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Non-variability of the fine-structure constant over cosmological time scales

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Abstract. — A statistical analysis of fine splitting of C IV, N V, O VI, Mg II, Al III and Si IV doublet absorption lines in quasar spectra is carried out in order to estimate a possible time variation of the fine-structure constant $\alpha = e^2/\hbar c$ over cosmological time scales $t \sim 10^{10}$ yr. The observational basis of the analysis is a catalogue* of 1414 pairs of wavelengths with redshifts $z = 0.2 - 3.7$, compiled from data published in 1980 – 1992. Robust statistical estimates like the "trimmed mean" are used as well as the least squares. No statistically significant time variation of α is found. The estimate $\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z = (-0.6 \pm 2.8)\ 10^{-4}$ is obtained. For the 95% significance level, an upper bound on the rate of a relative variation of the fine-structure constant is $|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z| < 5.6\ 10^{-4}$, which corresponds approximately to $|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}t| < 4\ 10^{-14}\ \mathrm{yr}^{-1}$. This limit represents the strongest up-to-date restriction on the possible time variation of α for the epoch $0.2 \le z \lesssim 4$.

Key words: quasars: general — catalogues — cosmology — quasars: absorption lines

1. Introduction

The problem of a possible time variation of fundamental constants has attracted attention since Dirac (1937) formulated his "large number hypothesis". Although the hypothesis was not confirmed in its original form, the interest to the problem arose again in the 1980s, when it was realized that Kaluza – Klein models of unified interactions may produce variations of fundamental constants, if the extra dimension scales in these models vary cosmologically (Chodos & Detweiler 1980; Freund 1982; Marciano 1984). "Superstring" theories also allow the fundamental constants to vary (e.g., Wu & Wang 1986). In this paper we consider possible variations of the fine-structure (i.e., electromagnetic coupling) constant[1] $\alpha = e^2/\hbar c$, where e, $\hbar$ and c are the electron charge, Planck constant and velocity of light, respectively.

Various tests of the fundamental constant variability differ in space-time regions of the Universe which they cover. Laboratory tests infer the possible variation of certain combinations of constants "here and now" from comparison of different frequency standards (Turneaure & Stein 1976; Domnin et al. 1986). However, as Marciano (1984) and Sisterna & Vucetich (1990) have noted, individual trends of the constants might compensate each other in some combination. Geophysical, geochemical and paleontological data impose constraints on a larger variety of the combinations over the past history of the Solar system, although most of these constraints are very indirect. Having analyzed critically a lot of such *local tests,* Sisterna & Vucetich (1990) derived restrictions on possible variation rates of individual physical constants for ages t less than a few billion years ago, which correspond approximately to cosmological redshifts $z \lesssim 0.2$. Specifically, they reported $\dot{\alpha}/\alpha = \mathrm{d}\ln\alpha/\mathrm{d}t = (-1.3 \pm 6.5)\ 10^{-16}\ \mathrm{yr}^{-1}$ (hereafter we use the form $\hat{x}\pm\hat{\sigma}(\hat{x})$, where $\sigma(\hat{x})$ is the root-mean-square (RMS) deviation of $\hat{x}$, and the "hat" denotes an estimate). The most crucial for this result were the tests based on the Oklo phenomenon (Shlyakhter 1976) and on the isotopic analysis of natural radioactive decay products in meteorites (Dyson 1972).

However the above estimate cannot be extended to earlier evolutionary stages of the Universe because the law of possible space-time variation of α is unknown (Marciano 1984). Another investigation is needed for higher cosmological redshifts.

Astrophysical tests, in contrast to the local ones, concern the values of fundamental constants in distant areas of the early Universe. Specifically, the extragalactic spectroscopy provides the means for direct estimation of the fine-structure constant in the early epochs. Evidently, the variation of α should be small, otherwise it

*The catalogue (Tables C1-C7) is only available in electronic form: see the editorial in A&AS 1994, Vol. 103, No. 1

[1]According to laboratory data (Kinoshita & Lindquist, 1990), $\alpha^{-1} = 137.03599222 \pm 0.00000094$

would not be possible to identify spectral lines of distant astronomical objects. Accurate measurements of the wavelengths can then provide quantitative constraints on the fine-structure variation rate, as Savedoff (1956) has pointed out. Following him, Bahcall et al. (1967) and Bahcall & Schmidt (1967) got the estimates from observations of resonance lines in spectra of quasi-stellar objects (QSOs), $\Delta\alpha/\alpha = (-2 \pm 5)\ 10^{-2}$ and $(1 \pm 2)\ 10^{-3}$ at $z = 1.95$ and 0.2, respectively. Here $\Delta\alpha = \alpha_z - \alpha_0$ is the difference between the fine-structure constants at given z and $z = 0$. Wolfe et al. (1976) derived an upper bound $|\dot{\alpha}/\alpha| < 4\ 10^{-12}\ \mathrm{yr}^{-1}$ from an observation by Burbidge et al. (1976) of Mg II absorption doublet at $z = 0.524$. With the Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1} accepted by Wolfe et al. (1976), their result is equivalent to $|\Delta\alpha/\alpha| < 0.04$ at this redshift. Two orders of magnitude stronger constraints are available for the *combination* $\alpha^2 g m_e/m_p$, where g, m_e and m_p are the gyromagnetic factor of the proton and the electron and proton masses, respectively (Wolfe & Davis 1979; Tubbs & Wolfe 1980).

The authors of above-cited works used observations of a small number of systems. Therefore they were unable to eliminate possible errors caused by shifting an observed center of a spectral line due to occasional blending, which often results in the presence of multiple subcomponents.

A common way to reduce this uncertainty consists in using a higher spectral resolution. However reaching a higher resolution with keeping an acceptable signal-to-noise ratio supposes a substantially longer observation time. Now there exists a small number of observations with a resolution of 0.1 Å, appropriate to the most accurate $\Delta\alpha/\alpha$ estimation. All these observations relate to moderate redshifts only. However, the presence of "unseen" subcomponents distorting a spectral line cannot be fully ruled out even at very high resolution, since they may fall together inside the real linewidth.

Fortunately, there is a way to circumvent the difficulty related to the differences of radial velocities of various subsystems in a blend. Since distant multicomponent absorption systems belong to distant galaxies, each of them being oriented arbitrarily with respect to the line of sight, the probability of any blend structure is equal to that of the symmetrical one. Therefore the above-mentioned uncertainty can be reduced by a statistical analysis of many observations of various absorption systems. This has become possible in recent years when many QSO spectra have been obtained.

