Electron screening effect on stellar thermonuclear fusion

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We study the impact of plasma correlation effects on nonresonant thermonuclear reactions for various stellar objects, namely in the liquid envelopes of neutron stars, and the interiors of white dwarfs, low-mass stars, and substellar objects. We examine in particular the effect of electron screening on the enhancement of thermonuclear reactions in dense plasmas within and beyond the linear mixing rule approximation as well as the corrections due to quantum effects at high density. In addition, we examine some recent unconventional theoretical results on stellar thermonuclear fusions and show that these scenarios do not apply to stellar conditions.

1 Introduction

Thermonuclear reactions play a crucial role in stellar evolution. Nuclear fusion rates in stellar interiors can be significantly enhanced over the binary Gamow [1] rates because of the many-body screening effect in the dense plasma (first recognized by E. Schatzman [2]; for reviews, see [3, 4]).

In the envelopes of neutron stars (NSs) and interiors of white dwarfs (WDs), where the electrons are strongly degenerate, the screening effect is usually treated under the assumption that the electron gas can be considered as a uniform "rigid" background, and the screening is provided solely by ions. On the other hand, in ordinary stars this effect is often treated with Salpeter formula, which implies Debye screening (see, e.g., Ref. [5] and references therein). The latter approximation is applicable in a gaseous phase. In the present article, we consider the electron screening effect on nuclear fusion at arbitrary electron degeneracy and arbitrary Coulomb coupling of ions in gaseous and liquid plasmas.

The influence of the electron polarization on the enhancement of nuclear reaction rates has been studied in some detail in several papers [3, 6–9]. At the time of those studies, uncertainties in the reaction rates due to other factors, viz. quantum effects and deviations from the linear mixing rule in strongly coupled plasmas, as well as theoretical uncertainties in the nuclear effective potentials at short distances, were more important than the electron-screening effects. For this reason, more recent works were aimed at reducing these uncertainties and mostly neglected the electron polarization (e.g., [10–12]). In this paper, we show, however, that the effect of the electron-polarization on the enhancement factor of the nuclear reaction rates is typically of the same order of magnitude as the other recently proposed corrections.

In Sect. 2 we compare different approximations for the enhancement factors. In Sect. 3 we describe the results of the application of the electron-screening correction to the nuclear reaction rates in stellar conditions. In Sect. 4 we discuss the origin of discrepancies between our results and some other results published recently. Sect. 5 is devoted to the conclusion.

2 Theory

A review of the theory of nuclear fusion in stars with extensive bibliography was given in the Nobel lecture by Fowler [13]. One should discriminate between the reactions related to nuclear resonances and nonresonant

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reactions. We consider only the latter ones. It is customary to write the cross section of binary nuclear fusion reactions in the form

$$\sigma(E) = \mathrm{e}^{-2\pi\eta}\, S(E)/E, \tag{1}$$

where E is the center-of-mass energy of the reacting nuclei "1" and "2",

$$\eta = \sqrt{E_R/E}, \qquad E_R = (Z_1 Z_2 e^2)^2\, m_{12}/2\hbar^2, \tag{2}$$

$Z_j e$ is the charge of nucleus "j", e is the elementary charge, $m_{12} = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and $S(E)$ is a function called "astrophysical factor." Then the reaction rate (the number of fusion events per unit time in unit volume) in the absence of plasma screening is given by

$$R_{12} = w_{12}\, n_1 n_2 \sqrt{\frac{2}{m_{12}}} \int_0^\infty \mathrm{e}^{-2\pi\eta} S(E)\, w(E)\, \mathrm{d}E \bigg/ \int_0^\infty w(E)\, \sqrt{E}\, \mathrm{d}E, \tag{3}$$

where n_j is the number density of the ions of type "j", $w(E)$ is the statistical distribution function of the center-of-mass energies of the reacting nuclei, and the factor w_{12} accounts for statistics: $w_{12} = \frac{1}{2}$, if nuclei "1" and "2" are identical; otherwise $w_{12} = 1$. With Boltzmann statistics, $w(E) = w_\mathrm{B}(E) \equiv T^{-1}\exp(-E/T)$, where T is temperature in energy units.

In order to take the plasma screening into account, it is convenient to write the radial pair-distribution function for ions in the form [14]

$$g_{12}(r) = \exp\left(-Z_1 Z_2 e^2/rT\right)\, \exp\left[H_{12}(r)/T\right], \tag{4}$$

where the first factor is the Boltzmann formula for an ideal gas, while the second one shows how the probability of separation of two chosen ions is affected by the surrounding plasma particles.

