

# PUZZLE OF THE CONSTANCY OF FUNDAMENTAL CONSTANTS

D. A. Varshalovich, A. Y. Potekhin, and A. V. Ivanchik  
*Ioffe Physical-Technical Institute, 194021 St. Petersburg, Russia*

## Abstract

We discuss experiments and observations aimed at testing the possible space-time variability of fundamental physical constants, predicted by the modern theory. Specifically, we consider two of the dimensionless physical parameters which are important for atomic and molecular physics: the fine-structure constant and the electron-to-proton mass ratio. We review the current status of such experiments and critically analyze recent claims of a detection of the variability of the fine-structure constant on the cosmological time scale. We stress that such a detection remains to be checked by future experiments and observations. The tightest of the firmly established upper limits read that the considered constants could not vary by more than 0.015% on the scale  $\sim 10^{10}$  years.

**Key words:** fundamental constants; cosmology

**PACS numbers:** 06.20.Jr, 06.30.-k, 98.80.Es

## 1 Introduction

Possible variability of fundamental physical constants is one of the topical problems of contemporary physics. The modern theory (Supersymmetric Grand Unification Theory – SUSY GUT, Superstring models, etc.) has established that the coupling constant values which characterize different kinds of interactions (i) are “running” with the energy transfer and (ii) may be different in different regions of the Universe and vary in the course of cosmological evolution (e.g., Ref. [1]). The energy dependence of the coupling parameters has been reliably confirmed by high-energy experiments (see, e.g., Ref. [2]), whereas the space-time variability of their low-energy limits so far escapes detection.

Note that a numerical value of any dimensional physical parameter depends on arbitrary choice of physical units. In turn, there is no way to determine the units in a remote space-time region other than through the fundamental constants. Therefore it is meaningless to speak of a variation of a dimensional physical constant without specifying which of the other physical parameters are *defined* to be invariable. Usually, while speaking of variability of a dimensional physical parameter, one *implies* that *all* the other fundamental constants are fixed. So did Milne [3] and Dirac [4] in their pioneering papers devoted to a possible change of the gravitational constant  $G$ . More recently, a number of authors considered cosmological theories with a time varying speed of light  $c$  (e.g., Ref. [5]). However, if we adopt the standard definition of meter [6] as the length of path traveled by light in vacuum in  $1/299\,792\,458$  s, then  $c = 2.997\,924\,58 \times 10^{10}$  cm s $^{-1}$  identically. Similarly, one cannot speak of variability of the electron mass  $m_e$  or charge  $e$  while using the Hartree units ( $\hbar = e = m_e = 1$ ), most natural in atomic physics. Thus, only *dimensionless* combinations of the physical parameters are truly fundamental, and only such combinations will be considered hereafter.

At present, the most promising candidate for the theory which is able to unify gravity with all other interactions is the Superstring theory, which treats gravity in a way consistent with quantum mechanics. All versions of the theory predict existence of the dilaton – a scalar partner to the tensorial graviton. Since the dilaton field  $\phi$  is generally not constant, the coupling constants and masses of elementary particles, being dependent on  $\phi$ , should vary in space and time. Thus, the existence of a weakly coupled massless

dilaton entails small, but non-zero, observable consequences such as Jordan–Brans–Dicke-type deviations from General Relativity and cosmological variations of the gauge coupling constants [7]. These variations depend on cosmological evolution of the dilaton field and may be non-monotonous as well as different in different space-time regions.

In this paper, we focus on the space-time variability of the low-energy limits of two fundamental constants which are of paramount importance for atomic and molecular spectroscopy:

- (i) the fine-structure constant  $\alpha = e^2/\hbar c$  (Sommerfeld parameter),
- (ii) the electron-to-proton mass ratio  $\mu = m_e/m_p$  (Born–Oppenheimer parameter).

The next section presents a compendium of the basic methods allowing one to obtain restrictions on possible variations of fundamental constants. In Sects. 3 and 4, we consider recent estimates of the values of  $\alpha$  and  $\mu$ , respectively, at cosmological redshifts  $z = 1\text{--}4$  which correspond to epochs  $\sim 7\text{--}13$  billion years ago. Conclusions are given in Sect. 5.

