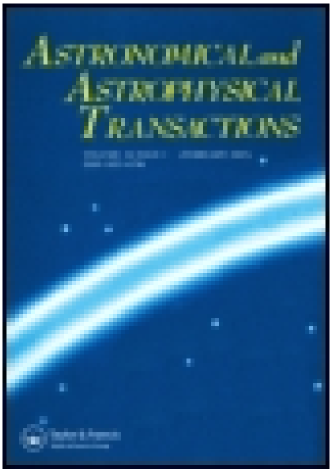

Astronomical & Astrophysical Transactions: The Journal of the Eurasian Astronomical Society

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gaat20

Photoionization of the hydrogen atom in a strong magnetic field

G. G. Pavlov [a] & A. Yu. Potekhin [a]

[a] A. F. Ioffe Institute of Physics and Technology , 194021, St. Petersburg, Russian Federation

Published online: 17 Aug 2006.

To cite this article: G. G. Pavlov & A. Yu. Potekhin (1994) Photoionization of the hydrogen atom in a strong magnetic field, Astronomical & Astrophysical Transactions: The Journal of the Eurasian Astronomical Society, 4:4, 261-267, DOI: 10.1080/10556799408205382

To link to this article: http://dx.doi.org/10.1080/10556799408205382

Astronomical and Astrophysical Transactions, 1994,
Vol. 4, pp. 261–267
Reprints available directly from the publisher.
Photocopying permitted by license only

PHOTOIONIZATION OF THE HYDROGEN ATOM IN A STRONG MAGNETIC FIELD

G. G. PAVLOV and A. YU. POTEKHIN

A. F. Ioffe Institute of Physics and Technology, 194021, St. Petersburg, Russian Federation

(Received 17 July, 1993)

Photoionization of the hydrogen atom in a strong magnetic field ($B \sim 10^{11} - 10^{13}$ G) is studied analytically and numerically for the ground and various excited atomic states and different polarizations of incident radiation.

KEY WORDS Neutron stars, atomic pocesses; magnetic fields

1. INTRODUCTION

An examination of neutron-star thermal spectra is an efficient tool for selecting neutron star models. Photoionization of atoms and ions is one of the most important radiative processes in the neutron star atmospheres which affects the spectra. Owing to the gravitational separation, the lightest elements present (particularly, hydrogen) can give the main contribution to the opacity of the surface layers. Since the electron cyclotron energy $\hbar\omega_B = \hbar B/m_e c$ is typically much larger than the Coulomb energy in the atmospheres of neutron stars (i.e., $\beta = \hbar\omega_B/4\,\mathrm{Ry} = B/4.7 \cdot 10^9\,G \gg 1$), the magnetic effects change drastically the atomic structure and cross sections of radiative processes (see, e.g., Garstang 1977).

Photoionization of hydrogenic atoms in strong magnetic fields was considered in a number of papers. Hasegawa and Howard (1961) and Gnedin *et al.* (1974) undertook approximate analytical studies of the photoionization cross section at $\ln\beta \gg 1$. Hasegawa and Howard considered the ground-state atom photoionization in the case when an incident photon is circularly polarized and propagates along the magnetic field. Gnedin *et al.* evaluated the cross sections in the Born approximation justified when photon energy is much higher than the electron binding energy. Schmitt *et al.* (1981) and Wunner *et al.* (1983) performed numerical calculations of the ground-state photoionization cross section and obtained asymptotics at high photon energies $\hbar\omega$. The asymptotics differ from those of Hasegawa and Howard (1961) and Gnedin *et al.* (1974). Moreover, according to Schmitt *et al.* (1981) and Wunner *et al.* (1983) the cross section for photons whose electric vector is perpendicular to the field becomes zero in the most important frequency range $\omega < \omega_B$, in disagreement with other authors. Note that a zero cross section would lead to an infinite mean free path of a photon propagating along the magnetic field. The non-zero cross sections were obtained recently by Miller and Neuhauser (1991). The latter authors, however, did not present frequency and polarization dependences of the cross sections. Thus, a more thorough analysis is required for an adequate study of radiative transfer in the atmospheres of strongly magnetized neutron stars.

In the present article, we study analytically and numerically the photoionization of the ground state and various excited states of atomic hydrogen in strong magnetic fields. The cross sections are computed and analysed in a wide frequency range and for different photon polarizations.