It is convenient to introduce parameters

$$\Gamma_{12} = Z_1 Z_2 e^2/a_{12}T, \qquad a_{12} = (a_1 + a_2)/2, \qquad \tau = 3(\pi^2 E_R/T)^{1/3}, \qquad \zeta = 3\Gamma_{12}/\tau. \tag{5}$$

where $a_j = (3Z_j/4\pi n_e)^{1/3}$ are the ion-sphere radii, and n_e is the electron number density. As shown in Ref. [4], under the condition $\zeta \ll 1$ the function $H_{12}(r)$ slowly varies on the scale of the classical turning point distance and the nuclei behave as classical particles. Then the reaction rate with allowance for the plasma screening is approximately given by $R_{12}\exp(h)$, where $h = H_{12}(0)/T$ and R_{12} is expressed by Eq. (3) [6]. Furthermore, one can prove [14–17] that $H_{12}(0)$ equals the difference between the excess free energies F_ex before and after an individual act of fusion. Here, $F_\mathrm{ex} = F - F_\mathrm{id}$, F is the total Helmholtz free energy, and F_id is the free energy of the ensemble of noninteracting ions and electrons. In the thermodynamic limit this gives the relation

$$h = \left(\frac{\partial}{\partial n_1} + \frac{\partial}{\partial n_2} - \frac{\partial}{\partial n_3}\right)\left[n_\mathrm{ion} f_\mathrm{ex}(\{n_j\}, n_e, T)\right], \tag{6}$$

where $n_\mathrm{ion} = \sum_j n_j$ is the total number density of ions, including number density n_3 of composite nuclei, which have charge number $Z_3 = Z_1 + Z_2$ and mass $m_3 \approx m_1 + m_2$, and $f_\mathrm{ex} \equiv F_\mathrm{ex}/n_\mathrm{ion}VT$ is the normalized excess energy.

In the linear-mixing approximation,

$$f_\mathrm{ex} \approx f_\mathrm{lm}(\{n_j\}, n_e, T) \equiv \sum_j x_j f_j(n_e, T). \tag{7}$$

Here, $x_j \equiv n_j/n_\mathrm{ion}$ denotes the number fractions, and $f_j(n_e, T)$ is f_ex for a plasma containing only the jth type of ions. In this approximation, the enhancement exponent h becomes

$$h_\mathrm{lm} = f_1(n_e, T) + f_2(n_e, T) - f_3(n_e, T). \tag{8}$$

In the model of a rigid electron background, this reduces to

$$h_\mathrm{lm,ii} = f_\mathrm{ii}(\Gamma_1) + f_\mathrm{ii}(\Gamma_2) - f_\mathrm{ii}(\Gamma_3), \tag{9}$$

where $f_{ii}(\Gamma)$ is the normalized excess free energy of the one-component plasma and $\Gamma_j = (Z_j e)^2/a_j T$ are coupling parameters of individual ion species. In the ion sphere approximation, $f_{ii}(\Gamma) = -0.9\,\Gamma$, and then $h_{lm,ii}$ becomes [6]

$$h_S = 0.9\,(\Gamma_3 - \Gamma_1 - \Gamma_2). \tag{10}$$

The linear mixing rule works in strongly coupled Coulomb plasmas, i.e., at $\Gamma_j \gg 1$ [18, 19]. In the opposite limit $\Gamma_j \ll 1$ ($\forall j$), the Debye-Hückel approximation is applicable: $F_{DH} = -VT/12\pi D^3$, where D is the screening length:

$$F_{ex} \approx -\frac{VT}{12\pi D^3}, \quad D^{-2} = k_{TF}^2 + D_{ion}^{-2}, \qquad D_{ion}^{-2} = \frac{4\pi e^2}{T}\sum_j n_j Z_j^2, \qquad k_{TF}^2 = 4\pi e^2 \frac{\partial n_e}{\partial \mu_e}, \tag{11}$$

where μ_e is the chemical potential of the electron Fermi gas. Using Eq. (24) of Ref. [20], one can write $k_{TF}(n_e, T)$ in an analytic form. In the two limiting approximations of nondegenerate electrons ($k_{TF}^2 \to 4\pi e^2 n_e/T$) and rigid background ($k_{TF} \to 0$), Eqs. (6) and (11) give the Salpeter formula [6]

$$h_{DH} = Z_1 Z_2 e^2/DT. \tag{12}$$

Salpeter and Van Horn [21] proposed a simple interpolation between the Debye-Hückel and strong-coupling limits:

$$h_{SVH} = \frac{h_S\, h_{DH}}{\sqrt{h_S^2 + h_{DH}^2}}, \tag{13}$$

where h_S and h_{DH} are given by Eqs. (10) and (12), respectively. A more elaborated approximation for the enhancement factor between the Debye-Hückel and strong-coupling limits was constructed for the rigid background model in Ref. [12].