## 2 Tests of variability of fundamental constants

Techniques used to investigate time variation of the fundamental constants may be divided into extragalactic and local methods. The latter ones include astronomical methods related to the Galaxy and the Solar system, geophysical methods, and laboratory measurements.

### 2.1 Local tests

#### 2.1.1 Laboratory measurements

Laboratory tests are based on comparison of different frequency standards, depending on different combinations of the fundamental constants. Were these combinations changing differently, the frequency standards would eventually discord with each other. An interest in this possibility has been repeatedly excited since relative frequency drift was observed by several research groups using long term comparisons of different frequency standards. For instance, a comparison of frequencies of He-Ne/CH<sub>4</sub> lasers, H masers, and Hg<sup>+</sup> clocks with a Cs standard [8, 9, 10, 11] has revealed relative drifts. Since the considered frequency standards have a different dependence on  $\alpha$  via relativistic contributions of order  $\alpha^2$ , the observed drift might be attributed to changing of the fine-structure constant. However, the more modern was the experiment, the smaller was the drift. Taking into account that the drift may be also related to some aging processes in experimental equipment, Prestage et al. [11] concluded that the current laboratory data provide only an upper limit  $|\dot{\alpha}/\alpha| \leq 3.7 \times 10^{-14} \text{ yr}^{-1}$ .

#### 2.1.2 Analysis of the Oklo phenomenon

The most stringent limits to variation of the fine-structure constant  $\alpha$  and the coupling constant of the strong interaction  $\alpha_s$  have been originally inferred by Shlyakhter [12] from results of an analysis of the isotope ratio  $^{149}\text{Sm}/^{147}\text{Sm}$  in the ore body of the Oklo site in Gabon, West Africa. This ratio turned out to be considerably lower than the standard one, which is believed to have occurred due to operation of the natural uranium fission reactor about  $2 \times 10^9$  yr ago in those ores. One of the nuclear reactions in the fission chain was the resonance capture of neutrons by  $^{149}\text{Sm}$  nuclei. Actually, the rate of the neutron capture reaction is sensitive to the energy of the relevant nuclear resonance level  $E_r$ , which depends on the strong and electromagnetic interaction. Since the capture has been efficient  $2 \times 10^9$  yr ago, it means that the position of the resonance has not shifted by more than its width (very narrow) during the elapsed time. At variable  $\alpha$  and invariable  $\alpha_s$  (which is just a model assumption), the shift of the resonance level would be determined by changing the difference between the Coulomb energies of the ground-state nucleus  $^{149}\text{Sm}$  and the nucleus  $^{150}\text{Sm}^*$  excited to the level  $E_r$ . Unfortunately, there is no experimental data for the Coulomb energy of the excited  $^{150}\text{Sm}^*$  in question. Using order-of-magnitude estimates, Shlyakhter [12] concluded that  $|\dot{\alpha}/\alpha| \lesssim 10^{-17} \text{ yr}^{-1}$ . From an opposite model assumption that  $\alpha_s$  is changing whereas  $\alpha = \text{constant}$ , he derived a bound  $|\dot{\alpha}_s/\alpha_s| \lesssim 10^{-19} \text{ yr}^{-1}$ .

Damour and Dyson [13] performed a more careful analysis, which resulted in the upper bound  $|\dot{\alpha}/\alpha| \lesssim 7 \times 10^{-17} \text{ yr}^{-1}$ . They have assumed that the Coulomb energy difference between the nuclear states of  $^{149}\text{Sm}$  and  $^{150}\text{Sm}^*$  in question is not less than that between the *ground* states of  $^{149}\text{Sm}$  and  $^{150}\text{Sm}$ . The latter energy difference has been estimated from isotope shifts and equals  $\approx 1 \text{ MeV}$ . However, it looks unnatural that a weakly bound neutron ( $\approx 0.1 \text{ eV}$ ), captured by a  $^{149}\text{Sm}$  nucleus to form the highly excited state  $^{150}\text{Sm}^*$ , can so strongly affect the Coulomb energy. Moreover, heavy excited nuclei often have Coulomb energies smaller than those for their ground states (e.g., Ref. [14]). This indicates the possibility of violation of the basic assumption involved in Ref. [13], and therefore this method may possess a lower actual sensitivity. Furthermore, a correlation between  $\alpha$  and  $\alpha_s$  (which is likely in the frame of modern theory) might lead to considerable softening of the above-mentioned bound, as estimated by Sisterna and Vucetich [15].