Levshakov (1992a) first used this possibility. However his analysis of about 500 pairs of lines suffered from some shortcomings (see Sect. **3**.1). Hence his result, $\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z = (2 \pm 1)\ 10^{-4}$, and his claim of a tentative variation of α cannot be regarded as conclusive. Later Levshakov (1992b) presented another estimate, which revealed much stronger z-dependence of α. This result, which will be quoted in Sect. **4**.6, is not consistent with our analysis given below.

The aim of this paper is to extract the most reliable information on the value of the fine-structure constant at high redshifts from astrophysical observations. We have combined the advantages of the alkalilike absorption doublet method with those obtained when using robust statistical estimation. First, we have compiled a list of alkalilike absorption doublet wavelengths found in the QSO spectra. Then we have carried out a statistical analysis which enabled us to reduce the above-mentioned uncertainties. Moreover, we have shown that this analysis gives an upper limit on the fine-structure variation rate which is stronger than limits found from the highest-quality observations published in recent years (see Sect. **4**.6). As a result, we have obtained the strongest up-to-date constraint $|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z| < 5.6\ 10^{-4}$ for $0.2 \lesssim z \lesssim 4$ at the 95% significance level.

2. Basis of the analysis

2.1. *Method of alkalilike absorption doublets*

QSO spectra contain many absorption lines which correspond to the transitions $^2\mathrm{S}_{1/2} \rightarrow {}^2\mathrm{P}_{3/2}$ and $^2\mathrm{S}_{1/2} \rightarrow {}^2\mathrm{P}_{1/2}$ from the ground levels of alkalilike ions. Let λ_1 and λ_2 be the wavelengths of these transitions, and let $\bar{\lambda} = \frac{2}{3}\lambda_1 + \frac{1}{3}\lambda_2$ be their weighted mean value calculated with statistical weights of the excited states. The relative splitting,

$$y = (\lambda_2 - \lambda_1)/\bar{\lambda}, \tag{1}$$

is proportional to α^2, up to corrections of higher orders in α (Bethe & Salpeter 1977). Comparing a relative splitting y_z at some redshift z with the corresponding laboratory value y_{lab} at $z = 0$, one can infer a possible variation of α. Let us define

$$Y(z) = (y_z/y_{\mathrm{lab}})^{1/2} - 1. \tag{2}$$

Then $Y(z) = \Delta\alpha/\alpha$ and $\mathrm{d}Y/\mathrm{d}z = \alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z$.

This method was previously used by Bahcall et al. (1967) and Wolfe et al. (1976). It has some advantages. First, it is based on differences of absorption line wavelengths, which can be measured much more accurately than emission ones, because the latter are generally broad. Secondly, the alkalilike doublets correspond to transitions from a single level. Thus their relative separation is not affected by differences in radial velocity distributions of different ions. Thirdly, the small difference between the wavelengths of the doublet components reduces the effect of possible calibration errors in different spectral ranges.

2.2. *Laboratory data*

For y_{lab} calculation we have adopted the laboratory standards of λ_1 and λ_2 given by Striganov & Odintsova (1982) and listed in Table 1. The differences from the standards of Morton et al. (1988) are too small to affect noticeably our results and conclusions.

Possible errors in these wavelength values do not exceed several mÅ, which causes an uncertainty in the last digit of the $y_{\rm lab}$ values given in Table 1.

Table 1. Laboratory wavelength standards

Ion	λ_1 (Å)	λ_2 (Å)	$y_{\rm lab}$
C IV	1548.202	1550.774	1.6604 10^{-3}
N V	1238.821	1242.804	3.2116 10^{-3}
O VI	1031.912	1037.613	5.5146 10^{-3}
Mg II	2796.352	2803.530	2.5647 10^{-3}
Al III	1854.716	1862.790	4.3470 10^{-3}
Si IV	1393.755	1402.769	6.4535 10^{-3}

2.3. *Observational data*

We have composed a catalogue of alkalilike doublet wavelengths with $z > 0.2$ found in the QSO absorption spectra, using all the data published since 1980, after the beginning of large-scale QSO surveys with electronic detection[2]. We use this catalogue in our specific analysis (Sect. 4), but it may be used as a database for any other analysis.

We have taken confidently identified and resolved resonant doublet absorption lines of most frequently observed ions (C IV, N V, O VI, Mg II, Al III, and Si IV) from series of observations with the resolution of FWHM < 6 Å which is sufficiently high to resolve the components of most of the doublets, in particular at high redshifts.

The selected lines were observed with signal-to-noise ratio $S/N = 10 - 100$; typically, $S/N \approx 20$. The effective spectral resolution was FWHM = 0.08 – 1 Å in observations of 184 of the doublets, FWHM = 4 – 6 Å for 419 of them, and FWHM ≈ 1 – 3 Å for the remaining 711 pairs of lines (see the catalogue for the references). The first (high-resolution) sample is most homogeneous; its typical S/N ranges between 10 and 20. Below in Sects. 4.1 – 4.3 we present results obtained when processing it separately.

We have considered 1414 pairs of lines altogether.

The catalogue of doublet wavelengths is presented in six tables; a separate table is given for each ion. A redshift z_i and observed wavelengths $(\lambda_1)_i$ and $(\lambda_2)_i$ are listed for each doublet together with an observational value of Y_i calculated according to Eq. (2). Figure 1 displays z-distributions (number of pairs of lines ΔN per bin $\Delta z = 0.2$) for all doublets and for the most abundant C IV and Mg II doublets. Clearly different z-distributions of these ions result from a selection due to difference in their rest-frame wavelengths.

