EQUATION OF STATE AND OPACITIES FOR HYDROGEN ATMOSPHERES OF STRONGLY MAGNETIZED COOLING NEUTRON STARS

Alexander Y. POTEKHIN, Yuri A. SHIBANOV
Department of Theoretical Astrophysics, Ioffe Physical-Technical Institute, St.-Petersburg 194021, RUSSIA, palex@astro.ioffe.rssi.ru
Joseph VENTURA
Physics Department, University of Crete, and Institute for Electronic Structure and Laser, FORTH, 71003 Heraklion, Crete, GREECE

1. Introduction

Thermal radiation from surfaces of several radio pulsars has been detected recently in soft X-rays by *ROSAT* and *ASCA* (see Becker and Trümper, 1998). A number of the point-like X-ray sources has also been discovered and identified as radio-silent isolated cooling neutron stars (NSs) (see Caraveo et al., 1996; Walter et al., 1996; Vasisht et al., 1997). In several cases, the observations seem to be better explained if the NSs possess an envelope of matter of low atomic weight, presumably hydrogen (Page, 1997), and if NS atmosphere models are applied to fit the data (Pavlov and Zavlin, 1998). There are indications that this may be the case for NSs with different magnetic fields B: from weak, $B \ll 10^{10}$ G (Rajagopal and Romani, 1996; Zavlin and Pavlov, 1998; Zavlin et al., 1998), to strong, $B \sim 10^{11-13}$ G (Page et al., 1996; Zavlin et al., 1998), and superstrong, $B > 10^{14}$ G (Heyl and Hernquist, 1997). To justfy this, one needs in further observations and advanced atmosphere models of NSs with high B. Thermal motion of atoms accross the magnetic field induces an electric field in the frame of the atom. This affects atomic structure, atmospheric thermodynamics and opacities (Ventura et al., 1992; Pavlov and Mészáros, 1993), being a critical point in the construction of models and data interpretation. Here we study this problem for hydrogen plasma at temperatures $T \sim 10^{5.5} - 10^{6}$ K, densities $\rho \sim 10^{-3} - 10^{1}$ g cm^{-3}, and magnetic fields $B \sim 10^{12} - 10^{13}$ G, typical for atmospheres of middle-aged cooling NSs. We construct an analytic model of the plasma free energy and derive a generalized Saha equation which is used to obtain the opacities.

2. Method and results

We use a generalization of the method applied previously to the non-magnetic case (Potekhin, 1996a). The Helmholtz free energy of a volume V of the plasma is $F(V,T,\{N_\alpha\}) = F_{\rm id}^{(p)} + F_{\rm id}^{(e)} + F_{\rm id}^{(H)} + F_{\rm ex}$, where the subscripts "id" and "ex" refer to the ideal and excess part of F, (e, p, H) relate to the electrons, protons, and atoms, respectively, and $\{N_\alpha\}$ is the set of numbers of the particles.

For protons, $\beta F_{\rm id}^{(p)}/N_p = \ln(2\pi a_m^2 \lambda_p n_p) + \ln\left[1 - {\rm e}^{-\beta\hbar\omega_{cp}}\right] - 1$, where $\beta \equiv 1/k_B T$, $n_\alpha \equiv N_\alpha/V$, $\lambda_\alpha \equiv (2\pi\beta\hbar^2/m_\alpha)^{1/2}$, $a_m \equiv (\hbar c/eB)^{1/2}$, $\omega_{cp} \equiv eB/m_p c$. Being the same for free protons and atoms, proton spin and zero-point energy do not affect thermodynamics, therefore the corresponding terms are suppressed.