These analytic approximations can be compared to the accurate result. We write the normalized excess free energy in the form $f_{ex} = f_{lm} + f_{mix}$, where f_{lm} is given by Eq. (7), and f_{mix} is the correction to the linear-mixing. Then Eq. (6) gives

$$h_0 = h_{lm} + \left.\frac{\mathrm{d}f_{mix}(x_1 + \xi, x_2 + \xi, x_3 - \xi)}{\mathrm{d}\xi}\right|_{\xi=0}, \tag{14}$$
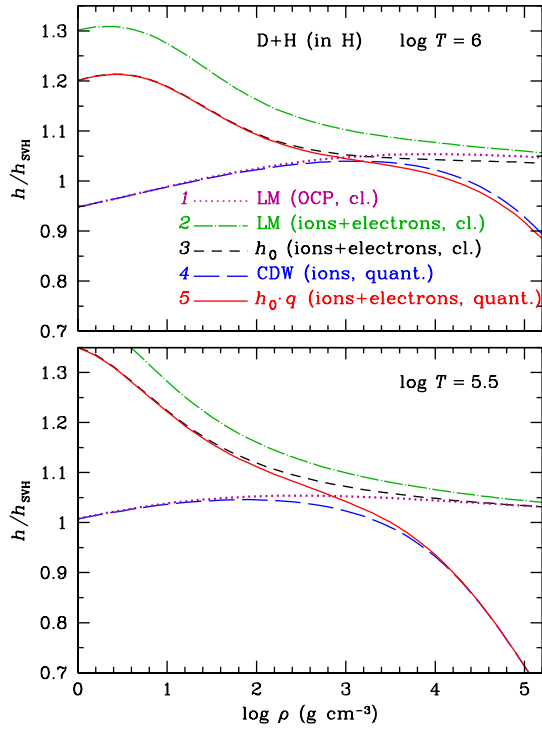
where h_{lm} is given by Eq. (8). The right-hand side of Eq. (14) can be written in an analytic form using our fitting formulae for $f_{ex}(n_e, T)$ and $f_{mix}(\{x_j\}, \{Z_j\}; n_e, T)$ (see [22] and references therein).

Equation (14) is obtained assuming that $H_{12}(r) \approx H_{12}(0)$, which is true for small values of the parameter ζ defined in Eq. (5). When this condition is not satisfied, the classical enhancement exponent h_0 should be corrected for the quantum effects. We denote this corrected value h_q. Alastuey and Jancovici [24] showed that $h_q < h_0$ and developed a perturbation expansion of h_q in powers of ζ. More recently, Militzer and Pollock [25, 26] performed simulations of the contact probabilities in the quantum regime and extended numerical results beyond the applicability range of the perturbation theory [24]. Chugunov and DeWitt [11] found that the quantum effects can be described in the linear-mixing, rigid-background approximation by substitution of $\tilde{\Gamma}_j = \Gamma_j/t_{12}$ instead of Γ_j into Eq. (9), where $t_{12} = \left[1 + c_1\zeta + c_2\zeta^2 + c_3\zeta^3\right]^{1/3}$, $c_1 = 0.013\,z^2$, $c_2 = 0.406\,z^{0.14}$, $c_3 = 0.062\,z^{0.19} + 1.8/\Gamma_{12}$, and $z = 4Z_1 Z_2/(Z_1 + Z_2)^2$. An analogous correction is not known for the polarizable background. Fortunately, the quantum effects are important only in the domain of high densities and relatively low temperatures, whereas the deviations from the linear-mixing and rigid-background approximations are most important in the opposite case. Therefore, in order to take all these effects into account, we multiply the classical expression (14) by factor $q = \tilde{h}_{lm,ii}/h_{lm,ii}$, where $h_{lm,ii}$ and $\tilde{h}_{lm,ii}$ are given by Eq. (9) with Γ_j and $\tilde{\Gamma}_j$, respectively.
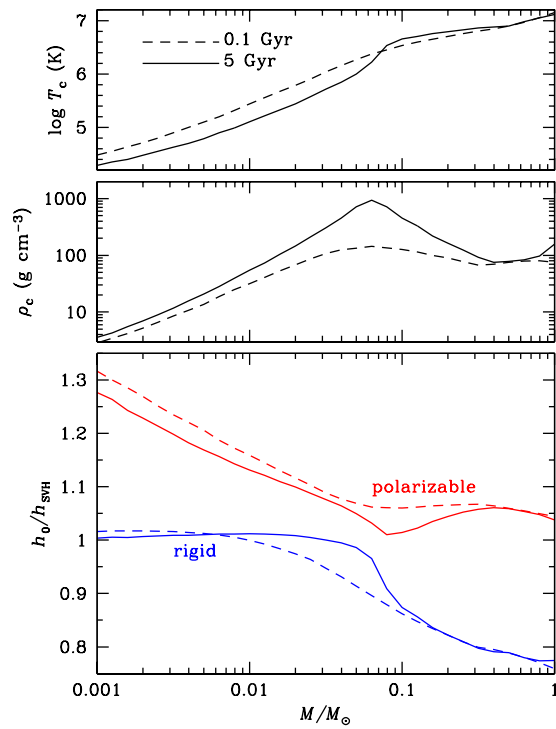
3 Results