[2]The wavelengths tables with their references compose the appendix to this paper which is distributed by electronic mail

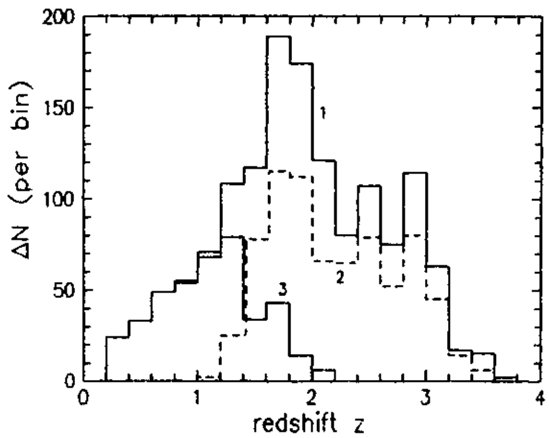

Fig. 1. Redshift distribution of alkalilike absorption doublets in QSO spectra from the catalogue of wavelengths: 1 – all the data, 2 – C IV doublets, 3 – Mg II doublets

3. Data processing

3.1. *Treatment of errors*

Any observational value Y_i has a random error ϵ_i arising from errors in λ_1 and λ_2 due to finite spectral resolution, noise, possible calibration errors, and blending. For the sake of confidence in the statistical analysis, the major attention should be paid to possible systematic errors.

First, a possible correlation between positions of optically thick and thin subcomponents of a blend could lead to errors in measured line centers, which might be correlated for any *single* absorption system. However we have argued in Sect. 1 that the resulting error is not systematic and can be reduced if various distant absorption systems are analyzed.

In a similar way, calibration errors are also randomized by including various series of observations in the analysis.

Second, if the isotopic composition varies with the cosmological time, then doublet separations might vary due to the isotopic shifts (Levshakov 1993). This possible variation is not greater than several mÅ, thus being well below final statistical errors of our analysis (Sect. 4).

Third, the laboratory value of the relative doublet separation, $y_{\rm lab}$, which is the standpoint of the analysis, is known with an error $\epsilon_{\rm lab}$, introducing an additional error ϵ_0 to Y_i:

$$Y_i = Y_{\rm ex}(z_i) + \epsilon_i + \epsilon_0, \tag{3}$$

$$\epsilon_0 \simeq \frac{\epsilon_{\rm lab}}{2y_{\rm lab}}(1+Y_i) + O(\epsilon_{\rm lab}^2) \approx \frac{\epsilon_{\rm lab}}{2y_{\rm lab}}, \tag{4}$$

where $Y_{\rm ex}$ is the exact value, and Y_i is measured one. The latter approximate equation follows from Eq. (2) and from the fact that $|Y_i| \ll 1$ for all observations. It assumes that ϵ_0 is independent of i for ions of each type. However the values of $\epsilon_{\rm lab}$ are different for different ions. Since different ions are observed at different typical redshifts (see Fig. 1), the systematic error ϵ_0 averaged over *a mixture* of various ions could depend indirectly on z. This indirect dependence could simulate a false trend for α. Hence a separate processing of each ions is required for an unambiguous interpretation.

This has not been taken into account by Levshakov (1992a) who mixed data for different ions and used one-parameter regression model $Y(z) = az$ which did not account for non-vanishing errors of laboratory wavelengths. Besides, he removed all the data which could give $|\Delta\alpha/\alpha| > 0.01$ but did not account for this removal in his least-square analysis. Therefore he underestimated the standard deviation.

Taking into account that $\lambda_2 - \lambda_1 \ll \bar{\lambda}$, one can easily show that, for uncorrelated errors,

$$\sigma(\epsilon_i) \approx 2^{-1/2}(\sigma(\lambda))_i/(\lambda_2 - \lambda_1)_i \tag{5}$$

Possible errors of the laboratory wavelengths λ_1 and λ_2 are about several mÅ (Striganov & Odintsova 1982), whereas typical errors $\sigma(\lambda)$ of astrophysical measurements vary from tens to hundreds of mÅ. For a typical redshift $z_i \sim 2$, it follows that $\sigma(\epsilon_i)/\sigma(\epsilon_0) \sim 10$. We have allowed for this by including the laboratory point $Y_0 = 0$ ($z_0 = 0$) into our data analysis with the relative weight $w_{\rm rel} = \sigma^2(\epsilon_i)/\sigma^2(\epsilon_0) \sim 100$ ($i > 0$).

Another possible systematic source of error is the non-linear λ-dependence of Y (Levshakov 1993), which gives rise to a shift in the statistical estimate roughly proportional to the variance $\sigma^2(Y)$. Below in Sect. 4.4 we calculate this shift and show that it is negligible in comparison with confidence intervals of our estimates.

3.2. *Regression model*

Most of the constant-variability models discussed in the literature treat physical constants as functions of cosmological time which is uniquely related to the cosmological redshift z. We have investigated the z-dependence of α with the aid of the simple linear regression

$$Y(z) = a + b(z - \bar{z}_0). \tag{6}$$

Here $\bar{z}_0 = N\bar{z}/(N + w_{\rm rel})$ is the weighted mean of z_i and $z_0 = 0$, where N is the number of observational points. Non-zero value of $\hat{b}$ would mean that α does vary. We allow also $\hat{Y}(0)$ to be non-zero in order to include the possibility of non-zero ϵ_0 in Eq. (3).

Since the z-dependence of α might be non-linear (e.g., Marciano 1984), contributions of Y_i in $\hat{b}$ might compensate each other at small and large z_i. For testing such a behavior of $\alpha(z)$ and checking thus the applicability of the regression model (6), we estimated centers of distributions of subsets $\{Y_i : z_i > z_{\rm min}\}$ with different $z_{\rm min} > 0$. These estimates may be additionally important because they supply information about different redshift ranges.

3.3. *Statistical techniques*

If the error distribution were gaussian then the least-square (LS) regression analysis would be most efficient. In practice, however, the data are essentially inhomogeneous, and their actual distribution differs from gaussian. In this case robust estimation should be applied. From a large variety of robust estimates (e.g., Lehmann 1983) the "trimmed mean" (TM) combines simplicity with efficiency and gives reliable confidence intervals even if the error distribution is unknown. TM estimates were originally applied to the statistical location problem. They consisted in the LS processing after removing $[\beta N]$ smallest and $[\beta N]$ largest values from a set of N data, where $0 < \beta < 0.5$, and $[x]$ means integer part of x. The LS and widely known median estimates may be regarded as the limiting cases of $\beta \to 0$ and $\beta \to 0.5$, respectively.

The TM estimates were extended to the linear regression analysis by Koenker & Bassett (1978) who proposed a suitable rule to remove $2[\beta N]$ observation points. Ruppert & Carroll (1980) proved the consistency of this method and presented expressions for the variance estimation (see Appendix).