For electrons, we take into account partial degeneracy and relativity: $F_{\rm id}^{(e)}/V = \mu n_e - P_{\rm id}^{(e)}$, where the pressure $P_{\rm id}^{(e)}$ is easily expressed through fitting formulae by Blinnikov et al. (1996), and n_e is related to the chemical potential μ through a fitting formula by Potekhin (1996b). For $F_{\rm ex}^{(ep)}$, we use an analytic expression (Chabrier and Potekhin, 1998) based on calculations by Chabrier (1990) at $B = 0$. The latter approximation is justified for the classical plasma (according to the Bohr-van Leeuwen theorem) but fails in the region where B suppresses strong electron degeneracy (that occurs only in subphotospheric layers).

For atoms, $\beta F_{\rm id}^{(H)} = \sum_{s\nu} \int {\rm d}^2 K_\perp N_{s\nu}(K_\perp)\,\{\ln[n_H \lambda_H^3 w_{s\nu}(K_\perp)/Z_w] - 1\}$, s and ν relate to electronic excitations, $N_{s\nu} = (\lambda_H/2\pi\hbar)^2 N_H w_{s\nu} {\rm e}^{\beta\chi_{s\nu}}/Z_w$ are the atomic occupancies per unit phase space of the transverse component $K_\perp$ of the pseudomomentum $\mathbf{K}$ which characterizes the atomic motion in the magnetic field, $w_{s\nu}(K_\perp)$ and $\chi_{s\nu}(K_\perp)$ are the occupation probabilities and binding energies of the moving atom, and $Z_w = (\lambda_H/2\pi\hbar)^2 \sum_{s\nu} \int {\rm d}^2 K_\perp w_{s\nu}(K_\perp) \exp[\beta\chi_{s\nu}(K_\perp)]$ is the internal partition function. Large values of $K_\perp$ correspond to the decentered states with large dipole moment created by the induced electric field. These states have been neglected in previous studies (e.g., Lai and Salpeter, 1997) but our results show that they are important at $T \gtrsim 10^{5.5}$ K. Generalization of the unbinding free energy (Potekhin, 1996a) to the magnetic case gives $\beta F_{\rm ex}^{(H)} = (N_H + N_e) \sum_{s\nu} \int\, {\rm d}^2 K_\perp N_{s\nu} v_{s\nu}/V$, where $v_{s\nu}(K_\perp) = (4\pi/3)(r_{s\nu}^{(ep)}(K_\perp))^3$ is an effective interaction volume and r_{ep} the root-mean-square electron-proton distance in the atom.

Minimization of F leads to the generalized Saha equation

$$n_H = \lambda_p \lambda_e \frac{(2\pi a_m^2)^2}{\lambda_H^3}\, n_e n_p\, \left[1 - \exp(-\beta\hbar\omega_{cp})\right]\, Z_w \exp(\Lambda_{\rm deg}), \qquad (1)$$

where $\ln w_{s\nu} \approx \beta(\partial F_{\rm ex}^{(ep)}/\partial N_p + \partial F_{\rm ex}^{(ep)}/\partial N_e - (n_e + n_H) v_{s\nu}(K_\perp)$, and $\Lambda_{\rm deg} = \beta[\mu + \partial\mu/\partial \ln n_e - \partial P_{\rm id}^{(e)}/\partial n_e] - \ln(2\pi n_e \lambda_e a_m^2)$ is the correction for electron degeneracy. In numerical solutions of Eq. (1) we use fitting formulae for $\chi_{s\nu}$ and $r_{s\nu}^{(ep)}$ (Potekhin, 1998). The equation of state (EOS) is calculated from $P = -(\partial F/\partial V)_{T,\{N_\alpha\}}$.