3.1 Degenerate stars

The electron-screening effects on the enhancement factors and ignition curves for carbon and oxygen fusion reactions in the liquid layers of WDs and NSs were studied in Ref. [27]. Under the typical conditions in these
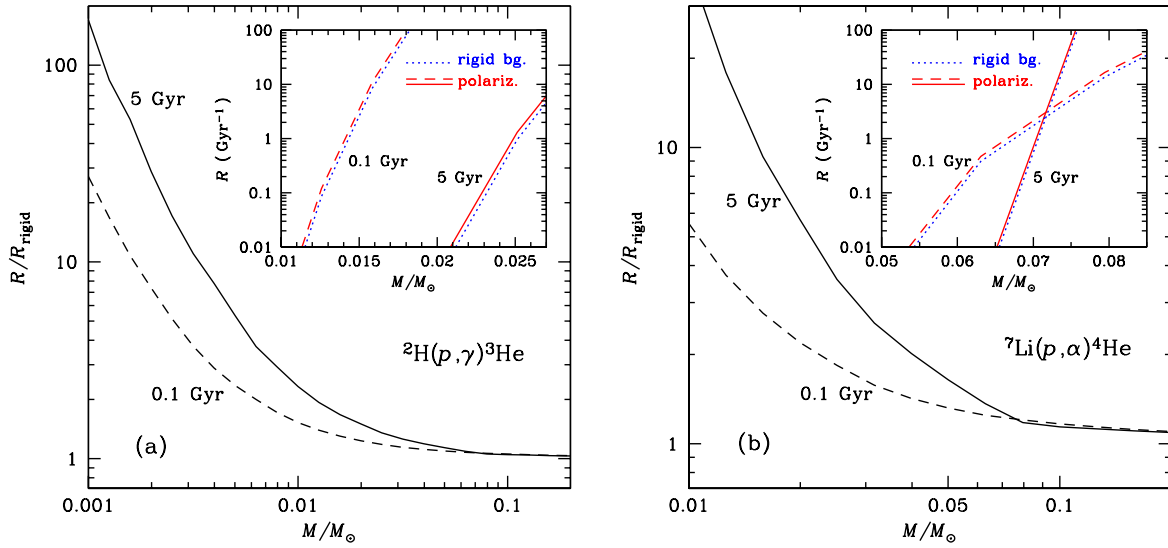
Fig. 1 Plasma enhancement exponents for deuterium fusion in hydrogen medium as a function of mass density at $T = 10^6$ K (upper panel) and $10^{5.5}$ K (lower panel) in different approximations. Three lines show results for classical nuclei: (*1*) h_0 in the linear-mixing, rigid background approximation [Eq. (9)] (dotted lines), (*2*) h_0 for the linear mixing with polarizable electron background [Eq. (8)] (dot-dashed lines), (*3*) h_0 beyond the linear-mixing approximation with a polarizable electron background [Eq. (14)], the most general classical approximation] (short dashes). The other two lines take into account quantum corrections: (*4*) the fit of Ref. [11] for h_q (long dashes), and (*5*) the approximation $h_q = q h_0$ (with q defined in the text), which includes both the ionic and electronic screening contributions and takes both the quantum effects and the deviations from the linear-mixing rule into account (solid lines).