Let the trimming level β vary in the interval $0 < \beta_1 < \beta < \beta_2 < 0.5$, and let the minimum of the variance estimate $\hat{\sigma}^2$ be at some β_0. As shown by Jaeckel (1971) for the location problem, the TM estimate with $\beta = \beta_0$ is asymptotically the best one for the (unknown) real distribution of errors ϵ_i. This estimate is *adaptive* since it adjusts to a sample distribution (Lehmann 1983). The interval $[\beta_1, \beta_2]$ should a priori include the true optimal value of β; thus β_2 cannot be small. On the other hand, β_2 should not be too close to 0.5, for the estimate $\hat{\sigma}^2$ to have a not too wide confidence interval. It is reasonable to take $\beta_1 = 0$, $\beta_2 = 0.25$ (Jaeckel 1971, Andrews et al. 1972).

4. Results and discussion

First we analyzed the distribution of observational data. Then we applied the LS and TM methods. Each of the methods was applied to the estimation of linear regression parameters as well as to the estimation of the data distribution center. The formulae used are given in Appendix.

4.1. *Data distribution*

We have investigated the distribution of the total sample of data as well as the distribution of subsets corresponding to separate resolution ranges and separate types of ions, except for those subsets whose number is too small ($N < 20$). In Table 2 we present the mean value $\bar{Y}$, standard deviation $\sigma(Y)$, coefficient of asymmetry $\gamma_1 = \mu_3/\sigma^3$ and excess $\gamma_2 = \mu_4/\sigma^4 - 3$ for each subset, where μ_n is the nth centered moment of the distribution considered. All these

Table 2. Distribution parameters

Ion	N	$\bar{Y}$ ($\times 10^{-4}$)	$\sigma(Y)$ ($\times 10^{-2}$)	γ_1	γ_2	$Q_N(\chi^2)$	$Q_{\rm DE}(\chi^2)$
			All the data (FWHM < 6 Å)				
C IV	741	23 ± 15	4.1 ± 0.2	0.2 ± 0.2	1.8 ± 0.7	$< 10^{-5}$	0.70
N V	46	-42 ± 46	3.1 ± 0.5	0.0 ± 0.6	0.9 ± 1.4	0.20	0.81
Mg II	404	-1 ± 18	3.6 ± 0.2	-0.1 ± 0.3	2.3 ± 1.8	$< 10^{-5}$	0.13
Al III	42	-3 ± 43	2.8 ± 0.4	0.0 ± 0.6	0.3 ± 1.3	0.23	0.38
Si IV	168	-14 ± 13	1.7 ± 0.2	-0.9 ± 1.1	7.0 ± 5.4	4×10^{-5}	0.66
			Data obtained with FWHM < 3 Å				
C IV	559	-5 ± 14	3.2 ± 0.2	-0.2 ± 0.3	3.0 ± 1.1	$< 10^{-5}$	0.16
N V	36	-77 ± 49	3.0 ± 0.5	0.8 ± 0.7	1.4 ± 1.8	0.30	0.72
Mg II	137	25 ± 19	2.2 ± 0.3	-0.3 ± 0.9	5.1 ± 3.0	$< 10^{-5}$	0.006
Si IV	132	-15 ± 14	1.6 ± 0.3	-1.0 ± 1.7	10.4 ± 8.7	2×10^{-4}	0.45
			Data obtained with FWHM ≤ 1 Å				
C IV	122	-14 ± 26	2.9 ± 0.4	-1.0 ± 1.1	6.1 ± 4.1	0.0012	0.64
Mg II	45	3 ± 12	0.8 ± 0.2	-1.1 ± 1.2	3.1 ± 3.4	0.011	0.25

quantities and their RMS deviations have been estimated from sample moments.

For all subsets in Table 2, the deviations of $\bar{Y}$ from zero do not exceed 2σ indicating that all observed differences of α_z from α_0 lie within the 95% confidence interval. This preliminary analysis yields an upper bound $|\Delta\alpha/\alpha| < 3\ 10^{-3}$ at an average redshift $\bar{z} \sim 2$.

Figure 2 presents histograms of sample distributions. The observational data are divided into $M = [N^{1/2}]$ intervals $\Delta Y_{\rm obs}$, where $[x]$ is the integer part of x; the gaussian probability of getting a point in each bin equals M^{-1}. Many of the distributions are clearly non-gaussian which is confirmed by the chi-square test. Residual probabilities $Q_N(\chi^2)$ are given in Table 2. In most of the cases the values $Q_N(\chi^2)$ are small enough to reject confidently the hypothesis of the gaussian distribution. This is not surprising: samples are inhomogeneous due to mixing the data of different resolution as well as due to different nature of errors (noise, blending, etc.). For a comparison, the last column of the table presents residual probabilities $Q_{\rm DE}(\chi^2)$ corresponding to the double-side exponential distribution, with the density $f_{\rm DE}(Y) = (\sqrt{2}\sigma)^{-1}\exp(-|Y|\sqrt{2}/\sigma)$. We see that this model is a good approximation for most of the sample distributions.

4.2. *Least-square estimates: effect of resolution*

We have obtained the standard LS estimates of the regression parameters a and b. First we have tried to account for the data inhomogeneity by weighting all data according to expected errors $\sigma_{\rm exp}(\lambda)$ pointed out by observers,

$$\sigma_{\rm exp}(\lambda) \approx (\sigma(W)/W)M/\sqrt{12} \propto (S/N)^{-1}W^{-1}F^2, \quad (7)$$

where W is the equivalent width, $M \approx 2.5F$ is an estimate of the full-width at zero-intensity of a line, and F is its observed full-width at half-maximum (Young et al. 1979, Levshakov 1992a). Equation (7) accounts for fluctuations of a weighted center of a line due to Poisson fluctuations of photon counts only, therefore values of $\sigma^2_{\rm exp}(\lambda)$ differ from actual variances of measured wavelengths $\sigma^2(\lambda)$. Nevertheless, the value of $\sigma^2_{\rm exp}(\lambda)$ can be regarded as a formal measure of an observed line quality. If this was not presented, we adopted a typical value depending on the resolution. Other LS estimates have been obtained with equal weights attributed to observational points, each of them being hundred times "lighter" than the laboratory zero point. In some cases RMS errors of the estimates have not been decreased by the weighting. Moreover, the variance estimates $\hat{\sigma}^2(\hat{a})$, $\hat{\sigma}^2(\hat{b})$ obtained from the scatter of real data have been several times larger than the expected values calculated directly from the $\sigma_{\rm exp}(\lambda)$ used. This indicates that the use of the quoted $\sigma_{\rm exp}(\lambda)$ is inadequate, probably due to a major contribution of occasional blending which could not be taken into account in the $\sigma_{\rm exp}(\lambda)$ estimate.

