COSMOLOGICAL VARIABILITY OF FUNDAMENTAL PHYSICAL CONSTANTS

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Abstract. Gamow was one of the pioneers who studied the possible variability of fundamental physical constants. Some versions of modern Grand Unification theories do predict such variability. The paper is concerned with three of the constants: the fine-structure constant α, the ratio of the proton mass m_p to the electron mass m_e, and the ratio of the neutron mass m_n to m_e. It is shown on the basis of the quasar spectra analysis, that all the three constants revealed no statistically significant variation over the last 90% of the life time of the Universe. At the 2σ significance level, the following upper bounds are obtained for the epoch corresponding to the cosmological redshifts $z \sim 2-3$: $\Delta\alpha/\alpha < 1.5 \times 10^{-3}$, $\Delta m_p/m_p < 2 \times 10^{-3}$, and $\Delta m/m < 3 \times 10^{-4}$, where Δx is a possible deviation of a quantity x from its present value, $m = m_p + m_n$, and the nucleon masses are in units of m_e. (According to new observational data which became known most recently, $\Delta m_p/m_p < 2 \times 10^{-4}$). In addition a possible anisotropy of the high-redshift fine splitting over the celestial sphere is checked. Within the relative statistical error $3\sigma < 1\%$ the values of α turned out to be the same in various quadrants of the celestial sphere, which corresponds to their equality in causally disconnected areas. However, at the 2σ level a tentative anisotropy of estimated $\Delta\alpha/\alpha$ values is found in directions that approximately coincide with the direction of the relic microwave background anisotropy.

The revealed constraints serve as criteria for selection of those theoretical models which predict variation of α, m_p or m_n with the cosmological time.

Key words: Cosmology – Quasar Spectra – Physical Constants

1. Historical Background

The problem of a possible time variation of fundamental physical constants has been put forward by Milne (1937) who considered a hypothetical variation of the gravitational constant G proportional to the cosmological time t. Simultaneously Dirac (1937) formulated his famous "large number hypothesis" and supposed that G is inversely proportional to t. Following Dirac arguments, Teller (1948) supposed that the fine-structure constant α varies as $(\ln t)^{-1}$. Dyson (1972) has pointed out that this form of α dependence could be supported by some considerations following from the renormalization rules in quantum electrodynamics.

In the 1960s it became clear that the original Dirac's idea leads to too high brightness of the Sun in the past, which contradicted to paleontological data. An attempt to save the idea was made by George Gamow (1967), who supposed that G is constant, but α is directly proportional to t (see Alpher (1973) and Chechev and Kramarovsky (1978) for a review).

The Gamow paper got an immediate reply from Bahcall *et al.* (1967) and Bahcall and Schmidt (1967), who pointed out that the proposed dependence

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$\alpha \propto t$ is definitely ruled out by spectral observations of quasi-stellar objects (quasars). Using the method of Savedoff (1956), they compared the observed redshifts z of the components of fine-structure doublets in spectra of distant quasars, and derived the estimates $\Delta\alpha/\alpha = (-2 \pm 5) \times 10^{-2}$ at $z = 1.95$ and $\Delta\alpha/\alpha = (-1 \pm 2) \times 10^{-3}$ at $z = 0.2$. Since the cosmological time t is proportional to $(1 + z)^{-a}$, where the power index a ranges most probably between 1 and 1.5, the Gamow law of α variation would lead to $\Delta\alpha/\alpha \sim 1$ for such redshifts, at clear variance to the observations.

It was the first example of the usage of quasar spectra for setting bounds on a possible variation rate of a fundamental physical constant. Later a similar bound, $|\Delta\alpha/\alpha| < 0.04$ at $z = 0.524$, was obtained by Wolfe *et al.* (1976). Meanwhile, more and more *local tests* appeared, which used geophysical, geochemical and paleontological data to set bounds on possible variation rates of certain combinations of fundamental constants. A variety of such tests has been analysed by Sisterna and Vucetich (1990), who took into account the possibility of simultaneous variations of different constant, entering the bounded combinations, and derived the most confident constraints on possible variation rates of the individual constants. However, all the local tests inevitably remain restricted to the space-time region of the Solar system, and cannot provide information about the behavior of the constants for redshifts $z > 0.2$.