Fig. 2 Physical conditions (at two ages) and enhancement exponents (in two approximations) at the centers of LMSs and SSOs with the solar abundance of heavy elements. In all three panels solid and dashed lines are drawn at the ages 5 Gyr and 100 Myr, respectively. The top and middle panels reproduce, respectively, the central temperature and density along the LMS-SSO mass range. The bottom panel shows the normalized enhancement exponent h/h_{SVH} for the deuterium burning along these temperatures and densities with (upper pair of curves) and without (lower pair of curves) account of electron polarization.

layers, the electron screening proved to increase the enhancement exponent h by several to tens percent, which translates into a factor of a few for the reaction rate $\propto e^h$. The deviations from the linear-mixing approximation have the opposite effect of a similar magnitude. Therefore, the corrections beyond the linear mixing [12] must be considered only together with the electron polarization. In some cases the two effects nearly compensate each other.

All the discussed corrections, except the quantum one, proved to have almost no effect on the positions of the carbon and oxygen ignition curves. In the WDs, the heat produced by the nuclear reactions is evacuated by neutrino emission. In this case, the position of the ignition curves may be even stronger affected by the current uncertainties in the neutrino reaction rates in dense plasma environment than by the departures from the linear-mixing approximation or by electron-polarization corrections. In the NSs, the heat is not only taken away by neutrinos, but also effectively sinks through the envelope. In the latter case, the one-zone approximation of the heat diffusion is often applied to the analysis of stability of the nuclear fusion (e.g., Refs. [28, 29]). We

Fig. 3 (a) Ratios of the deuterium burning rates with account of electron screening to the rates in the rigid-background model at the same temperatures and densities as in Fig. 2. The inset shows the absolute reaction rates per one nucleus with account of electron screening (solid line for age 5 Gyr, dashed line for 100 Myr) and for the rigid background (dotted lines). (b) The same for lithium burning.

have found [27] that it is more important to go beyond the one-zone approximation than to introduce all other corrections mentioned above. In magnetars (NSs with superstrong magnetic fields of $10^{14} - 10^{15}$ G) an account of the magnetic modification of the heat transport coefficients is equally significant.

3.2   Low-mass objects

As another astrophysical example, let us consider the electron-screening effect on nuclear fusion in low-mass stars (LMSs) and substellar objects (SSOs; see Ref. [30] for a review). Important indicators of the ages and masses of these objects are the so called lithium and deuterium tests, which are based on depletion of lithium and deuterium by nuclear burning.

Figure 1 displays the enhancement exponent h, normalized with respect to $h_{\rm SVH}$, for the reaction $p + d \rightarrow$ ^{3}He$+\gamma$, in different approximations. The accurate result is compared to the result of application of the linear-mixing approximation in the cases of polarizable electron background according to Eq. (8) and rigid background according to Eq. (9), with and without the quantum corrections.

We note that the simple Salpeter – Van Horn approximation (13) performs surprisingly well: its accuracy in the LMS-SSO conditions, as we see in Fig. 1, is within a few tens percent. We recall that in WD-NS conditions its accuracy is still better, typically a few percent [27]. The quantum effects in Fig. 1 are significant only at $\rho \gg 10^3$ g cm^{-3}. From the lower panel of Fig. 2 we see that such densities are not reached in the LMS-SSO conditions, therefore the quantum effects do not play role in theoretical models of nuclear fusion in LMSs and SSOs (unlike the NS-WD case [27]).

Figure 2 shows the dependences of LMS-SSO central densities and temperatures at two characteristic ages of these objects (the two lower panels, from Ref. [30]), together with the respective normalized enhancement exponents $h/h_{\rm SVH}$ with and without the rigid-background approximation. As previously, we see that the corrections beyond the Salpeter – Van Horn approximation are of the same magnitude for the rigid and polarizable electron backgrounds (to $\sim 30\%$ in this case), but of different sign.

This difference can translate into factors of a few for the reaction rate $R_{12} \propto {\rm e}^h$ in the objects of very small mass, because they are relatively cool and therefore have a large factor $h = H_{12}(0)/T$ in their central parts. Figure 3a demonstrates this for the SSO deuterium fusion. For masses $M \sim 10^{-2} M_\odot$, the minimum mass for D-fusion [30], where $M_\odot$ is the solar mass, the electron polarization effect changes R_{12} by a factor of $1.5 - 2$. This change does not significantly affect the deuterium depletion curves and therefore is unimportant for the

mass-age deuterium test. As seen from the inset in Fig. 3, the corresponding corrections are of the order of a few$\times 10^{-4} M_\odot$, which is astrophysically negligible.