2 BASIC EQUATIONS AND ANALYTICAL RESULTS

Let us consider a hydrogen atom in a magnetic field **B** directed along the z-axis. So far the atomic structure has been thoroughly investigated in the literature for the case when the atom does not move across **B**. In this case the wave function of the electron and proton relative motion obeys the Schrödinger equation with the Hamiltonian (Gor'kov and Dzyaloshinskii 1967)

$$H = \frac{m_p + m_e}{2m_e m_p} \cdot \boldsymbol{\pi}^2 + \frac{e^2}{2(m_e + m_p)c^2} (\mathbf{B} \times \mathbf{r})^2 - \frac{e^2}{r}, \tag{1}$$

where

$$\boldsymbol{\pi} = -i\hbar\nabla_{\mathbf{r}} + \frac{e}{2c}\frac{m_p - m_e}{m_p + m_e}\mathbf{B} \times \mathbf{r} \tag{2}$$

is the kinetic momentum operator, and m_e (m_p) is the electron (proton) mass.

Let us expand the eigenfunctions $\psi(\mathbf{r}) = \psi(\rho, \phi, z)$ of the Hamiltonian (1) in terms of the Landau states $\Phi_{Ns}(\rho, \phi)$ which are the transverse parts of the electron coordinate wave functions in the absence of the Coulombic field. They can be expressed in terms of Laguerre functions (e.g., Kaminker and Yakovlev, 1981). Here N numerates the Landau levels, $s = -N, -N+1, \ldots$ is negative of the conserving z-projection of the relative angular momentum; $a_L = (\hbar c/eB)^{1/2} = a_B(2\beta)^{-1/2}$ is the magnetic length, $\beta = \hbar\omega_B/4\,\mathrm{Ry} = \hbar^3 B/2m_e^2c^2e^3$, and a_B is the Bohr radius. For sufficiently large β, the *adiabatic approximation* is valid, and one term in this expansion significantly exceeds the others (see, *e.g.*, Simola and Virtamo 1978),

$$\psi(\mathbf{r}) = \psi_{Ns}^{ad}(\mathbf{r}) + \delta\psi_{Ns}(\mathbf{r}) \approx \psi_{Ns}^{ad}(\mathbf{r}), \tag{3}$$

$$\psi_{Ns}^{ad}(\mathbf{r}) = \Phi_{Ns}(\rho, \phi) g_{Ns}(z); \tag{4}$$

$$\delta\psi_{Ns}(\mathbf{r}) = \sum_{N'} \Phi_{N's}(\rho, \phi) h_{N'}^{Ns}(z). \tag{5}$$

The longitudinal wave function $g_{Ns}(z)$ satisfies one-dimensional Schrödinger equation with the potential $V_{Ns}(z) = \langle\Phi_{Ns}| - e^2/r\, |\Phi_{Ns}\rangle$ and (longitudinal) energy $\varepsilon = E - (N + sm_e/m_p + 1/2 + m_\sigma)\hbar\omega_B$, where E is the total energy, and $m_\sigma = \pm 1/2$ is the electron spin projection onto the z-axis. We take into account the $m_\sigma = -1/2$ states only because spin-flip transitions give small contribution to the photoionization cross sections at the field strengths of study (Wunner *et al.*, 1983).

Negative eigenvalues of the longitudinal energy form a discrete spectrum, $\varepsilon = -\varepsilon_{Nsn}$, where $n = 0, 1, 2, \ldots$ numerates the nodes of an eigenfunction $g_{Nsn}(z)$ with parity $(-1)^n$. At $\beta \to \infty$ the energies of the tightly bound states with $n = 0$

grow logarithmically, $\varepsilon_{Ns0} \sim (\ln \beta)^2$ Ry, whereas the energies of the hydrogen-like states with $n \geq 1$ group about the levels of the field-free hydrogen atom.

The photoionization cross section for the transition from a bound state $(0, s, n)$ to a state $(N_f, s_f, \varepsilon_f)$ of continuum can be written as

$$\sigma^{\mu}_{sn\to N_f s_f} = \pi\alpha\, |D_{-\mu}|^2 \frac{\hbar\omega}{\sqrt{\varepsilon_f\, \mathrm{Ry}}} \frac{L}{a_B}, \tag{6}$$

where $\mu = 0, \pm 1$ labels circular polarization modes ($e_0 = e_z$, $e_{\pm 1} = (e_x \pm ie_y)/\sqrt{2}$), $\mathbf{D}$ is a matrix element, $\alpha = e^2/\hbar c$, L is the z-extension of the periodicity volume of the final state, and

$$\varepsilon_f = \hbar\omega - [N_f + (s_f + N_f - s)m_e/m_p]\hbar\omega_B \tag{7}$$

is the photoelectron longitudinal energy. The term $(s_f - s)m_e/m_p$ at $N_f = 0$ is by no means negligible, as it may be comparable with ε_{0sn} (Herold *et al.* 1981).