The interest in the problem greatly increased during 1980s, thanks to new developments in the Kaluza–Klein and supergravity models of unification of all the physical interactions. The fundamental constants appear in these theories as a manifestation of the "internal space" scale-lengths in the extra dimensions, additional to the usual four space-time ones. Chodos and Detweiler (1980) obtained from the Kaluza–Klein theory the $\alpha/G \propto t$ law, which however, has already been shown to be invalid. Freund (1982) found that some modifications of the theory may lead to different variation laws. As an example, he derived an $\alpha/G \propto t^{-1/4}$ dependence. Marciano (1984) speculated about various dependences which could follow from different versions of the Kaluza–Klein and supergravity theories. In particular, he pointed out that such dependences might be non-monotonous. Wu and Wang (1986) investigated a version of the supergravity theory that led to $\dot{G}/G \sim -10^{11\pm1}$ yr^{-1} and $\dot{\alpha}/\alpha \propto \dot{G}/G$, where the dot denotes the time derivative, and the proportionality coefficient is unknown. As an alternative, they proposed a version of the theory with no observable variations of the fundamental constants. The paper of Wu and Wang (1986) has been critically analysed by Maeda (1988), who also found two possibilities: either $\alpha \propto t^{-4/3}$, or no observable variation.

This brief review makes it obvious that bounds on possible time-dependences of fundamental constants may serve as an important tool for checking the validity of different theoretical models of the Grand Unification.

Moreover, since the law of the supposed variation is generally unknown, it is worthwhile to obtain independent constraints for different cosmological epochs, corresponding to different cosmological redshifts z. Constraints for $z \sim 2$ would essentially supplement those obtained from the local tests at $z < 0.2$. In the present paper we use quasar spectroscopic data to derive the strongest up-to-date restrictions for $z \approx 2-3$ on $\Delta\alpha/\alpha$, $\Delta m_p/m_p$, and $\Delta m/m$, where Δx is a possible deviation of a quantity x from its present value, m_p and m_n are the proton and the neutron masses (in the Hartree system, i.e. in units of the electron mass), and $m = m_p + m_n$. In addition to the temporal dependence, the rich observational material on fine-structure doublets in quasar spectra enables us to study a possible spatial dependence of α in distant regions of the Universe.

2. Proton Mass at the Redshift $z = 2.811$

Electronic, vibrational and rotational energies of a molecule depend in different ways on its reduced mass M. In the first approximation, they are proportional to M^0, $M^{-1/2}$ and M^{-1}, respectively. Therefore, having compared ratios of wavelengths λ_i, λ_k of various electron-vibration-rotational lines in a quasar spectrum at some redshift z and in the laboratory (at $z = 0$), one may reveal a variation of M. This possibility has been used previously by Foltz *et al.* (1988) and Varshalovich and Levshakov (1993). Let us define the quantity

$$\eta_i = 1 - (\lambda_i)_0(1+\bar{z})(\lambda_i)_z^{-1}, \tag{1}$$

where $\bar{z}$ is the mean observed redshift of the wavelengths $(\lambda_i)_z$. Then, to an accuracy of higher orders in $\Delta M/M$,

$$\eta_i = \frac{\delta z}{1+z} + K_i \frac{\Delta M}{M}. \tag{2}$$

Here $\Delta M = M_z - M_0$ is the difference between the reduced mass in the epoch of a redshift z and its contemporary value, δz is a correction to $\bar{z}$ due to this difference, and $K_i = \partial \ln \lambda_i / \partial \ln M$ are the sensitivity coefficients of the wavelengths to M variation. For the H_2 molecule $M \propto m_p$. Therefore, if the proton mass for a redshift z were distinct from the contemporary value, then the measured η_i and K_i values should be linearly correlated.

The coefficients K_i were calculated previously by Varshalovich and Levshakov (1993) from the spectroscopic constants of the H_2 molecule, using theoretical considerations concerning M-dependences of these constants. In the present paper we use another way of calculating K_i, free of uncertainties connected with these theoretical considerations. For each of 18 unblended H_2 lines identified by Foltz *et al.* (1988) in the PKS 0528−250 spectrum, the coefficient K_i was determined by comparing the H_2 laboratory wavelength with the corresponding wavelengths for D_2 and T_2 molecules. The laboratory wavelengths were calculated using the spectroscopic constants presented by Huber and Herzberg (1979). The data on the HD wavelengths were used to additionally control the accuracy, which was shown to be satisfactory.

The wavelengths observed by Foltz *et al.* (1988) and the laboratory values are given in Table I together with the calculated sensitivities K_i.

