . Estimations of the values  $\Omega_{uncov}$  and f will be outlined below.

Let us demonstrate that the phenomenon of incomplete covering is amplified by cosmological properties of the space-time. Fig. 2 displays two dependencies of an angular size of a cosmologically distant object (with proper linear size l and redshift z) on the distance L = ct(z), where t(z) is the standard expression for the cosmological time of the light-signal propagation from the object to observer. Thus the green curve in Fig. 2 demonstrates the dependence of  $\theta(z)$  on the light-propagation distance L = ct(z) in expanding  $\Lambda$ CDM model. In the Euclidean (stationary) cosmological model it is a hyperbolic dependence  $\theta = l/L$  on the distance L to the object.

One can see that an angular size in the expanding Universe is larger than it would be expected in the stationary model, and it grows with L at L > 3 Gpc. This is well known phenomenon in the Friedmann cosmological models (e.g. [6]). Specific light trajectories in such models relatively to the observer reference frame are schematically drawn in Fig. 1.





Figure 2. Angular size (in arcsec)  $\theta(z)$  of an object with a proper transverse size l = 1 pc and cosmological redshift z versus distance L = ct; light passes from the object to the observer for cosmological time t(z) in expanding (green curve) and stationary (Euclidean; red curve) Universes.

Figure 3. Four types of coverage. Types of mutual relations between cloud (grey circles) and QSO BLR (red circles) in the observer reference frame: 1) is full coverage (widely known case), 2) – crescent-like coverage, 3) – an "annular" coverage, 4) – full lack of coverage.

Thus, if BLR transverse size  $l_q$  is comparable with the cloud size  $l_c$ , then the phenomenon of partial coverage may occur for essentially different  $z_q$  and  $z_c$ . The condition L > 3 Gpc corresponds to  $z > z_* \approx 1.64$ . For detected H<sub>2</sub> absorption systems in QSO spectra both values  $z_{em} = z_q$  and  $z_{abs} = z_c$  typically exceed the value  $z_*$ , so it is necessary to take into account the cosmological expansion.

Assuming that BLRs and clouds are spherical one can determine  $\Omega_{uncov}$  (see Eq. (1)) using an estimation of two partly overlapping circle areas. A circle conventionally representing the BLR may be characterized by the radius  $\theta_q$  and a circle representing the cloud – by the radius  $\theta_c$ . Then the square of the BRL circle is  $\pi \theta_q^2 = \Omega_q$ . Let us introduce also an overlapping square S (in units of rad<sup>2</sup>) of the BLR and cloud circles. Then one can define  $f = 1 - S/\Omega_q$ , and additionally  $\rho = \theta_c/\theta_q$ ,  $\kappa = l_c/l_q$ . In these terms we have  $\rho(\kappa, z_q, z_c) = \kappa d_A(z_q)/d_A(z_c)$ ,  $f = 1 - s/\pi$ , where  $s = S/\theta_q^2$ . Note that  $\rho$  may be either > or <  $\kappa$ . It is useful to introduce also a relative angular deviation  $\delta = \Delta \theta/\theta_q$ , where  $\Delta \theta$  is an angular distance between line-of-sights to the centres of BRL and cloud. In our schematic approach  $\delta$  characterizes a distance between circle centres. One can compose the resulting expression for  $f = f(\kappa, \delta, z_q, z_c)$  as function of four parameters  $\kappa$ ,  $\delta$ ,  $z_q$  and  $z_c$ .

Fig. 3 displays all possible relative positions of two conventional circles referred to the BLR and cloud; it is suitable to represent all values in units  $\theta_q$ , then the radius of the BLR is 1.