### 2.1.3 Some other local tests

Geophysical, geochemical, and paleontological data impose constraints on a possible changing of various combinations of fundamental constants over the past history of the Solar system, however most of these constraints are very indirect. A number of other methods are based on stellar and planetary models. The radii of the planets and stars and the reaction rates in them are influenced by values of the fundamental constants, which offers a possibility to check variability of the constants by studying, for example, lunar and Earth's secular accelerations. This was done using satellite data, tidal records, and ancient eclipses. Another possibility is offered by analyzing the data on binary pulsars and the luminosity of faint stars. Most of these have relatively low sensitivity. Their common weak point is the dependence on a model of a fairly complex phenomenon, involving many physical effects.

An analysis of natural long-lived  $\alpha$ - and  $\beta$ -decayers in geological minerals and meteorites is much more sensitive. For instance, a strong bound,  $|\dot{\alpha}/\alpha| < 5 \times 10^{-15} \text{ yr}^{-1}$ , was obtained by Dyson [16] from an isotopic analysis of natural  $\alpha$ - and  $\beta$ -decay products in Earth's ores and meteorites.

Having critically reviewed the wealth of the local tests, taking into account possible correlated synchronous changes of different physical constants, Sisterna and Vucetich [15] derived restrictions on possible variation rates of individual physical constants for ages  $t$  less than a few billion years ago, which correspond to cosmological redshifts  $z \lesssim 0.2$ . In particular, they have arrived at the estimate  $\dot{\alpha}/\alpha = (-1.3 \pm 6.5) \times 10^{-16} \text{ yr}^{-1}$ .

All the local methods listed above give estimates for only a narrow space-time region around the Solar system. For example, the epoch of the Oklo reactor ( $1.8 \times 10^9$  years ago) corresponds to the cosmological redshift  $z \approx 0.1$ . These tests cannot be extended to earlier evolutionary stages of the Universe, because the possible variation of the fundamental constants is, in general, unknown and may be oscillating [17, 7]. Another investigation is needed for higher cosmological redshifts.

## 2.2 Extragalactic tests

Extragalactic tests, in contrast to the local ones, concern values of the fundamental constants in distant areas of the early Universe. A test which relates to the earliest epoch is based on the standard model of the primordial nucleosynthesis. The amount of  $^4\text{He}$  produced in the Big Bang is mainly determined by the neutron-to-proton number ratio at the freezing-out of  $n \leftrightarrow p$  reactions. The freezing-out temperature  $T_f$  is determined by the competition between the expansion rate of the Universe and the  $\beta$ -decay rate. A comparison of the observed primordial helium mass fraction,  $Y_p = 0.24 \pm 0.01$ , with a theoretical value allows one to obtain restrictions on the difference between the neutron and proton masses at the epoch of the nucleosynthesis and, through it, to estimate relative variation of the curvature radius  $R$  of extra dimensions in multidimensional Kaluza–Klein-like theories which in turn is related to the  $\alpha$  value [18, 19]. However, as noted above, different coupling constants might change simultaneously. For example, increasing the constant of the weak interactions  $G_F$  would cause a weak freezing-out at a lower temperature, hence a decrease in the primordial  $^4\text{He}$  abundance. This process would compete with the one described above, therefore, it reduces sensitivity of the estimates. Finally, the restrictions would

be different for different cosmological models since the expansion rate of the Universe depends on the cosmological constant  $\Lambda$ .

The most unambiguous estimation of the atomic and molecular constants at early epochs and in distant regions of the Universe can be performed using the extragalactic spectroscopy. Accurate measurements of the wavelengths in spectra of distant objects provide quantitative constraints on the variation rates of the physical constants. This opportunity has been first noted and used by Savedoff [20], and in recent years exploited by many researchers (see, e.g., Refs. [21, 22] and references therein). At present, the extragalactic spectroscopy enables one to probe the physical conditions in the Universe up to cosmological redshifts  $z \lesssim 4$ , which correspond, by order of magnitude, to the scales  $\sim 10^{10}$  yr in time and  $\sim 10^9$  parsec in space. In the following sections, we review briefly the studies of the space-time variability of the fine-structure constant  $\alpha$  and the electron-to-proton mass ratio  $\mu$ , based on the latter method.