Table 1

i	$(\lambda_i)_z$ (Å)	Transition	$\lambda_i(H_2)$ (Å)	$\lambda_i(D_2)$ (Å)	$\lambda_i(T_2)$ (Å)	K_i ($\times 10^{-3}$)
1	4225.06	L(0–0)R(1)	1108.633	1103.351	1101.021	–7.19
2	4230.50	L(0–0)P(1)	1110.062	1104.071	1101.501	–8.44
3	4162.77	L(1–0)R(0)	1092.194	1091.765	1091.565	–0.55
4	4164.48	L(1–0)R(1)	1092.732	1092.026	1091.736	–1.07
5	4169.99	L(1–0)P(1)	1094.051	1092.704	1092.195	–2.21
6	4105.15	L(2–0)R(0)	1077.137	1080.882	1082.584	5.08
7	4107.32	L(2–0)R(1)	1077.698	1081.153	1082.760	4.53
8	4121.28	L(2–0)R(3)	1081.265	1082.978	1083.993	1.33
9	4057.10	L(3–0)P(1)	1064.605	1071.312	1074.498	8.65
10	3998.92	L(4–0)R(0)	1049.367	1060.360	1065.482	14.9
11	4001.53	L(4–0)R(1)	1049.959	1060.645	1065.667	14.3
12	4005.43	L(4–0)P(1)	1051.032	1061.227	1066.072	13.4
13	4007.03	L(4–0)R(2)	1051.498	1061.407	1066.169	12.8
14	4014.04	L(4–0)P(2)	1053.283	1062.377	1066.847	11.3
15	3862.37	L(7–0)R(1)	1013.437	1032.655	1041.934	25.8
16	3778.37	L(9–0)R(0)	991.373	1015.301	1027.025	32.3
17	3780.76	L(9–0)R(1)	992.012	1015.610	1027.218	31.7
18	3753.26	L(10–0)P(2)	984.832	1009.000	1021.213	31.3

We have applied the trimmed-mean method (Lehmann, 1983) to the data from Table I, in order to estimate the linear regression coefficient of η_i (Equation (1)) with respect to K_i. According to Equation(2), it provides the $\Delta m_p/m_p$ estimate. The optimal trimming level, equal to 3 points from each side of the sample, gives us the estimate $\Delta m_p/m_p = (6.8 \pm 5.2) \times 10^{-4}$, that yields a 2σ upper bound

$$|\Delta m_p/m_p| < 0.002 \quad \text{at} \quad z = 2.811. \qquad (3)$$

A more stringent constraint claimed by Foltz *et al.* (1988) appeared to be not justified (see Varshalovich and Levshakov, 1993)*.

* *Addition in proof*: After the paper was accepted for publication, much stronger constraint was obtained by Lanzetta *et al.* (1995), who applied the dependence (2) with the coefficients partly listed in Table I to processing a modern high-resolution spectrum of PKS 0528–250: $|\Delta m_p/m_p| < 2 \times 10^{-4}$ at $z = 2.8108$.

3. Nucleon Masses at the Redshift $z \sim 2$

While the spectral lines of H_2 give us information on the proton mass m_p, spectra of heavier molecules can be used to conclude about the neutron mass m_n as well. At present we know two spectral systems with rotational microwave radiolines of $^{12}C^{16}O$ observed along with optical spectral lines of ions. One of these rotational lines ($J = 3 - 2$ in the galaxy IRAS F10214+4724) has been identified with a sufficient confidence (Brown and Vanden Bout, 1991). The optical redshift $z = 2.286 \pm 0.001$ of this galaxy has been determined by Rowan-Robinson *et al.* (1991) from narrow emission lines of ions. These lines correspond to electron transitions and practically do not depend on the nuclei masses, while the rotational transition frequency of CO is proportional to M^{-1}. With a sufficient accuracy one may assume $M \propto m = m_p + m_n$. Therefore the difference of the optical and radio redshifts of the same galaxy is approximately

$$\Delta z = \frac{\Delta m}{m}(1 + z). \tag{4}$$

According to Brown and Vanden Bout (1991), the maximum of the CO line corresponds to a redshift $z = 2.2867 \pm 0.0003$. However, taking into account that the line could be complex and a significant radio signal was observed in the channels ranging from -115 to $+250$ km s^{-1}, we adopt a wider confidence interval, $z = 2.2862 \pm 0.0006$. Combining radio and optical redshift errors, we have $\Delta z = (0.2 \pm 1.2) \times 10^{-3}$, and according to Equation (4)

$$\Delta m/m = (0.6 \pm 3.7) \times 10^{-4} \quad \text{at} \quad z = 2.286. \tag{5}$$