- 1) Full coverage. All flux from the QSO goes through the cloud, f = 0. The condition is:  $1 + \delta \leq \rho$ .
- 2) Crescent coverage. The condition for this case is:  $|\rho 1| < \delta < \rho + 1$ . After some geometric calculations with  $f(s) = 1 s/\pi$  we obtain

$$f = 1 - \frac{\rho^2}{\pi} \arccos \frac{\rho^2 + \delta^2 - 1}{2\rho\delta} - \frac{1}{\pi} \arccos \frac{1 + \delta^2 - \rho^2}{2\delta} + \frac{1}{2\pi} \sqrt{\left[\rho^2 - (\delta - 1)^2\right] \left[(\delta + 1)^2 - \rho^2\right]}.$$
(3)

- 3) Annular coverage. The angular size of the cloud should be less than the BLR size. The condition is  $\rho + \delta \leq 1$ ;  $\rho \leq 1$ ,  $\delta \leq 1$ . In this case we get  $f = 1 \rho^2$ .
- 4) Full noncoverage. Relative angular deviation is large and all flux from the QSO goes by the cloud, i.e. the cloud remains unobservable in the QSO spectrum, i.e. f = 1. The condition of this case is  $1 + \rho \leq \delta$ .

#### 3. Distribution of noncoverage factor

Let us determine a distribution of noncoverage factor f, i.e. probability  $P(f > f_0)$  to detect the absorption systems in QSO spectra with noncoverage factor exceeding a value  $f_0$ . We fix  $z_q$  and  $z_c$  and treat the parameters  $\kappa$  and  $\delta$  as arbitrary ones. Firstly note that the very fact of absorption-lines registration excludes the case (4) with f = 1, i.e the probability of full noncoverage P(f = 1) = 0. According to the conditions of Section 2 it is true when  $\delta < 1 + \rho(\kappa, z_q, z_c)$ . We assume the uniform distribution of the deviation parameter  $\delta$  and introduce the probability P and probability density function  $\Phi_{\delta}$  according to the standard definition:  $P(\delta' < \delta < \delta' + d\delta') = \Phi_{\delta}(\delta') d\delta'$ . In such a way we use:

$$\Phi_{\delta}(\delta') = \frac{\Theta(1 + \rho(\kappa, z_q, z_c) - \delta')}{1 + \rho(\kappa, z_q, z_c)},\tag{4}$$

where  $\Theta(x)$  is the Heaviside function.

Distribution of  $\kappa$  is determined by distributions of  $l_c$  and  $l_q$ . Hereafter we assume that  $l_q = 0.1$  pc and consider only dependence of the distribution on  $l_c$ , which is assumed [7] to be  $l_c \sim 0.1 \div 10$  pc. The probability density function  $\Phi_{\kappa}$  for  $\kappa$ -distribution  $P(\kappa' < \kappa < \kappa' + d\kappa') =$ 

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 $\Phi_{\kappa}(\kappa') \, d\kappa'$  may be represented as  $\Phi_{\kappa}(\kappa') = \psi(\kappa')\Theta(\kappa'-\kappa_{min})\Theta(\kappa_{max}-\kappa')$ , where  $\psi(\kappa')$  describes a distribution function over cloud sizes  $l_c$ ; below we consider the function  $\psi(\kappa)$  as a power-law distribution in a general form:  $\psi(\kappa) = (1-\beta)\kappa^{-\beta} (\kappa_{max}^{1-\beta} - \kappa_{min}^{1-\beta})^{-1}$ , uniform distribution over  $\kappa$ corresponding to  $\beta = 0$ .