Figure 3b shows the rates of the reaction $^7\mathrm{Li} + p \rightarrow 2\,^4\mathrm{He}$. This case is similar to the previous one, with the difference that the latter reaction takes place for more massive objects. At very small masses the difference in the reaction rates calculated with and without the allowance for the electron polarization is huge, but astrophysically unimportant, because these rates are so low that one may neglect this reaction altogether. At contrast, for $M \approx 0.07\,M_\odot$ this reaction is crucially important for stellar diagnostics, but in this case the polarization correction is smaller, and it translates into a negligible correction for the mass-age relation.

4   Remarks on some controversial approaches

4.1   Yukawa potential

For an arbitrary degree of degeneracy (but at not too strong Coulomb coupling; see [23]), the screened interaction between ions is approximately described by the Yukawa potential $(Z_1 Z_2 e^2 / r) \exp(-r/D)$, where D is given in Eq. (11). Pollock and Militzer [25] studied the contact probabilities of Yukawa systems with the intention to simulate the electron-screening effect (see also Ref. [16]). Based on these simulations, they arrived at the conclusion that electron screening "reduces the enhancement effect," in obvious contradiction with our findings above and with the earlier results [3, 8, 9, 14, 21].

We note, however, that the Yukawa model corresponds to the Thomas-Fermi limit, $\epsilon(k) \sim 1 + (k_{\mathrm{TF}}/k)^2$, for the static dielectric function $\epsilon(k)$, which is only justified at $k \ll k_{\mathrm{TF}}$ (see, e.g., [31]). Therefore, this model is inappropriate at short distances (i.e., large wavenumbers k). In particular, it is not applicable for the evaluation of the screening potential at zero separation, $H_{12}(0)$. Therefore, a Yukawa system cannot correctly reproduce the effect of electron polarization on the nuclear fusion rates. This fact was recognized by Ichimaru [4], who mentioned two opposite effects of electron screening: first, the binary repulsive potentials between reacting nuclei are reduced by electrons ("short-range effect"), which increases $H_{12}(0)$; second, the reduction of particle interactions by the screening affects the many-body correlation function in such a way that it decreases $H_{12}(0)$ ("long-range effect"). In real electron-ion plasmas (without the Yukawa approximation) the first effect overpowers the second one. The Yukawa model grasps the second effect, but misses the first, dominant one.

4.2   "Quantum tail" in energy distribution

Starostin and coworkers [32–36] noted that in dense plasmas, in addition to the potential lowering that results in the enhancement factor discussed above, there is another effect capable to modify the reaction rates. Because a state with definite momentum has a finite lifetime, its momentum distribution (or, equivalently, the distribution of kinetic energies) is broadened due to Heisenberg uncertainty principle. Therefore the probability to find a pair of nuclei in a state with a high center-of-mass energy is larger than predicted by the Boltzmann statistics. These use in Eq. (3), instead of $w_\mathrm{B}(E)$, a modified distribution function
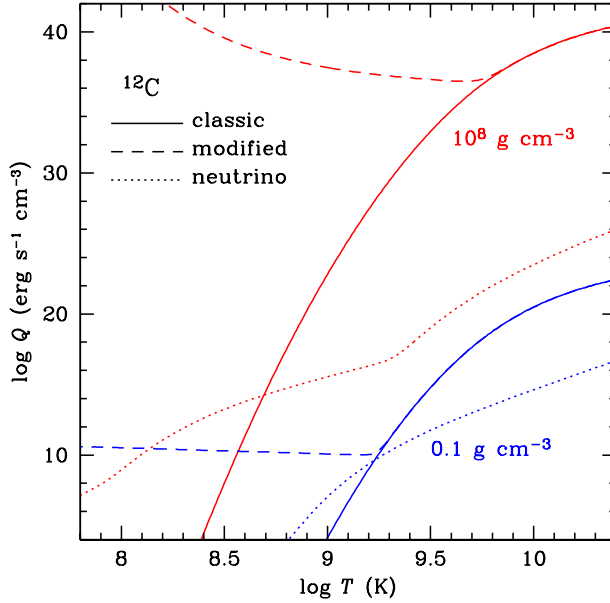
$$w_\mathrm{mod}(E_p) = \int_0^\infty w_\mathrm{B}(E)\,\phi(E, E_p)\,\mathrm{d}E, \tag{15}$$

where $\phi(E)$ describes the quantum broadening of the kinetic energy, defined through the particles' momenta p. The authors suggest to use for this broadening the spectral function [37, 38]

$$\phi(E, E_p) = (\gamma/\pi) \left/ \left[(E - E_p - \Delta)^2 + \gamma^2 \right] \right., \tag{16}$$