### 3 Non-variability of $\alpha$

We have already mentioned in Sect. 2.1.1 that several laboratory tests hinted at a tentative time variation of  $\alpha$ , but were later refuted by measurements at a higher level of accuracy. A similar situation has occurred for extragalactic tests at larger space-time scales.

Bahcall and Schmidt [23] were the first to use spectral observations of distant quasars to set a bound on the variability of the fine-structure constant. They have obtained an estimate  $\Delta\alpha/\alpha = (-2 \pm 5) \times 10^{-2}$  at  $z = 1.95$ . Later statistical analyses [24, 25] of fine-structure doublet lines in quasar spectra appeared to indicate a tentative variation of  $\alpha$  (of the order of  $\sim 0.3\%$  at the cosmological redshift  $z \sim 2$ ). However, this tentative variation has been shown to result from a statistical bias [26].

Another statistical examination of the fine-doublet wavelengths of absorption lines in quasar spectra [21] indicated a tentative (at the  $2-3\sigma$  level) variability of  $\alpha$  values by  $\sim 0.1\%$  over the celestial sphere (as function of angle) at redshifts  $z \sim 2-3$ . However, this result has not been confirmed by a later analysis [22], which was based on higher-quality spectra and yielded an order of magnitude higher precision.

Quite recently, Webb et al. [27] have estimated  $\alpha$  by comparing wavelengths of Fe II and Mg II fine-splitting spectral lines in extragalactic spectra and in the laboratory. Their result suggests a time-variation of  $\alpha$  at the incredibly high accuracy level of  $\sim 10^{-3}\%$ : the authors' estimate reads  $\Delta\alpha/\alpha = (-1.9 \pm 0.5) \times 10^{-5}$  at  $z = 1.0-1.6$ . Note, however, two important sources of a possible systematic error which could mimic the effect: (a) Fe II and Mg II lines used are situated in different orders of the echelle-spectra, so relative shifts in calibration of the different orders can affect the result of comparison, and (b) if the isotopic composition varies during the evolution of the Universe, then the average doublet separations should vary due to the isotopic shifts. Were the relative abundances of Mg isotopes changing during the cosmological evolution, the Mg II lines would be subject to an additional  $z$ -dependent shift relative to the Fe II lines, quite sufficient to simulate the variation of  $\alpha$  (this shift can be easily estimated from recent laboratory measurements [28]).

The method based on the fine splitting of a line of the same ion species is not affected by these two uncertainty sources. We have studied the fine splitting of the doublet lines of Si IV, C IV, Mg II and other ions, observed in spectra of distant quasars. According to quantum electrodynamics, the relative splitting of these lines  $\delta\lambda/\lambda$  is proportional to  $\alpha^2$  (neglecting small relativistic corrections, recently estimated by Dzuba et al. [29]). We have selected the results of high-resolution observations [30, 31, 32], most suitable for an analysis of the variation of  $\alpha$ . According to our analysis, presented elsewhere [33], the most reliable estimate of the possible deviation of the fine-structure constant at  $z = 2-4$  from its present ( $z = 0$ ) value:

$$\Delta\alpha/\alpha = (-4.6 \pm 4.3 [\text{stat}] \pm 1.4 [\text{syst}]) \times 10^{-5}. \quad (1)$$

Thus, only an upper bound can be derived at present for the long-term variability of  $\alpha$ :

$$|\dot{\alpha}/\alpha| < 1.4 \times 10^{-14} \text{ yr}^{-1} \quad (2)$$

(at the 95% confidence level).

## 4 Non-variability of $\mu$

The dimensionless Born–Oppenheimer constant  $\mu = m_e/m_p$  approximately equals the ratio of the constant of electromagnetic interaction  $\alpha = e^2/\hbar c \approx 1/137$  to the constant of strong interaction  $\alpha_s = g^2/\hbar c \sim 14$ , where  $g$  is the effective coupling constant calculated from the amplitude of  $\pi$ -meson–nucleon scattering at low energy.

An early limit on the possible variation of this constant,  $|\dot{\mu}/\mu| < 1.2 \times 10^{-10} \text{ yr}^{-1}$ , has been derived from the concordance of K–Ar and Rb–Sr geochemical ages [34]. The first astrophysical bound [35], based on the agreement between redshifts of atomic hydrogen and other lines in quasar absorption spectra, turned out to be twice stronger.

