Pulsar Astronomy — 2000 and Beyond
ASP Conference Series, Vol. 3×10^8, *1999*
M. Kramer, N. Wex, and R. Wielebinski, eds.

Emission of particles and photons in the pulsar polar cap

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Abstract. We study emission of particles and photons from the pulsar polar cap. The Goldreich-Julian model for the regime of free emission of charged particles from the neutron star surface is used. In this case the electric field is generated due to the general relativistic effect of dragging of inertial frames. The spectra and shapes of gamma-ray pulses, the parameters of the electron-positron plasma and the intensity of X-ray emission from hot spots in the polar region of radio pulsars are discussed. The effect of non dipole field on X-ray emission of polar caps is considered. It is shown that increasing magnetic line curvature leads to much smaller temperatures and X-ray luminosities of polar caps as compared with pure dipole field.

1. Introduction

The electric fields of the rotating neutron stars (radio pulsars) with a dipole magnetic field (in the "free emission" regime of charged particles) is determined by the general relativistic effect of drag of inertial reference frames(Muslimov and Tsygan 1990, 1991, 1992; Beskin 1990). The general relativistic electrodynamical model for radio pulsars with the dipole magnetic field for the first time was presented by Muslimov and Tsygan on IAU Colloquium No.128 held in Lagow, Poland (1990). We generalized this model for the arbitrary axial symmetric magnetic field (Pal'shin and Tsygan, 1999).

Let us consider a rotating neutron star (NS) surrounded by a corotating magnetosphere, that was introduced in the physics of radio pulsars by Goldreich and Julian (1969). The calculations of Jones (1985), Neuhauser, Koonin and Langanke (1987), yield the work function of ions and electrons for NS surface is 200-300 eV (for magnetic field 10^{12} G). For the NS with the surface temperature $T > 2 \times 10^5$ K, the thermal emission of electrons and ions is important. It produced the "free emission" regime of charged particles.

2. The electrodynamical model

In the reference frame corotating with a NS the electrostatic potential in quasistatic region ($\Omega r \ll c$) of open field lines satisfies the Poisson equation in General Relativity (Muslimov & Tsygan 1990, 1991, 1992):

$$\nabla \cdot \left(\frac{1}{\alpha} \nabla \Phi \right) = -4\pi (\rho + \rho_{eff}) \tag{1}$$

$$\rho_{eff} = \frac{1}{4\pi}\nabla\cdot\left(\alpha\overrightarrow{g}\times\overrightarrow{B}\right) = \frac{\alpha\overrightarrow{B}}{4\pi}\nabla\times\overrightarrow{g}$$

$$\overrightarrow{g} = \frac{1}{c\alpha^2}\left[1-\kappa\left(\frac{a}{r}\right)^3\right]\overrightarrow{\Omega}\times\overrightarrow{r}$$

$$\overrightarrow{E} = -\nabla\Phi;\ \Phi\mid_s = 0;\ \left(\overrightarrow{E}\cdot\overrightarrow{B}\right)\mid_{r=a} = 0$$

Here ρ_{eff} - is the Goldreich-Julian charge density in General Relativity, ρ- is the charge density of the particles; $\alpha = \sqrt{1-r_g/r}$ is the gravitational redshift function, $r_g = 2GM/c^2$ is the gravitational radius of NS of mass M; $\kappa = (r_g/a)\cdot(I/Ma^2) = \varepsilon\cdot\beta$, $\varepsilon = r_g/a$ = 0.3-0.4 is the compactness parameter, a - is the stellar radius (more exact is the radial coordinate of the surface) and $\beta = I/Ma^2$ is the stellar moment of inertia in the units of Ma^2; that is κ =0.12-0.16. The differential operators is taken in three-dimensional curvilinear coordinates for Schwarzschild metric.

Let us consider the arbitrary axial symmetric magnetic field and the thin tube of open field lines. Within a small-angle approximation for the boundary condition at the surface and the tube bottom, $\Phi\mid_s = 0$, the electric field is determined by the local charge density $(\rho+\rho_{eff})$. For the "free emission" regime ($\overrightarrow{E}\cdot\overrightarrow{B}\mid_{\eta=1} = 0$) on the surface of the star will be valid (to within $\theta_0 \ll 1$) the condition:

$$(\rho+\rho_{eff})\mid_{\eta=1} = 0 \tag{2}$$

(for the dipole magnetic field it was found in the paper (MT, 1992). Where $\eta = r/a$ is a distance from the star in the units of stellar radius.