where $\gamma = \gamma(p) = \hbar \nu_\mathrm{coll}$ is a collisional width, ν_coll is an effective collision frequency, and Δ is the collisional energy shift, which is inessential for our discussion and will be suppressed. The use of the Lorentz profile (16) in Eq. (15) implies the necessary condition $\gamma \ll T$, which is satisfied in all examples discussed hereafter. For Coulomb scattering of a light particle with charge $Z_1 e$, momentum p and velocity v from ambient heavy particles with charges $Z_2 e$, the collision frequency is $\nu_\mathrm{coll} = n_2 \langle \sigma_\mathrm{coll} v \rangle = \left[4\pi n_2 (Z_1 Z_2 e^2)^2 / p^2 v \right] \Lambda(p)$, where σ_coll is an effective collisional cross section and $\Lambda(p)$ is a Coulomb logarithm. For estimates we will use this formula for any particle masses and adopt the nonmagnetic nonrelativistic limit of the transport Coulomb logarithm from

Fig. 4 Power of the carbon fusion reaction as function of temperature T at constant density ρ according to the conventional calculation (solid lines) and with statistical averaging over momenta including "quantum tails" in their distribution according to Eq. (15) (dashed lines). For comparison, neutrino emission power is shown by dotted lines. The upper line of each type corresponds to $\rho = 10^8$ g cm^{-3} and the lower line to $\rho = 0.1$ g cm^{-3}.

Ref. [39]: $\Lambda(p) = 0.5 \ln(1+u) - 0.5 u/(1+u)$, where $u = \left[(D_{\mathrm{ion}}^2 + (2a_i/3)^2)^{-1} + k_{\mathrm{TF}}^2 \right] (\hbar/2p)^2$ (numerically, $\Lambda \sim 1-10$ in the examples below).

In the limit $\gamma \to 0$, the function ϕ turns into a Dirac delta function, and the Boltzmann distribution is recovered. However, for finite γ and for energies much higher than the thermal energy, so that

$$E_p/T \gg \ln(\pi E_p^2/\gamma T), \tag{17}$$

the exponential decay of $w_{\mathrm{B}}(E)$ is overpowered by so called "quantum tail" $w_{\mathrm{qt}}(E_p) \to \gamma T/\pi E_p^2$. For nonrelativistic Coulomb particles, $\gamma(p) \propto E_p^{-3/2}$, therefore the tail decays as $w_{\mathrm{qt}} \propto E_p^{-7/2}$.[1]

This approach was criticized by Bahcall et al. [5] and by Zubarev [41]. Bahcall et al. merely state that the term $d^3 p_1 d^3 p_2$ in the quantum-mechanical expression $R \propto \int \int d^3 p_1 d^3 p_2 \exp(-E/T) |\langle f|H|i \rangle|^2$ for the reaction rate represents the density of states and should not be confused with the expectation values of particle momenta, distributed according to $w_{\mathrm{mod}}(E_p)$. While the statement is certainly true in general, the argument misses the point because it says nothing about $\langle f|H|i \rangle$, which needs not be just a first-order perturbation matrix element. Indeed, the kinetic Green function derivation of the distribution (15) [34, 38] implies that multiple scattering is taken into account in addition to the fusion matrix element *per se*. Thus one may say that $\phi(E)$ is effectively contained in $|\langle f|H|i \rangle|^2$ in the above expression for R.

Zubarev [41] put forward another argument. He noted that the pp-fusion rates calculated with $w_{\mathrm{B}}(E)$ and with $w_{\mathrm{mod}}(E)$ differ by three orders of magnitude for the Sun, and concluded that in the second case "one has neglected the coupling between the various probability amplitudes of velocity which is introduced by the quantum uncertainty." These statements are refuted in [36], but if there were indeed such a large difference, the second (nonstandard) method of averaging must have a flaw. Indeed, the agreement between the current stellar evolution theory and observations does not leave room for such a huge change of the basic reaction rates.

Let us consider the fusion reaction ^{12}C+^{12}C $\to$ ^{24}Mg, which plays a crucial role in the theory of WDs, red giants, and accreting NSs. We calculate the astrophysical factor $S(E)$ for this reaction using the most recent effective potential model [42] derived from laboratory results. In Fig. 4 we show thermonuclear heat rates Q_{nuc} per unit volume as functions of T at densities $\rho = 10^8$ g cm^{-3} and 0.1 g cm^{-3}, calculated with using the classical statistical weight $w_{\mathrm{B}}(E)$ and the modified weight $w_{\mathrm{mod}}(E)$. The conventional results show steep decrease with