Orders-of-magnitude more precise analysis has become possible due to discovery [36] of a system of  $\text{H}_2$  absorption lines in the spectrum of quasar PKS 0528–250 at  $z = 2.811$ . A study of this system yields information about physical conditions and, in particular, the value of  $\mu$  at this redshift (corresponding to the epoch when the Universe was several times younger than now). A possibility of distinguishing between the cosmological redshift of spectral wavelengths and shifts due to a variation of  $\mu$  arises from the fact that the electronic, vibrational, and rotational energies of  $\text{H}_2$  each undergo a different dependence on the reduced mass of the molecule. Hence comparing ratios of wavelengths  $\lambda_i$  of various  $\text{H}_2$  electron–vibration–rotational lines in a quasar spectrum at some redshift  $z$  and in laboratory (at  $z = 0$ ), we can trace variation of  $\mu$ . We have calculated [21, 37] sensitivity coefficients  $K_i$  of the wavelengths  $\lambda_i$  with respect to possible variation of  $\mu$  and applied a linear regression analysis to the measured redshifts of individual lines  $z_i$  as function of  $K_i$ . If the proton mass in the epoch of line formation were different from the present value, the measured  $z_i$  and  $K_i$  values would correlate:

$$\frac{z_i}{z_k} = \frac{(\lambda_i/\lambda_k)_z}{(\lambda_i/\lambda_k)_0} \simeq 1 + (K_i - K_k) \left( \frac{\Delta\mu}{\mu} \right). \quad (3)$$

We have performed a  $z$ -to- $K$  regression analysis using a modern high-resolution spectrum of PKS 0528–250. Eighty-two of the  $\text{H}_2$  lines have been identified. The resulting parameter estimate and  $1\sigma$  uncertainty is

$$\Delta\mu/\mu = (-11.5 \pm 7.6 [\text{stat}] \pm 1.9 [\text{syst}]) \times 10^{-5}. \quad (4)$$

The  $2\sigma$  confidence bound on  $\Delta\mu/\mu$  reads

$$|\Delta\mu/\mu| < 2.0 \times 10^{-4}. \quad (5)$$

Assuming that the age of the Universe is  $\sim 1.5 \times 10^{10} \text{ yr}$  the redshift of the  $\text{H}_2$  absorption system  $z = 2.81080$  corresponds to the elapsed time  $\approx 1.3 \times 10^{10} \text{ yr}$  (in the standard cosmological model). Therefore we arrive at the restriction

$$|\dot{\mu}/\mu| < 1.5 \times 10^{-14} \text{ yr}^{-1} \quad (6)$$

on the variation rate of  $\mu$ , averaged over 90% of the lifetime of the Universe.

## 5 Conclusions

Despite the theoretical prediction that fundamental constants of Nature should vary, no statistically significant variation of any of the constants has been reliably detected up to date, according to our point of view substantiated above. The upper limits obtained indicate that the constants of electroweak and strong interactions did not significantly change over the last 90% of the history of the Universe. The striking tightness of these limits is really astonishing and has already ruled out some theoretical models (see Refs. [21, 22, 35]). A more elaborated theory (e.g., Ref. [7]) cannot be ruled out yet, but its parameters can be severely restricted (e.g., see Ref. [22]). This shows that more precise measurements and observations and their accurate statistical analyses are required in order to detect the expected variations of the fundamental constants.

**Acknowledgments.** This work was performed in frames of the Russian State Program “Fundamental Metrology” and supported by the grant RFBR 99-02-18232.