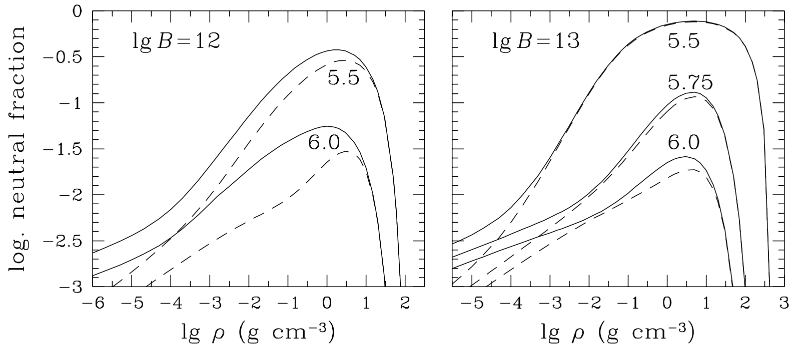

Fig. 1. Fraction of atoms (solid lines — total, dashed lines — ground-state) that contrubute to the opacities at different B shown at the panels. Numbers near curves are $\lg T$. At high ρ neutrals dissociate due to the pressure ionization effect.

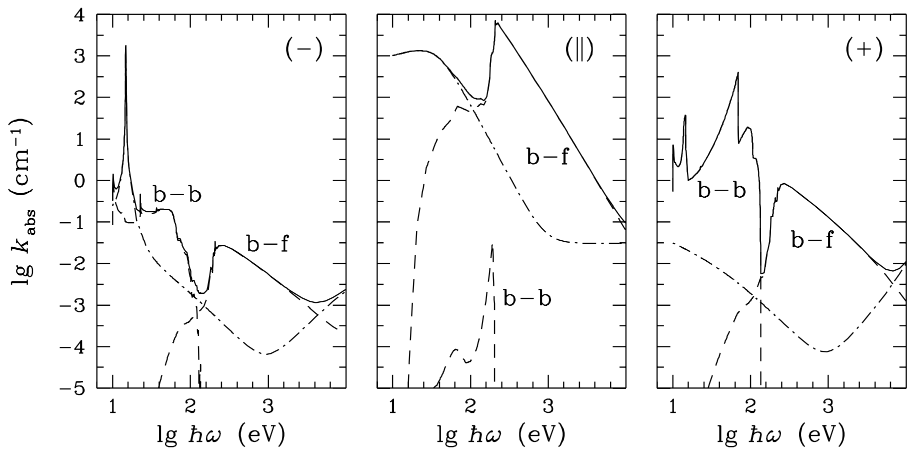

Fig. 2. Opacities of hydrogen plasma at $B = 2.35 \times 10^{12}$ G, $T = 3.16 \times 10^5$ K, and $\rho = 0.1$ g cm^{-3} for the radiation with left (left panel), longitudinal (middle panel) and right (right panel) polarization. Dashed lines represent the contribution of ground-state atoms, dot-dashed lines — the opacity of the charged component, and solid line — the sum of both.

Fig. 1 shows the neutral fraction $n_H^{\rm opt}/(n_H + n_p)$ that contributes to the atomic opacities ($n_{s\nu}^{\rm opt}(K_\perp) \sim n_{s\nu}(K_\perp)\exp[-4^3 n_p v_{s\nu}(K_\perp)]$, cf. Potekhin, 1996a). The total n_H contributing to the EOS can be much larger in the low and high density limits. The different dependence of $n_H^{\rm opt}$ on B at high and low T is explained by the competition of change in $\chi_{s\nu}$ and the phase space occupied by electrons and atoms with B. The opacities shown in Fig. 2 are obtained using the bound-bound and bound-free cross sections calculated previously (Potekhin and Pavlov, 1997 and ref. therein), and absorption coefficient of the ionized fraction calculated following Shibanov et al. (1992). The moving bound species contribute largely in the opacities of typical cooling NS atmospheres, their spectra are considerably different from those of the static-atom approximation (cf. Pavlov et al., 1995). This impels us to revise the previous atmosphere models at $T \lesssim 10^6$ K. To do that properly one has to include the absorption by excited atoms, which can be substantial at relatively high $T \sim 10^6$ K (see Fig. 1), and the bound-quasifree transitions (cf. Stehlé and Jacuemot, 1993) which are expected to fill the gap seen in the absorption for circular polarizations between b-b and b-f contributions.

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