We assume that the parameter  $\delta$  is fixed and calculate the reciprocal function  $\kappa(f, \delta, z_q, z_c)$ . Actually, the reciprocal function  $\kappa(f, \delta)$  at certain  $z_q$  and  $z_c$  can be represented analytically in five regions completely covering the whole domain of permissible values f and  $\delta$ . Our calculations show that  $\kappa(f, \delta)$  in all cases is a decreasing function of f, i.e.  $\kappa < \kappa_0 = \kappa(f_0)$  when  $f > f_0$ , and f = 1 at  $\kappa = 0$ . On the contrary, the greater  $\delta$ , the greater  $f \leq 1$  can be obtained. Summing up all points discussed above one can write

$$P(f > f_0) = \int_0^\infty \int_0^{\kappa(f_0, \delta, z_q, z_c)} \Phi_\kappa(\kappa') \ \Phi_\delta(\delta') d\kappa' d\delta'.$$
(5)

Resulting dependencies  $P(f > f_0)$  are given in Figs. 4, 5, where  $P(f > 0) = 1 - \mathcal{P}(f = 0) \neq 1$ , and  $\mathcal{P}(f = 0)$  is a probability of full coverage.



Figure 4.  $P(f > f_0)$  for uniform distribution of cloud sizes at  $z_q = 2.57$ ,  $z_c = 2.33$ . Coloured curves correspond to different  $\kappa_{max}$ .



Figure 5.  $P(f > f_0)$  for power-low distribution of cloud sizes at  $z_q = 2.57$ ,  $z_c = 2.33$ . Coloured curves correspond to different  $\beta$ .

#### 4. Results and conclusions

In the present paper we investigate the probability of the partial coverage of a QSO BLR by an intervening H<sub>2</sub>-cloud. Partial coverage is estimated by *noncoverage factor*, i.e. the ratio of a solid angle comprising the whole BLR emission to a solid angle of an uncovered part. A *distribution of noncoverage factor* was calculated for fixed redshifts of the QSO and cloud in the cases of uniform and power-law distributions of cloud sizes.

Using the obtained value,  $P(f > f_0)$ , one can estimate the probability to reveal the incomplete coverage at a chosen level  $f \ge 0.02$  which might affect the absorption lines analysis and obtained physical conditions (see [2]). It is shown that in the most unfavourable case of uniform  $\kappa$ distribution and sufficiently large  $\kappa_{max} = 100$  the probability to reveal the lowest noncoverage factor is  $P(f > 0.02) \ge 0.08$ . In more realistic cases the probability can reach values exceeding 0.3. It means that at the analysis of QSO spectra one should expect the possibility of the incomplete coverage phenomenon. Two molecular clouds with actually estimated linear sizes has  $l_c \sim 0.1 \div 0.2$  pc. It means that appropriate  $\kappa \sim 1 \div 2$  at  $l_q \sim 0.1$ . It is not likely that all  $\sim 20$  registered molecular H<sub>2</sub>-clouds at high redshifts correspond to a small sizes. So, it is very important to accumulate some additional information on the distribution of the clouds, as well as the QSO BLRs, over their sizes. In the case of the power-law distribution it would increase minimal P(f > 0.02)estimation. It is good reason to investigate QSO absorption-line spectra with systematic search for the partial coverage. Being revealed the partial coverage phenomenon may lead to essential revision of interstellar medium parameters at high redshift  $z \sim 2 - 4$ .

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## References

- [1] Ivanchik A V, Petitjean P, Balashev S A, Srianand R, Varshalovich D A, Ledoux C and Noeterdaeme P, 2010, MNRAS, **404**, 1583
- [2] Balashev S A, Petitjean P, Ivanchik A V, Ledoux C, Srianand R, Noterdaeme P and Varshalovich D A, 2011, MNRAS, 418, 357
- Balashev S A, Klimenko V V, Ivanchik A V, Ledoux C, Noterdaeme P, Petitjean P, Varshalovich D A, 2013, MNRAS (in preparation)
- [4] Dave R, Oppenheimer B D, Katz N, Kollomeier J A and Weinberg D H, 2010, MNRAS, 408, 2051
- [5] Kayser R, Helbig P, Schramm T, 1997, A& A, **318**, 680
- [6] Zel'dovich Ya B, Novikov I D, 1983, The Structure and Evolution of the Universe, Univ. Chicago Press, Chicago.
- [7] Rauch M, 1998, ARA& A, 36, 267