Another observation relates to an absorption system ($z = 1.9438$) in the spectrum of the quasar PKS 1157 + 014. Valtts *et al.* (1993) measured an emission rotational line of CO ($J = 2 - 1$) in this system. The identification is not too confident, but if it is confirmed by future observations, it will enable to set a more stringent constraint on $\Delta m/m$. The detected radioline corresponds to radial velocities (20 ± 17) km s^{-1}, while the width of the HI absorption line (which determines the optical redshift) is 18 km s^{-1}. The estimate follows,

$$\frac{\Delta m}{m} = \frac{20 \pm 30 \text{ km s}^{-1}}{3 \times 10^5 \text{ km s}^{-1}} = (0.7 \pm 1.0) \times 10^{-4} \quad \text{at} \quad z = 1.944. \tag{6}$$

Thus, in the cosmological epoch that corresponds to the redshifts $z \sim 2$, the sum of the nucleon masses might deviate from the contemporary value by no more than by 0.03%. If we adopt for that epoch an upper bound $|\Delta(m_n - m_p)/(m_n - m_p)| < 0.05$, which follows from a consideration of primordial nucleosynthesis by Kolb *et al.* (1986), then a constraint on the proton mass variation follows from the estimate (6),

$$|\Delta m_p/m_p| < 0.0003 \quad \text{at} \quad z = 1.944, \tag{7}$$

which is seven times stronger than the upper limit (3).

4. Fine-Structure Constant at the Redshift $z \sim 2-3$

Comparison of different redshifted wavelengths of the same absorption system in a quasar spectra, the method used in the previous sections to bound the nucleon masses at high redshifts, can be used also to estimate the variation rate of the fine-structure constant α. Quasar spectra contain many absorption lines which correspond to the transitions ${}^2S_{1/2} \rightarrow {}^2P_{3/2}$ (wavelength λ_1) and ${}^2S_{1/2} \rightarrow {}^2P_{1/2}$ (wavelength λ_2). The relative splitting at the redshift z,

$$y(z) = (\lambda_2 - \lambda_1)_z/\bar{\lambda}_z, \tag{8}$$

is proportional to α^2. Let us define

$$Y(z) = \frac{1}{2}\left(\frac{y(z)}{y(0)} - 1\right). \tag{9}$$

Then $Y(z) \approx \Delta\alpha/\alpha$, assuming $Y(z)$ to be much less than unity.

This method, rather similar to that previously described for $\Delta m/m$ estimation, appears to be even much more advantageous for $\Delta\alpha/\alpha$ estimation owing to the following two circumstances. First, fine-splitted lines of alkali-like ions are much more abundant in quasar spectra than the above molecular lines. Second, the *absorption* alkali-like doublet components correspond to transitions from a single level. Thus they definitely belong to the same cloud of interstellar medium, their relative separation being not affected by differences in radial velocities of different clouds.

We have composed a catalogue of alkali-like doublet wavelengths with $z > 0.2$ found in the quasar absorption spectra, using all the data published since 1980, after the beginning of large-scale quasar surveys with electronic detection. The catalogue, published in an electronic form by Potekhin and Varshalovich (1994) (hereafter Paper I), is now supplemented by the most recent data (Wolfe *et al.*, 1993, Lu *et al.*, 1993, Petitjean *et al.*, 1994) and serves as the data base of our present analysis. The large number of measured wavelengths ($\sim$ 1500 pairs of lines) allows one to make advantage of modern methods of statistics, in order to refine the estimate of $\Delta\alpha/\alpha$. (See Paper I for full details of the used robust statistical techniques, as well as for an analysis of possible error sources and for a discussion of previous examples of statistical treatment of the problem by Levshakov (1992, 1993)).

Our updated analysis of the regression model

$$Y(z) = a + b(z - \bar{z}) \tag{10}$$

fully confirmed the previous results (Paper I), and yielded the restriction

$$|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}z| < 6 \times 10^{-4} \tag{11}$$

at the 2σ significance level. Thus, $\Delta\alpha/\alpha < 1.5 \times 10^{-3}$ at $z \sim 2.5$.

Our separate analysis of modern high-quality spectroscopic data of Petitjean *et al.* (1994), again yields the same restriction (11) (see for details Varshalovich and Potekhin, 1994, hereafter Paper II).

In the present work we have tried also two other regression models,

$$Y(z) = a + b(x - \bar{x}), \qquad x = \ln(1+z),$$

$$\text{and } Y(z) = a + b(\xi - \bar{\xi}), \qquad \xi = (1+z)^{-3/2}, \tag{13}$$

and obtained the restrictions

$$|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}x| < 1.2 \times 10^{-3}, \tag{14}$$

$$|\alpha^{-1}\mathrm{d}\alpha/\mathrm{d}\xi| < 1.5 \times 10^{-3} \tag{15}$$

(see Paper II for more details).