Table 3 gives results for the non-weighted LS analysis. Its second and third parts present results for more uniform data subsets with instrumental FWHM < 3 Å and FWHM ≤ 1 Å. The latter subset includes 13% of all lines only. Despite this reduction, RMS errors do not increase by more than a factor of 1.7, demonstrating the advantage of high-resolution results (although with lower S/N). On the other hand, these results show that a large number of data with moderate resolution may give more accurate results than a small number of high-resolution data (see also Sect. 4.6).

Different types of ions have been processed separately. The most accurate estimates are those for Si IV. This agrees with Eq. (5): this doublet has the largest redshifted $(\lambda_2 - \lambda_1)$. The overall estimate of the slope b for all ions (calculated with weights proportional to $\hat{\sigma}^{-2}(\hat{b})$) is $(-2.7 \pm 6.5)\ 10^{-4}$. Analogous estimates for a and for more uniform data subsets with FWHM < 3 Å and FWHM < 1 Å are also shown in Table 3.

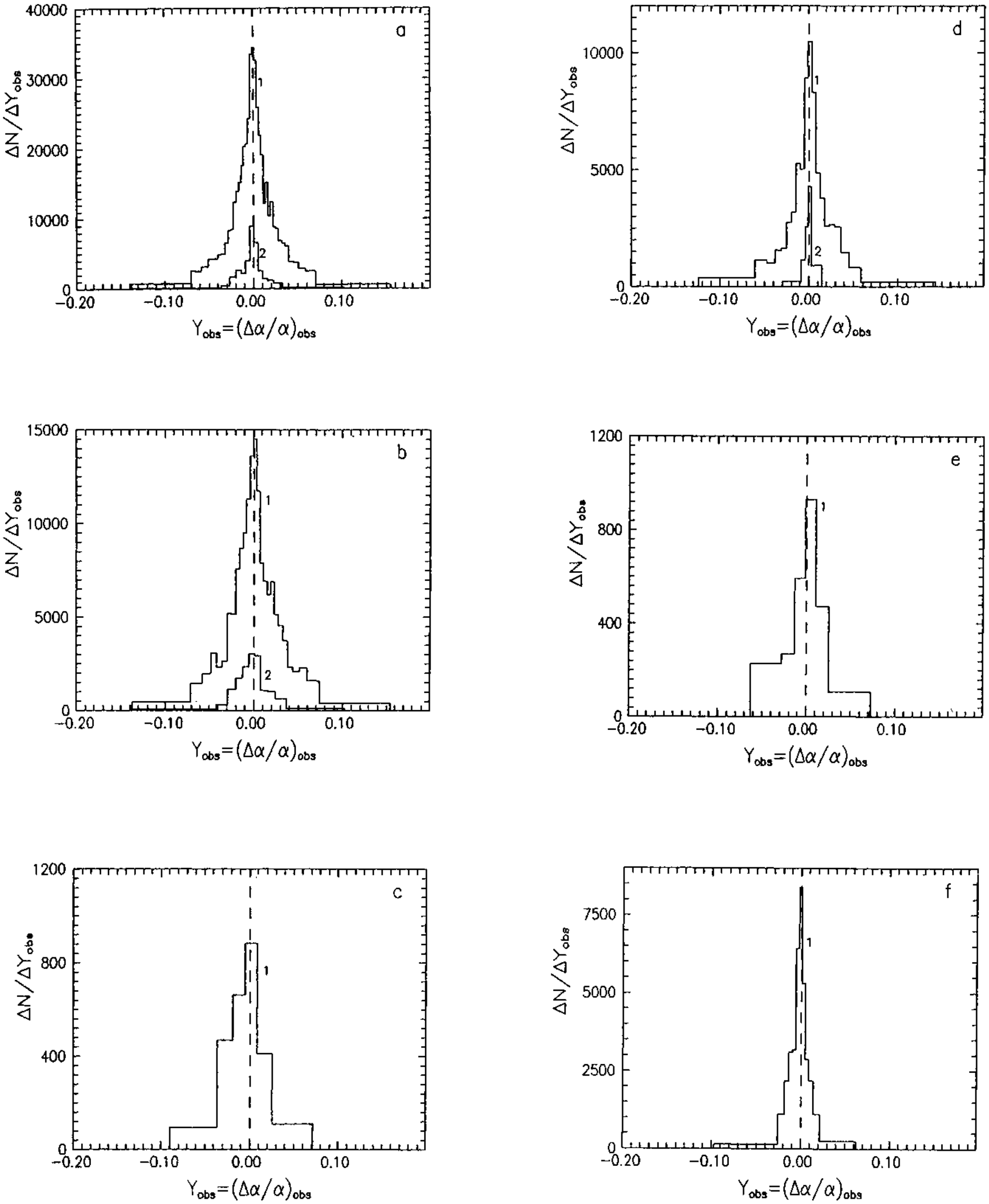

Fig. 2. Sample distributions of observed relative fine splitting: all the data a), C IV b), N V c), Mg II d), Al III e), and Si IV f); 1 – data with instrumental FWHM < 6 Å, 2 – data with instrumental FWHM ≤ 1 Å