Using the commutation relations of $\boldsymbol{\pi}$ and $\mathbf{r}$ with the Hamiltonian (1) and neglecting the corrections $\sim m_e/m_p$, the matrix element D_μ in (6) can be written either in the 'velocity form'

$$\mathbf{D}^{(\pi)} = (im_e\omega)^{-1}\langle \psi_f |\, \exp(i\mathbf{qr})\boldsymbol{\pi}\, | \psi \rangle, \tag{8}$$

or in the 'length form'

$$\mathbf{D}^{(r)} = \left\langle \psi_f \left| \mathbf{r}\exp(i\mathbf{qr})\left(1 - \frac{\hbar q^2}{2m_e\omega} - \frac{\mathbf{q}\cdot\boldsymbol{\pi}}{m_e\omega}\right) \right| \psi \right\rangle, \tag{9}$$

where $\mathbf{q}$ is the wave vector. Both forms would be equivalent if ψ_f and ψ were exact wave functions. However, under the adiabatic approximation (3), these forms may lead to qualitatively different results. For instance, Schmitt *et al.* (1981), Wunner *et al.* (1983) and Mega *et al.* (1984) used the velocity form and obtained zero cross section for transverse polarizations $\mu = \pm 1$ at $\omega < \omega_B$. On the other hand, Hasegawa and Howard (1961), Gnedin *et al.* (1974), and Miller and Neuhauser (1991) used the length form and obtained finite cross sections.

Let us suppose that at some ω the adiabatic approximation for the wave functions (3) holds also for the longitudinal matrix elements, *i.e.*,

$$|\langle g_{N's'} \,|\, h^{N_s}_{N'\nu} \rangle| \ll |\langle g_{N's'} \,|\, g_{N_s} \rangle|, \tag{10}$$

and find the first correction to the adiabatic $D^{(\pi)}_{\pm 1}$. We obtain that the adiabatic length form is justified not too far from the Landau thresholds of the photoionization cross section, including the most important region $\omega \ll \omega_B$, which corresponds to $N_f = 0$. In this region it coincides with the first *non-adiabatic correction* to the velocity form, whereas at $N_f > 0$ both forms coincide approximately in the adiabatic approximation if

$$|\omega - N_f\omega_B| \ll \omega_B. \tag{11}$$

On the contrary, the two forms differ essentially outside the frequency range (11). It is easy to show that in this case the assumption (10) cannot be satisfied, which indicates the illegacy of application of the adiabatic approximation to calculating the matrix elements, in both forms. However, when this happens, the cross sections themselves are very small.

Thus we choose the length form (9) of $\mathbf{D}$. Making use of the well-known relations of Laguerre functions (*e.g.*, Kaminker and Yakovlev 1981), the transverse integrations (over ρ and ϕ) in (9) can be performed analytically in the adiabatic approximation. Thus the problem is reduced to the calculation of the longitudinal matrix elements. Numerical results are presented in the next section. An analytical evaluation is possible in the limit of $\omega \to \infty$. Using the method of Hasegawa and Howard (1961), we obtain: $\sigma^{\pm 1} \sim \omega^{-3/2}$. On the other hand, the method of Schmitt *et al.* (1981) leads to $\sigma^{\pm 1} \sim \omega^{-2s-7/2}$. In both cases $\sigma^0 \sim \sigma^1/\omega$. An estimation of the neglected terms shows that, in fact, the asymptotics are valid in different frequency ranges. The asymptotic behaviour $\omega^{-3/2}$ takes place in the range $1 \ll \hbar\omega/\mathrm{Ry} \ll \sqrt{\beta}$, which exists at extremely high β, whereas $\omega^{-2s-7/2}$ is valid at $\hbar\omega \gg 4\beta\,\mathrm{Ry} = \hbar\omega_B$.