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[1] The authors [36] obtain $w_{\mathrm{qt}} \propto E_p^{-4}$, because they replace $v = \sqrt{2E_p/m_{12}}$ by $\sqrt{2E/m_1}$ in $\langle \sigma_{\mathrm{coll}} v \rangle$, which introduces an error in Eq. (15) at $E_p \gg T$. However, this difference in the power index does not qualitatively change any results or conclusions. We note in passing that the parallel to Kimball's power law p^{-8} [40], drawn in [33–36], appears ungrounded, because the Kimball's result is specific to the distribution of fast particles scattered by a potential with asymptotic behavior $\sim r^{-1}$ at $r \to 0$, whereas the collisional broadening is present for any scattering potential [37].

decreasing temperature below $T \lesssim 10^9$ K. At contrast, the modified calculation gives a minimum at $T \gtrsim 10^9$ K and then an increase of the rate with decreasing T. The origin of this behavior is mathematically obvious. At low T, the denominator in Eq. (3) remains determined by the Boltzmann part of $w(E)$ and provides the normalization $\propto \sqrt{T}$, while the largest contribution to the numerator comes from the integration of $Se^{-2\pi\eta}$ with the weight $w \approx w_{qt}$ over the energies above the Coulomb barrier, which is temperature-independent. Thus the ratio becomes $\propto T^{-1/2}$. The latter dependence is well discerned in Fig. 4 for $T \lesssim 10^9$ K and $\rho = 0.1$ g cm^{-3}. At $\rho = 10^8$ g cm^{-3}, there is a stronger increase of Q_{nuc} with decreasing T, caused by the increase of the factor e^h.

From the physics point of view, this enhancement of nuclear power at low T is unacceptable. For example, it is incompatible with the existence of carbon WDs. To show this, we have additionally plotted in Fig. 4 neutrino emission rates Q_ν, calculated following Ref. [43] as the sum of the power carried away by neutrino emission due to annihilation of electron-positron pairs, plasmon decay, and bremsstrahlung. The intersection $Q_{\mathrm{nuc}} = Q_\nu$ is the *ignition point*, beyond which nuclear burning becomes unstable. We see that with the modified statistical averaging the intersection is absent, i.e., the burning is always unstable. If it were true, all carbon WDs should have exploded, but they do exist. The modified calculation also predicts cold fusion at the normal conditions, which does not happen.

What is the basic flaw of the "quantum-tail" calculation? As can be seen, for example, from Ref. [34], Eq. (15) is related to a perturbation correction of the order $\hbar^2$ to the Maxwellian distribution. Different forms of this correction can be equivalent to the same order. For example, the original Wigner expansion of his probability function in powers of $\hbar^2$ [44] can be rewritten in several ways: $\exp(-E_p/T)\left[1 + \hbar^2 g_2 E_p/T^4 + \ldots\right] \sim \exp[-E_p/T + \hbar^2 g_2 E_p/T^4 + \ldots] \sim \exp[-E_p/(T + \hbar^2 g_2/T^3 + \ldots)]$ (cf. [45], § 33). Here, g_2 is a coefficient involving average products of derivatives of the interaction potential. These different forms, however, are not equivalent if $\hbar^2 g_2 E_p/T^4$ is large, which indicates that applicability of this correction is restricted to relatively low energies or high temperatures. However, the "quantum-tail" contribution to the numerator of Eq. (3) at low T comes mainly from high energies, whose difference from T exceeds the collisional width γ by many orders of magnitude. Therefore we think that such evaluation of Eq. (3) falls beyond the applicability range of the distribution (15).

The quantum uncertainty of particles' momenta is conjugate to the quantum uncertainty of their coordinates. The thermonuclear fusion enhancement and the cold (pycnonuclear) fusion due to these quantum uncertainties are well known (e.g., [21]). However, these effects are important only at very high densities (e.g., at $\rho \gg 10^9$ g cm^{-3} for the carbon fusion [10]).

5 Conclusions

We have studied the effects of electron screening on thermonuclear reactions in dense plasmas and compared different approximations to determine plasma enhancement factors for the nuclear fusion rates. The electron screening always increases the enhancement effect. The opposite conclusion may come from using the Yukawa potential model, which is inappropriate to calculate the contact probability for fusing nuclei. The method of taking quantum uncertainties in particles' momenta by a convolution of Boltzmann and Lorentz distributions, suggested in some publications, leads to physically unreasonable results. We argue that it may be a consequence of violation of applicability conditions of underlying theory.

Although the electron polarization correction can increase a fusion rate by orders of magnitude, we find that it does not significantly affect theoretical models of WDs, NSs, LMSs, and SSOs.

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