## References

- [1] L.B. Okun, Usp. Fiz. Nauk **161** (Physics–Uspekhi **34**), 177 (1991)
- [2] L.B. Okun, Physics–Uspekhi **41**, 553 (1998)
- [3] E. Milne, Proc. R. Soc. London **A158**, 324 (1937)
- [4] P.A.M. Dirac, Nature, **139**, 323 (1937); Proc. R. Soc. London **A165**, 199 (1938)
- [5] A. Albrecht, J. Mageijo, Phys. Rev. D **59**, 043516 (1999)
- [6] B.W. Petley, Nature **303**, 373 (1983)
- [7] T. Damour, A.M. Polyakov, Nucl. Phys. B **423**, 532 (1994)
- [8] Yu.S. Domnin, A.N. Malimon, V.M. Tatarenkov, P.S. Schumyanskii, Pisma v Zhurn. Eksp. i Teor. Fiz. (Sov. Phys. JETP Lett.), **43**, 167 (1986)
- [9] N.A. Demidov, E.M. Ezhov, B.A. Sakharov, B.A. Uljanov, A. Bauch, B. Fisher, in *Proc. of 6th European Frequency and Time Forum* (European Space Agency, 1992) 409
- [10] L.A. Breakiron, in *Proc. of the 25th Annual Precise Time Interval Applications and Planning Meeting*, NASA Conference Publication No. 3267, 401 (1993)
- [11] J.D. Prestage, R.L. Tjoelker, L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995)
- [12] A.I. Shlyakhter, Nature **25**, 340 (1976)
- [13] T. Damour, F.J. Dyson, Nucl. Phys. B **480**, 37 (1996)
- [14] G.M. Kalvius, G.K. Shenoy, Atomic and Nuclear Data Tables, **14**, 639 (1974)
- [15] P.D. Sisterna, H. Vucetich, Phys. Rev. D **41**, 1034 (1990)
- [16] F.J. Dyson, in *Aspects of Quantum Theory*, eds. A. Salam, E.P. Wigner, (Cambridge Univ. Press, Cambridge 1972) 213
- [17] W.J. Marciano, Phys. Rev. Lett. **52**, 489 (1984)
- [18] E.W. Kolb, M.J. Perry, T.P. Walker, Phys. Rev. D **33**, 869 (1986)
- [19] J.D. Barrow, Phys. Rev. D **35**, 1805 (1987)
- [20] M.P. Savedoff, Nature **178**, 3511 (1956)
- [21] D.A. Varshalovich, A.Y. Potekhin, Space Sci. Rev. **74**, 259 (1995)
- [22] A.V. Ivanchik, A.Y. Potekhin, D.A. Varshalovich, Astron. Astrophys. **343**, 439 (1999)
- [23] J.N. Bahcall, M. Schmidt, Phys. Rev. Lett. **19**, 1294 (1967)
- [24] S.A. Levshakov, ESO Workshop & Conf. Proc. **40**, 139 (1992)
- [25] S.A. Levshakov, Vistas in Astronomy **37**, 535 (1993)
- [26] A.Y. Potekhin, D.A. Varshalovich, Astron. Astrophys. Suppl. Ser. **104**, 89 (1994)
- [27] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999)
- [28] J.C. Pickering, A.P. Thorne, J.K. Webb, Mon. Not. Roy. Astron. Soc. **300**, 131 (1998)
- [29] V.A. Dzuba, V.V. Flambaum, J.K. Webb, Phys. Rev. A (submitted); e-print: physics/9808021 (1999)
- [30] P. Petitjean, M. Rauch, R.F. Carswell, Astron. Astrophys. **291**, 29 (1994)
- [31] D.A. Varshalovich, V.E. Panchuk, A.V. Ivanchik, Astron. Lett. **22**, 6
- [32] P.J. Outram, B.J. Boyle, R.F. Carswell, P.C. Hewett, R.E. Williams, Mon. Not. Roy. Astron. Soc. **305**, 685 (1999); <http://www.ast.cam.ac.uk/AAO/hdfs/>
- [33] D.A. Varshalovich, A.Y. Potekhin, A.V. Ivanchik, in *Proc. 18th Int. Conf. on X-ray and Inner-Shell Processes “X-99”* (AIP Press, Chicago 2000) in press
- [34] A. Yahil, in *The Interaction between Science and Philosophy*, ed. Y. Elkana (Humanities Press, 1975) 27
- [35] B. Pagel, Mon. Not. Roy. Astron. Soc. **179**, 81 (1977)
- [36] S.A. Levshakov, D.A. Varshalovich, Mon. Not. Roy. Astron. Soc. **212**, 517 (1985)
- [37] A.Y. Potekhin, A.V. Ivanchik, D.A. Varshalovich, Lanzetta K.M., Baldwin J.A., Williger G.M., Carswell R.F., Astrophys. J. **505**, 523 (1998)