3. NUMERICAL RESULTS

In our numerical calculations, the wave functions of discrete spectrum and continuum have been computed by the methods similar to those used by Simola and Virtamo (1978) and Schmitt *et al.* (1981). The photoionization cross sections have been determined in the adiabatic approximation with the aid of Eq. (9). Figure 1 shows partial cross sections $\sigma_{0sn \to 0,s_f}$ as functions of photoelectron

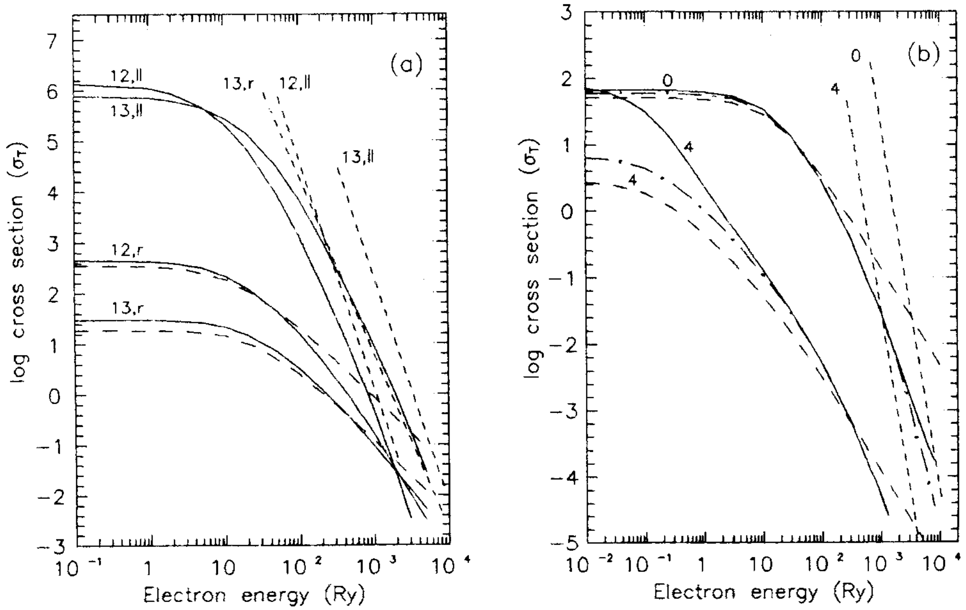

Figure 1 Partial cross section $\sigma^{\mu}_{s \to s+\mu}$ vs. photoelectron longitudinal energy for different magnetic fields, photon incident angles and polarizations. Solid, long-dash, and short-dash curves show numerical results, analytical approximation of Hasegawa and Howard (1961), and high-energy asymptotics of Schmitt *et al.* (1981), respectively. (a) Transitions from the ground state ($s = n = 0$). Curves are labelled by log B (Gauss). $\|$ and r denote $\mu = 0$, $\theta = 90°$, and $\mu = +1$, $\theta = 0°$, respectively. (b) Transitions from excited states ($s = 1$, $n = 0$ and 4 (figures near the curves)) for $B = 10^{12.5}$ G, $\theta = 0°$, $\mu = +1$. Dash-and-dotted curves present numerical results in the dipole, infinite-proton-mass approximation.