Within a small-angle approximation the equation (**??**) in a spherical coordinate system with the polar axis directed along the magnetic axis takes the form:

$$\frac{\alpha}{\eta^2}\frac{\partial}{\partial\eta}\left(\eta^2\frac{\partial\Phi}{\partial\eta}\right) + \frac{1}{\alpha\eta^2\vartheta}\frac{\partial}{\partial\vartheta}\left(\vartheta\frac{\partial\Phi}{\partial\vartheta}\right) = -4\pi a^2\left(\rho+\rho_{eff}\right) \tag{3}$$

$$\rho_{eff} \approx \frac{\overrightarrow{\Omega}\cdot\overrightarrow{B}}{2\pi\alpha c}\left(1-\frac{\kappa}{\eta^3}\right) \approx \frac{\Omega B\cos\chi}{2\pi\alpha c}\left(1-\frac{\kappa}{\eta^3}\right)$$

χ- is the angle between the magnetic and rotational axes. Taking into account the continuity equation $\nabla\cdot(\alpha\overrightarrow{j}) = 0$, (where $\overrightarrow{j} = \rho c\overrightarrow{B}/B$), and the Maxwell equation $\nabla\cdot\overrightarrow{B} = 0$ we obtain: $\overrightarrow{B}\cdot\nabla(\alpha\rho/B) = 0$. The magnetic field is near to radial and have to depend on only the angle . Using (**??**), one can obtain $\alpha\rho/B$ as a function of ϑ . The charge density of the relativistic particles approximate can be written as:

$$\rho \approx -\frac{\Omega B\cos\chi}{2\pi\alpha c}\left(1-\kappa\right) \tag{4}$$

$$\rho+\rho_{eff} \approx \kappa\frac{\Omega B\cos\chi}{2\pi\alpha c}\left(1-\frac{1}{\eta^3}\right)$$

Along all length the tube (except the region near the tube bottom) the first term on the left-hand side of the equation (**??**) can be neglected. In this case the solution of which satisfies the boundary conditions is:

$$\Phi\left(\eta,\xi\right)=\kappa\frac{1}{2}\left(\frac{\Omega a}{c}\right)a\eta^{2}\theta^{2}(\eta)B(\eta)\left(1-\frac{1}{\eta^{3}}\right)\left(1-\xi^{2}\right)\cos\chi \tag{5}$$

For $\cos\chi=0$ we have to use the results of Scharlemann, Arons and Fawley 1978; Arons and Scharlemann 1979 (or Muslimov & Tsygan 1990, 1991, 1992 for Schwarzschild metric).

The coordinate $\xi\equiv\vartheta/\theta(\eta)$ describes position of a point across the cone of open magnetic field lines $(0\leq\xi\leq1)$, where $\theta(\eta)$ - is the polar angle of the last open line. The solution (**??**) is valid for arbitrary magnetic field (in Schwarzschild metric) and was received in the work of Pal'shin and Tsygan (1999). The value $\theta(1)$ for $\cos\chi\neq1$ is determined by angular velocity of the star and the structure of magnetic field in the Schwarzschild metric. For the case $\cos\chi=1$, the angle $\theta(1)$ is determined by the energy density of particles accelerated by the stellar electric field. The value angle $\theta(1)$ may also depend on external conditions, for example external magnetic field. In the paper of Tsygan (1997) was shown that an external magnetic field, of strength at least comparable with that of the pulsar at the light cylinder, will modify the structure of the funnel of the open field lines.

Note the lucky circumstance that allow us to determine the function $B(\eta)\eta^{2}\theta^{2}(\eta)$. From the equation $\nabla\cdot\vec{B}=0$ it takes place the conservation of magnetic flux of the tube $\pi a^{2}\eta^{2}\theta^{2}(\eta)B(\eta)=F=const.$ The value of the flux may be obtained on the light radius were the field are determined of the magnetic moment of NS.

$$F=\int\limits_{(2\pi)}\vec{B}\mid_{r=r_L}\cdot\vec{n}\,dS=\frac{2\pi m\Omega}{c} \tag{6}$$

$$\eta^{2}\theta^{2}(\eta)B(\eta)=\frac{2m}{a^{3}}\left(\frac{\Omega a}{c}\right)=\theta_0^{2}B_0,$$

where $B_0=2m/a^{3}$ - is the "effective" dipole magnetic field on the pole of NS, and $\theta_0=\sqrt{\Omega a/c}$ - is the "effective" angle of the open magnetic field lines. Then the electrostatic potential can be written in the form:

$$\Phi\left(\eta,\xi\right)=\kappa\frac{1}{2}\left(\frac{\Omega a}{c}\right)a\theta_0^{2}B_0\left(1-\frac{1}{\eta^{3}}\right)\left(1-\xi^{2}\right)\cos\chi. \tag{7}$$

3. Gamma-ray and X-ray emission in the pulsar polar caps

The electrons emitted from the surface of the NS in the polar cap region are accelerated by the potential Φ along the magnetic field to the Lorentz factor $\gamma(\eta,\xi)=e\Phi(\eta,\xi)/mc^{2}=5\times10^{6}\div5\times10^{7}$. As they move along the curved magnetic field force lines, electrons emit gamma-quanta "curvature radiation", that create in magnetic field the electron-positron pairs. The electron-positron plasma screens out the longitudinal electric field at some altitude η_p , thereby producing a "gap" in which particles acceleration occurs (Sturrock 1971; Ruderman and Sutherland 1975).