Table 3. Least-square regression estimates

Ion	N	$\bar{z}$	a (10^{-4})	b (10^{-4})
		FWHM < 6 Å		
C IV	741	2.2	21 ± 14	16 ± 16
N V	46	2.4	-13 ± 26	-23 ± 23
O VI	13	2.7	-4 ± 26	-13 ± 29
Mg II	404	1.1	-1 ± 16	-31 ± 27
Al III	42	2.0	-1 ± 23	3 ± 25
Si IV	168	2.4	-9 ± 11	-2.4 ± 8.6
Overall	1414	1.9	-0.4 ± 6.6	-2.7 ± 6.5
		FWHM < 3 Å		
C IV	559	2.1	-4 ± 13	-10 ± 14
N V	36	2.3	-20 ± 25	-40 ± 24
O VI	13	2.7	-4 ± 26	-13 ± 29
Mg II	137	0.9	14 ± 14	0 ± 25
Al III	18	1.8	11 ± 15	41 ± 22
Si IV	132	2.3	-8 ± 11	-3.5 ± 8.9
Overall	895	2.0	-0.7 ± 6.0	-3.9 ± 6.4
		FWHM ≤ 1 Å		
C IV	123	1.7	-7 ± 20	-4 ± 23
Mg II	45	1.0	0.8 ± 6.8	5 ± 15
Si IV	17	1.9	-2.8 ± 9.6	-9 ± 14
Overall	184	1.5	-0.9 ± 5.3	-2.8 ± 9.4

Table 4. Trimmed-mean regression estimates

Ion	N	$\bar{z}$	a ($\times 10^{-4}$)	b ($\times 10^{-4}$)	β
		FWHM < 6 Å			
C IV	741	2.2	11.7 ± 9.3	9.0 ± 8.9	0.25
N V	46	2.4	-9 ± 11	-20 ± 11	0.21
O VI	13	2.7	1.5 ± 5.9	10.2 ± 8.9	0.21
Mg II	404	1.1	6 ± 11	-14 ± 17	0.25
Al III	42	2.0	2 ± 12	0 ± 15	0.21
Si IV	168	2.4	-2.9 ± 4.5	-3.1 ± 3.5	0.25
Overall	1414	1.9	0.2 ± 3.0	-1.7 ± 2.9	
		FWHM < 3 Å			
C IV	559	2.1	-0.8 ± 8.7	-2.4 ± 8.3	0.23
N V	36	2.3	-18 ± 19	-38 ± 20	0.12
O VI	13	2.7	1.5 ± 5.9	10.2 ± 8.9	0.21
Mg II	137	0.9	9.1 ± 8.6	14 ± 16	0.24
Al III	18	1.8	11.4 ± 9.8	40 ± 15	0.06
Si IV	132	2.3	-2.6 ± 4.0	-3.4 ± 3.3	0.24
Overall	895	2.0	0.5 ± 2.8	-0.6 ± 2.8	
		FWHM ≤ 1 Å			
C IV	123	1.7	-4 ± 12	-5 ± 14	0.25
Mg II	45	1.0	3.1 ± 5.8	6 ± 13	0.03
Si IV	17	1.9	-1.7 ± 2.7	-9.8 ± 5.3	0.22
Overall	184	1.5	-1.7 ± 2.6	-7.1 ± 4.6	

4.3. *Trimmed-mean estimates: effect of non-gaussian statistics*

Table 4 presents results of the TM analysis of the regression parameters with the trimming level β which minimizes the estimate of the slope variance, $\hat{\sigma}^2(\hat{b})$, in the interval $0 < \beta < 0.25$. If the error distribution were gaussian then the optimal trimming level β_0 would be close to zero, and the TM and LS estimates would coincide. The actual β_0 is close to 0.25, in agreement with the non-gaussian nature of errors.

The TM-method filters efficiently a sample contaminated by less accurate data (Lehmann 1983). A comparison of Tables 3 and 4 reveals a noticeable decrease of RMS errors. According to Eq. (A9) of Appendix, the most accurate overall TM-estimate of the slope, $b = (-0.6 \pm 2.8)\ 10^{-4}$, yields the restriction $|b| < 5.6\ 10^{-4}$ at the 95% significance level.

4.4. *Shift due to non-linear wavelength-dependence*

Let us estimate the possible systematic error mentioned at the end of Sect. 3.1. Substituting Eq. (1) into Eq. (2), expanding in Taylor series over λ_1, λ_2, and calculating the mean, one easily finds

$$\bar{Y} - Y_{\text{ex}} = -\frac{1}{2}\sigma^2(Y)(1 + O(\sigma(Y)) + O(y_{\text{lab}})), \qquad (8)$$

where Y_{ex} is the exact value of Y. From the third and the fourth columns of Table 2 we see that in all the cases the shift (8) does not exceed $\sigma(\bar{Y})$.

Furthermore, it is easy to show that trimmed-mean estimates $\bar{Y}_\beta$ deviate from Y_{ex} less then $\bar{Y}$ does. In particular, if we adopt the f_{DE} probability density, in consistency with the results of Sect. 4.1, then

$$\bar{Y}_\beta - Y_{\text{ex}} = -\frac{1}{2}\sigma^2(Y) \left(1 - 2\beta\left(1 - \ln 2\beta + \frac{1}{2}(\ln 2\beta)^2\right)\right)/(1 - 2\beta) \qquad (9)$$

with the same accuracy as in Eq. (8). The corresponding shift of the slope estimate $\hat{b}$ is about $(\bar{Y}_\beta - Y_{\text{ex}})/\bar{z}$. Taking the numerical values from Tables 2 and 4, one obtains the shifts less then 10^{-4} for each data subset, which are fully negligible in comparison with RMS statistical errors.

4.5. *Stability of estimates*

The numbers N of processed doublet data for the C IV, Mg II and Si IV ions are quite sufficient for obtaining accurate estimates. In contrast, adding or removing one or several doublets of N V, O VI and Al III causes variations of $\hat{b}$ and $\hat{\sigma}(\hat{b})$ up to 50% with respect to the tabulated values, because the confidence intervals are rather wide. Therefore one should not attribute a real physical meaning to the $2.7\hat{\sigma}$ and $2.0\hat{\sigma}$ deviations of $\hat{b}$ from zero for N V and Al III in the second part of Table 4. However this uncertainty does not affect the overall estimate since it is mainly determined by the most accurate Si IV data.

4.6. *Comparison with selected data processing*

Consideration of the subsets of different resolution in Tables 2 – 4 reveals that the lowest statistical errors are at FWHM < 3 Å. Further lowering the FWHM threshold does not diminish resulting statistical errors due to a correspondingly small number of available observations.