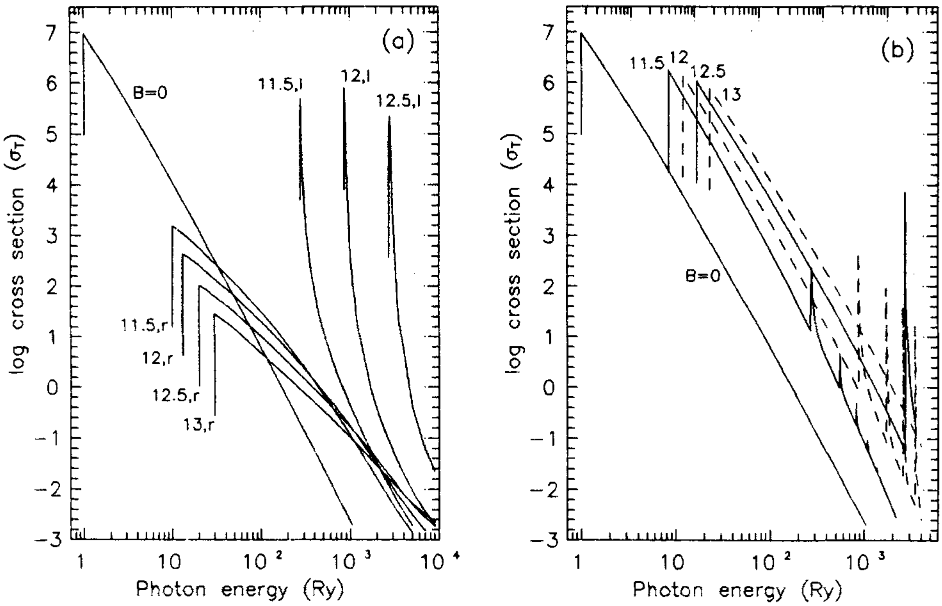

Figure 2 Total photoionization cross section as a function of photon energy (in Ry) for the ground state ($s = n = 0$) atom at various $\log B$ (figures near the curves). (a) $\theta = 0°$; right (r) and left (l) circular polarizations. (b) $\theta = 90°$, longitudinal polarization.

longitudinal energy ε_f at several values of B for photons with right circular polarization at $\theta = 0°$, and linear polarization parallel to **B** at $\theta = 90°$. A comparison of the numerical cross sections with analytical ones shows that, in accordance with the above analysis, the analytical dependence obtained by the method of Hasegawa and Howard (1961) is accurate at low and intermediate $\hbar\omega$, whereas the asymptotics of Schmitt *et al.* (1981) is valid at $\omega \gg \omega_B$. Figure 1b demonstrates also the effect of changing the relation between ω and ε_f due to the finite proton mass (see Eq. (7)). The effect is especially significant for hydrogen-like initial levels.

Figure 2 shows frequency dependences of the total photoionization cross sections of the ground-state atom for a polarized light propagating at $\theta = 0°$ or $\theta = 90°$ and for the magnetic field varying from $10^{11.5}$ to 10^{13} G. For comparison, the field-free cross sections are also presented. The peaks in Figure 2b are associated with the transitions to the excited Landau states.

Figure 3 demonstrates photoionization cross section of an excited atom by photons propagating across **B**. The 'jumps' at the ionization thresholds correspond to the transitions to the states with $s_f = s - 1$, which may occur for $s > 0$. According to Eq. (7) their thresholds are redshifted with respect to the thresholds of the transitions to $s_f = s$ and $s_f = s + 1$ (which give the main maxima in the cross sections for the polarizations parallel and perpendicular to **B**, respectively).

If an atom in an odd state is ionized by a photon whose electric field is polarized parallel to **B**, then the cross section displays sharp absorption-like resonances (see Figure 3b). These resonances, previously mentioned by Mega *et al.* (1984), occur at photon energies where the dipole matrix element becomes