In this case the expression (**??**) is valid (for $(\eta_p - 1) \gg \theta(\eta_p)$) in the region $(1+\theta(1)) < \eta < (\eta_p - \theta(\eta_p))$. In the region $1 < \eta < \eta_p$ the separation of charge particles in electron-positron plasma takes place. All positrons born in the "gap" region are accelerated toward the surface of the star, the secondary electrons - from the star. Concentration of the primary electrons also are changed. Let us estimate the charge $\rho_+(1)$ density of the returning positrons and the new charge density $\rho(1)$ of primary electrons from the compensation of longitudinal electric field on the both boundary of "gap":

$$\begin{aligned} \rho(1) + \rho_+(1) + \rho_{eff}(1) &= 0 \\ \rho(\eta_p) + \rho_-(\eta_p) + \rho_{eff}(\eta_p) &= 0 \end{aligned} \tag{8}$$

From the condition $\alpha\rho/B = const(\eta)$ follows: $\alpha(\eta_p)\rho(\eta_p)/B(\eta_p) = \alpha(1)\rho(1)/B(1)$. In addition the equality the currents of positrons for $\eta = 1$ and electrons for $\eta = \eta_p$ may be fulfilled $\rho_+(1)\alpha(1)\theta^2(1) = -\rho_-(\eta_p)\alpha(\eta_p)\eta_p^2\theta^2(\eta_p)$. The solution of (**??**) is:

$$\begin{aligned} \rho_+(1) &= \rho_{eff}(1)\frac{\kappa}{2(1-\kappa)}\left(1 - \frac{1}{\eta_p^3}\right) \\ \rho(1) &= -\rho_{eff}(1)\left[1 + \frac{\kappa}{2(1-\kappa)}\left(1 - \frac{1}{\eta_p^3}\right)\right] \end{aligned} \tag{9}$$

Further, for simplicity, we will take into account only linear terms on the parameter $\varepsilon = r_g/a$ (or κ). It is enough to use the expression (**??**) for Φ but take the calculations in the flat space-time. The pulsar energy loss rate because of the acceleration of charged particles on polar caps are:

$$L = 2c\rho(1)\theta^2(1)2\pi a^2 \int_0^1 \Phi(\eta_p, \xi)\xi d\xi = 3\frac{\kappa}{2}\left(1 - \frac{1}{\eta_p^3}\right) L_m \cos^2\chi, \tag{10}$$

where L_m is $(1/6)B_0^2 a^2 c(\Omega a/c)^4$.

The deceleration of positrons in the surface layer of the NS will eventually heat the surface. We can estimate the temperature of a point on the surface from the condition that the flux of warm X-ray emission be equal to the flux of positron energy: $\sigma T^4(\xi) = \rho_+(1)c\Phi(\eta_p, \xi)$. We neglect the heat flux directed inward toward the star. The X-ray luminosity of the hot spots is:

$$L_x \approx (\rho_+(1)/\rho(1))L = 3\left[\frac{\kappa}{2}\left(1 - \frac{1}{\eta_p^3}\right)\right]^2 L_m \cos^2\chi \tag{11}$$

For $\theta_0 = \sqrt{\Omega a/c} \ll 1$ as the potential Φ, the energy loss rate L as the X-ray luminosity L_X (for $1/\eta_p^3 \ll 1$) depend on only the dipole component of NS magnetic field. However, the magnetic anomalies in the gamma-ray emission, the temperature of the hot spots, the multiplication factor of the electron-positron plasma, the value of η_p are displayed .

We assume that, for the generation of pulsar radio emission, the secondary plasma-multiplication factor must be not less than $(\kappa/2)(1-1/\eta_p^3)$ (for producing a "gap").

Let us present the results of numerical calculations the height of the "gap" ($z_p = \eta_p - 1$) and the temperature as function ξ for the purely dipole magnetic field (Fig. 1), and also the X-ray luminosity from pulsar polar caps(Fig. 3). Fig. 2 shows the shape and the spectrum of gamma-pulse for PSR 1706-44 (also for the purely dipole field). The results for non dipole field is shown on Fig. 4. We used the model, in which the magnetic field represents as sum of dipole field and additional magnetic moment $\overrightarrow{m}_1 \perp \overrightarrow{m}$, that placed at the depth of $0.1a$ from the surface in polar region of NS (Pal'shin & Tsygan, 1998). The parameter α equal the ratio of the magnetic fields of dipoles $\overrightarrow{m_1}$ $\overrightarrow{m}$.

The results of our calculations (Pal'shin & Tsygan, 1998, 1999) are slightly different from the results of Pal'shin & Tsygan, 1997, Muslimov & Harding, 1997 and Harding & Muslimov, 1998 and can explain the observed intensity of X-ray emission from the polar caps of radio pulsars.

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