One may guess whether additional selection criteria should be used in order to improve the accuracy. Evidently, non-saturated lines with low FWHM and high S/N are preferable. The $\sigma_{\rm exp}(\lambda)$ could serve as a formal cumulative measure of an apparent quality of a line. For example, let us consider MgII doublets with $\sigma_{\rm exp}(\lambda) < 0.03$ Å obtained with instrumental FWHM ≈ 0.1 Å and listed in Table 5. The standard LS estimate gives $\Delta\alpha/\alpha = (1.2 \pm 0.4)\ 10^{-3}$ at $z \approx 0.4$, the 95% confidence intervals being $(-0.2, 2.5)\ 10^{-3}$ for the mean and $(0.3, 1.6)\ 10^{-3}$ for the standard deviation.

Table 5. Very high resolution data

QSO	z	λ_1	λ_2	ref.	Y
1100−264	0.3563	3792.66	3802.44	a	0.00229
	0.3589	3800.10	3809.87	a	0.00079
	0.3591	3800.68	3810.44	a	0.00021
2128−123	0.4298	3998.25	4008.54	b	0.00131

(a) Carswell et al. (1991), (b) Lanzetta & Bowen (1992)

A larger sample has been presented by Levshakov (1992b). He selected 36 pairs of relatively strong, isolated lines with $z > 1$ "which do not lie i) in the Lyman-alpha forest, ii) on steep emission line profiles or near strong absorption troughs, iii) at the edges of observed spectral regions, and iv) near strong night sky lines". The result he reported was $\Delta\alpha/\alpha = (3 \pm 1)\ 10^{-3}$ at $z \sim 2$. Moreover, one of the robust estimates he used gave $\Delta\alpha/\alpha = (3.6 \pm 0.8)\ 10^{-3}$. However one cannot be sure that his selection procedure has not biased the sample.

The standard deviations in both these examples are greater than that of our analysis presented in Sect. 4.3. Assuming that the standard deviation of $\bar{Y} \sim b\bar{z}$ is proportional to $\sigma(\epsilon_i)/\sqrt{N}$, where $\sigma(\epsilon_i)$ is defined by Eq. (5), one finds that the standard deviation of the slope $\sigma(\hat{b})$ is roughly proportional to $\sigma(\lambda)/[\bar{z}(\lambda_2-\lambda_1)_z N^{1/2}]$. Hence one may conclude from the above results that our standard deviation $\sigma(\hat{b}) \approx 3\ 10^{-4}$ from Table 4 might be achieved, for example, if one could obtain and process several observations of various Si IV systems at $\bar{z} \sim 3$ with a quality as high as that of the data listed in Table 5. However even in this hypotetical case the number of such observations should not be small, in order to randomize a possible error caused by the occasional blending, as we have argued in Sect. 1.

Table 6. Trimmed-mean location estimates

Ion	N	$\bar{z}$	β_0	$\bar{Y}_\beta$ ($\times 10^{-4}$)
		$z_{\rm min} = 1$		
C IV	741	2.2	0.25	12 ± 11
N V	46	2.4	0.11	-46 ± 35
O VI	13	2.7	0.23	24 ± 39
Mg II	244	1.4	0.24	12 ± 15
Al III	41	2.0	0.24	9 ± 35
Si IV	168	2.4	0.25	-8.7 ± 8.6
Overall	1253	2.1		-3.2 ± 5.9
		$z_{\rm min} = 2$		
C IV	411	2.6	0.24	31 ± 17
N V	37	2.5	0.14	-35 ± 43
O VI	13	2.7	0.23	24 ± 39
Al III	17	2.4	0.18	-17 ± 37
Si IV	112	2.6	0.25	-9 ± 10
Overall	590	2.6		-3.0 ± 6.2
		$z_{\rm min} = 3$		
C IV	69	3.2	0.19	62 ± 37
N V	7	3.2	0.14	-161 ± 83
Si IV	23	3.3	0.17	17 ± 31
Overall	99	3.2		21 ± 23

4.7. *Possible test for differences of laboratory wavelengths*

The two-parameter regression (6) supplies information not only on the slope b but on the total shift a as well. If this shift were significant, then, according to Eq. (2), the estimate $\hat{Y}(0) = \hat{a} - \hat{b}\bar{z}_0$ could give a correction to the laboratory splitting for ions of given type. However our analysis reveals no statistically significant deviation of $Y(0)$ from $Y_0 = 0$.

4.8. *Model-independent test*

To test the applicability of our regression model we have obtained the TM-location estimates $\bar{Y}_\beta$ (again optimizing β) for Y_i data with $z_i > z_{\rm min}$. These estimates do not depend on a regression model. Some of the results are given in Table 6. No statistically significant deviation of $\bar{Y}_\beta$ from zero is observed for any $z_{\rm min}$. In particular, from the last part of Table 6 we get $|\Delta\alpha/\alpha| < 6\ 10^{-3}$ for $\bar{z} = 3.2$ at the 95% significance level.

4.9. *Constraints on theoretical models*

Our results enable one to select theoretical models which predict variability of the fine-structure constant. For example, let us consider the power law $\alpha = \alpha_0(t_c/t_0)^{\rm p}$, where t_0 is the present age of the Universe and t_c is the cosmological time ($t = t_0 - t_c$ is the look-back time). The standard cosmological model (with the average density of matter close to the critical value and the Λ-term equal to zero) relates t_c to z as $t_c = \frac{2}{3}H_0^{-1}(1+z)^{-3/2}$. Comparing this with the last part of Table 6, one gets the restriction $|p| < 0.003$ which excludes the value $p = 1$ of Gamow (1967) and Chodos & Detweiler (1980) as well as the value $p = -1/4$ of Freund (1982). Taking into account other parts of Table 6, one can impose restrictions on more complicated models (e.g., Marciano 1984).

5. Conclusions

Our statistical analysis based on the catalogue of 1414 absorption doublets in the QSO absorption spectra at cosmological redshifts $z > 0.2$ reveals no statistically significant variation of the fine-structure constant α with z. The tentative variation of α reported by Levshakov (1992b) is not confirmed. Our estimate $\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z = (-0.6 \pm 2.8)\ 10^{-4}$ and the 95%-significance restriction $|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z| < 5.6\ 10^{-4}$ are by two orders of magnitude more stringent than corresponding results of Bahcall & Schmidt (1967), Bahcall et al. (1967), and Wolfe et al. (1976). The analysis of a large number of absorbing systems minimizes the uncertainty caused by occasional blending.