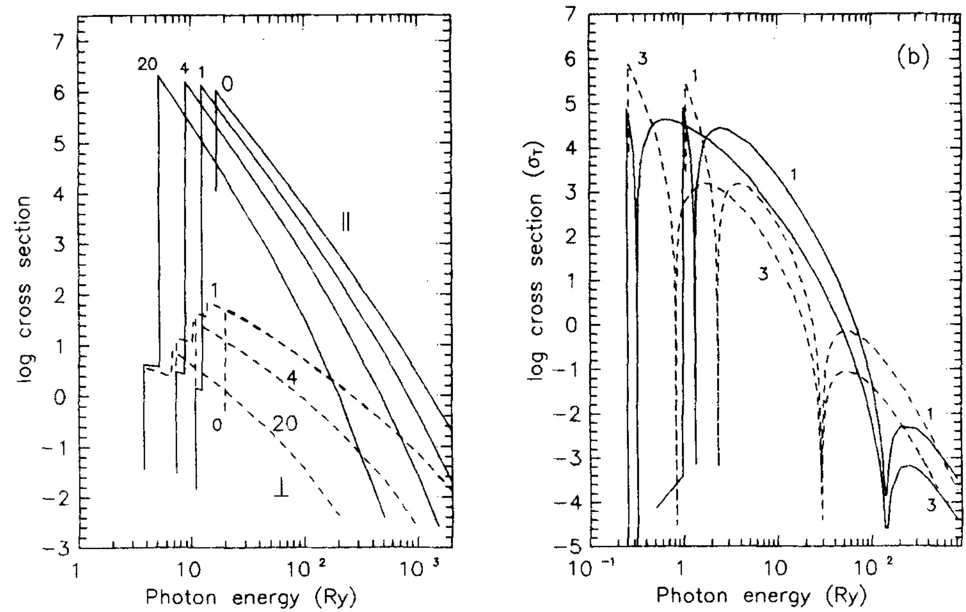

Figure 3 Spectra of total cross sections for excited states at $B = 10^{12.5}$ G, $\theta = 90°$. (a) Tightly bound initial states $n = 0$, $s = 0$, 1, 4, 20 (figures near the curves), longitudinal and transverse photon polarizations (solid and dashed lines, respectively). (b) Hydrogen-like states $n = 1$, 3 (figures near the curves), longitudinal polarization. Solid lines correspond to $s = 1$, and dashed lines, to $s = 4$.

zero. Their finite depth shown in Figure 3b comes from the contribution of higher multipoles.

4. CONCLUSIONS

A study of the photoionization cross sections of the hydrogen atom in strong magnetic fields has allowed us to resolve some contradictions between earlier results of other authors. It is shown that the discrepancy between high-energy asymptotics of partial cross sections of Hasegawa *et al.* (1961) and Gnedin *et al.* (1974) and of Schmitt *et al.* (1981) and Wunner *et al.* (1983) comes from assuming different relations between ω and ω_B in the asymptotic region: the former authors assumed Ry $\ll \hbar\omega \ll \hbar\omega_B$, while the latter, Ry $\ll \hbar\omega_B \ll \hbar\omega$. The latter authors obtained zero cross sections for the transverse and circular polarizations at $\omega < \omega_B$ by using the 'velocity form' of matrix elements. This is a poor choice at low energies in the adiabatic approximation. We have shown that in this case the 'length form' is more appropriate.

The photoionization cross sections have been computed for atoms in various states and for different photon polarizations and propagation directions. The field range $B = 10^{11.5} - 10^{13}$ G has been considered. The calculation has been performed without using the dipole approximation. This leads to the violation of some dipole-transition selection rules and to the appearance of new resonant peaks in the frequency dependences of the cross sections. The finite proton mass, which has also been taken into account, shifts the ionization thresholds of the partial cross sections. The shifts produce characteristic 'jumps' at the thresholds of the total cross sections.

The present study is planned to be a part of the work aimed at obtaining the opacities in atmospheres of strongly magnetized neutron stars under realistic conditions.

Acknowledgements

A. Yu. P. is grateful to the American Astronomical Society for the financial support of the work.

References

Garstang, R. H. (1977). *Rep. Progr. Phys.* **40,** 105.
Gnedin, Y. N. and Pavlov, G. G. (1974). *Sov. Phys. JETP* **38,** 903.
Gnedin, Y. N., Pavlov, G. G. and Tsygan, A. I. (1974). *Zh. Eksp. Teor. Fiz.* (*USSR*) **66,** 421 (*Sov. Phys. JETP* **39,** 301).
Gor'kov, L. P. and Dzyaloshinskii, I. E. (1967). *Zh. Eksp. Teor. Fiz.* (*USSR*) (*Sov. Phys. JETP*) **53,** 717.
Hasegawa, H. and Howard, R. E. (1961). *J. Phys. Chem. Solids* **21,** 179.
Herold, H., Ruder, H. and Wunner, G. (1981). *J. Phys. B: At. Mol. Phys.* **14,** 751.
Kaminker, A. D. and Yakovlev, D. G. (1981). *Teor. Matem. Fiz.* (*USSR*) **49,** 248.
Mega, C., Herold, H., Rosner, W., Ruder, H. and Wunner, G. (1984). *Phys. Rev.* A **30,** 1507.
Miller, M. C. and Neuhauser, D. (1991). *Mon. Not. Roy. Astron. Soc.* **253,** 107.
Rösner, W., Herold, H., Ruder, H. and Wunner, G. (1983). *Phys. Rev.* **A28,** 2071.
Rösner, W., Wunner, G., Herold, H. and Ruder, H. (1984). *J. Phys. B: At. Mol. Phys.* **17,** 29.
Schmitt, W., Herold, H., Ruder, H. and Wunner, G. (1981). *Astron. Astrophys.* **94,** 194.
Simola, J. and Virtamo, J. (1978). *J. Phys. B: At. Mol. Phys.* **11,** 3309.
Wunner, G., Ruder, H., Herold, H. and Schmitt, W. (1983). *Astron. Astrophys.* **117,** 156.