For the Hubble constant $H_0 = 75$ km s^{-1} Mpc^{-1} (Jacoby et al. 1992), the above limit yields $|\dot{\alpha}/\alpha| < 4\ 10^{-14}$ yr^{-1}, if one extrapolates it to the present epoch ($z \ll 1$). This limit is less restrictive than the limit $|\dot{\alpha}/\alpha| < 1.4\ 10^{-15}$ yr^{-1} reported by Sisterna & Vucetich (1990) for the present epoch. However, our constraint is different in that it covers earlier evolutionary stages and more distant regions of the Universe. The latter is not trivial, because some of these regions were causally disconnected at the epochs of line formation (cf. Tubbs & Wolfe 1980).

Estimates of the center of distribution of empirical values $\Delta\alpha/\alpha$ at $z > z_{\min}$ (with $z_{\min}$ treated as a parameter) also show no statistically significant difference between α_z and α_0. This indicates that the absence of any significant trend of α does not depend on the linear regression model adopted. From observations of 99 absorption systems with redshifts $3.0 < z < 3.7$, we have obtained $\Delta\alpha/\alpha = (2.1 \pm 2.3)\ 10^{-3}$ at $z = 3.2$.

The trimmed-mean method with the trimming level optimization used in our estimates is robust and adaptive (Lehmann 1983). Therefore it is unlikely that our upper limit on the variation rate of the fine-structure constant can be significantly improved without involving other observational data.

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A. Appendix: trimmed-mean estimation

Let β be a trimming level, $0 < \beta < 0.5$, $n = [\beta N]$ be the number of observations removed "from the high end" and "from the low end" of the sample (according to the rule defined below) and the $N_\beta = N - 2n$ be the number of observations Y_i' kept. Then the TM estimate of a regression parameters is the LS estimate applied to the remaining data. For the location problem, the n smallest and n largest data points can be removed, and the estimate is the mean

$$\bar{Y}_\beta = \frac{1}{N_\beta}\sum_{i=1}^{N_\beta} Y_i' \,. \tag{A1}$$

Let $Y_{\min}$ and $Y_{\max}$ be the smallest and the largest of the kept data Y_i'. Then the TM variance of $\bar{Y}_\beta$ can be estimated as (e.g., Lehmann 1983)

$$\begin{aligned}\hat{\sigma}^2(\bar{Y}_\beta) \;=\; & \left(N(1-2\beta)^2\right)^{-1}\left(Q_\beta(\bar{Y}_\beta)/(N-1)+\right.\\ & \left.\beta c_1^2 + \beta c_2^2 - \beta^2(c_1+c_2)^2\right)\,,\end{aligned} \tag{A2}$$

where $c_1 = Y_{\min} - \bar{Y}_\beta$, $c_2 = Y_{\max} - \bar{Y}_\beta$, and Q_β is the residual sum of squares (RSS)

$$Q_\beta(Y) = \sum_{i=1}^{N_\beta} (Y_i' - Y)^2 \,. \tag{A3}$$

Now let us consider the linear regression (6) following Ruppert & Carroll (1980). Including a weighted zero point $Y_0' = Y_0 = 0$, we have $(N+1)$ data points in total; $2n$ of them are to be removed. Koenker & Bassett (1978) have defined, for $0 < \gamma < 1$, the regression quantiles $a(\gamma)$ and $b(\gamma)$ as those a and b which minimize the function

$$r_\gamma(a,b) = \sum_{i=0}^{N} \rho_\gamma(Y_i - a - b(z_i - \bar{z})), \tag{A4}$$

where

$$\rho_\gamma(x) = \begin{cases}\gamma x & \text{if } x > 0\\ (\gamma-1)x & \text{otherwise}\end{cases} \tag{A5}$$

Let us remove all the data which do not satisfy the inequality

$$a(1-\beta)+b(1-\beta)\cdot(z-\bar{z}) \le Y_i' \le a(\beta)+b(\beta)\cdot(z-\bar{z}). \tag{A6}$$

To ensure that the point Y_0 is kept, we minimize (A4) under the restrictions $a(1-\beta) < b(1-\beta)\bar{z}$ and $a(\beta) > b(\beta)\bar{z}$.

Now the TM estimates of the parameters a and b in (6) are

$$\hat{a}_\beta \;=\; \bar{Y}' \equiv \sum_{i=0}^{N_\beta} w_i Y_i' \,, \tag{A7}$$

$$\hat{b}_\beta \;=\; \frac{\sum_{i=0}^{N_\beta} w_i\,(z_i' - \bar{z}_0')\,(Y_i' - \bar{Y}')}{\sum_{i=0}^{N_\beta} w_i\,(z_i' - \bar{z}_0')^2}\,, \tag{A8}$$

where w_i is the weight. In our case $w_0 = w_{\rm rel}/(N_\beta + w_{\rm rel})$ and $w_i = 1/(N_\beta + w_{\rm rel})$, $i > 0$.

According to Ruppert & Carroll (1980), these estimates have asymptotically gaussian distribution. Therefore an upper limit b_0 of $|b|$ at a significance level P_0 can be found from the equation

$$\frac{1}{\sqrt{2\pi}\hat\sigma(\hat b_\beta)} \int_{-b_0}^{b_0} \exp\left(- \left(x - \hat b_\beta\right)^2 / 2\hat\sigma^2(\hat b_\beta)\right) \mathrm{d}x = P_0 . \tag{A9}$$

The variances may be estimated as (Ruppert & Carroll 1980)

$$\hat\sigma^2(\hat a_\beta) = \frac{1}{(N+1)(1-2\beta)^2} \Big(Q_\beta(\hat a_\beta, \hat b_\beta)/(N-1) + \beta c_1^2 + \beta c_2^2 - \beta^2 (c_1 + c_2)^2 \Big) , \tag{A10}$$

$$\hat\sigma^2(\hat b_\beta) = \hat\sigma^2(\hat a_\beta) \left(\sum_{i=0}^{N_\beta} w_i' (z_i' - \bar z_0')^2 \right)^{-1} , \tag{A11}$$

where $c_1 = a(1-\beta) - \hat a_\beta$, $c_2 = a(\beta) - \hat a_\beta$, and the RSS is

$$Q_\beta(a,b) = (N_\beta + 1) \sum_{i=0}^{N_\beta} w_i \left(Y_i' - a - b(z_i' - \bar z)\right)^2 . \tag{A12}$$

In the particular case of $\beta = 0$ Eqs. (A7) - (A12) yield the standard LS estimates.

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