The consequences for the theoretical models are the following. In the standard model of the Universe expansion, the constraint (15) means that $\dot{\alpha}/\alpha < 10^{-13}$ yr^{-1} over the last 90% of the life time of the Universe. For the power law $\alpha \propto t^n$, the restriction $|n| < 8 \times 10^{-4}$ follows from Equation (14). This restriction excludes the power-law models of Gamow (1967), Freund (1982) and Maeda (1988), listed in Section 1. Moreover, Teller's (1948) hypothesis $\alpha^{-1} \propto \ln(t/\tau)$ can be definitely excluded now. Indeed, Equation (14) can be satisfied only if $|\ln(t_0/\tau)| > 10^3$, where $t_0 \sim 1.5 \times 10^{10}$ yr is the present age of the Universe. However, even the smallest possible value of the parameter τ, namely the Planck time $\tau = 5 \times 10^{-44}$ s, gives $|\ln(t_0/\tau)| = 140$. Therefore the logarithmic dependence cannot be realized.

5. Tentative Anisotropy of the Fine Splitting in Quasar Spectra

The data of our doublet-wavelength catalogue allow one to study not only the z-dependence of α, averaged over the celestial sphere, but also a possible spatial anisotropy of fine-splitting values at large z. With this aim we divided the celestial sphere into 8 quadrants and estimated the regression parameters (10) in each quadrant separately. No statistically significant deviation of the slope b from zero has been revealed. Within the relative statistical error $3\sigma < 1\%$ the values of α turned out to be the same in various quadrants (see Paper II for the detailed data). It means that, within the 1% accuracy, these values were the same in various distant areas of the Universe. This is not trivial, because some of these areas were causally disconnected at the epochs of line formation (cf. Tubbs and Wolfe, 1980).

However, when we united the quadrants by four to form hemispheres, in order to lower the mean square residuals, we revealed unexpected deviations of the average fine splittings. These results (number of doublets N, mean redshift $\bar{z}$, and the slope b of the linear regression (10) for two hemispheres: (a) $9^{\rm h} < \alpha_{1950} \leq 21^{\rm h}$, and ($b$) $\alpha_{1950} \leq 9^{\rm h}$ and $\alpha_{1950} > 21^{\rm h}$, $-90^o \leq \delta_{1950} \leq 90^o$) are presented in Table 2.

Table 2

Ion		Hemisphere (a)		Hemisphere (b)	
	$\bar{z}$	N	b, 10^{-4}	N	b, 10^{-4}
CIV	2.0	255	-1 ± 10	285	17 ± 8
NV	2.3	24	-28 ± 31	24	-10 ± 12
MgII	1.0	57	14 ± 24	71	9 ± 10
CIV+NV+MgII:			-1 ± 9		9 ± 6
SiIV	2.2	71	-12 ± 4	64	7 ± 3

Deviations at a level higher than 2σ (although lower than 3σ) are observed for the subsample of SiIV doublet lines. The sign of the deviation is opposite in the opposite hemispheres. The symmetry axis of the hemispheres approximately coincides with the axis of the relic microwave background anisotropy. Three other ions presented in the table have smaller laboratory fine splittings $y(0)$, resulting in larger root-mean-square errors. Therefore, the deviations for these ions are statistically insignificant, although the signs of their cumulative deviations coincide with those of the SiIV estimates. There could be three possible ways to explain this result: (i) just an occasional statistical deviation, (ii) spatial variations of α, or (iii) some optical dispersion effect in a moving gravitating matter distributed along the line of sight between a quasar and the observer. We leave the final explanation to further studies.

6. Summary

George Gamow was one of the pioneers who analysed the problem of time-variability of fundamental physical constants. Although his original considerations based on Dirac's "large number hypothesis" are rejected by experimental data, the problem remains a hot point of contemporary physics. Different versions of the theories of unification of all the physical interactions, developed in the 1980s, predicted different variation laws of fundamental constants. Therefore experimental or observational constraints on such variations may serve as criteria for deciding between these theoretical models.

Quasar spectra are an important source of our knowledge of the physical conditions at the early cosmological epochs, related to the redshifts $z \sim 2-3$. In particular, values of physical constants can be extracted from quasar spectroscopic data and compared with the laboratory values at the present epoch. This information is of importance, being independent of a local tests, because the latter, being restricted to the Solar system, may cover only ten times smaller range of the redshifts.

Our present analysis based on quasar spectroscopic data is concerned with the fine-structure constant α and the ratios of the proton mass m_p and neutron mass m_n to the electron mass m_e. The latter two ratios could be changeable if the constants of strong and/or electroweak interactions would suffer some variations, whereas the former one (α) is itself the key parameter of quantum electrodynamics. (Notice that the time-space variabilities of the coupling constants under discussion should not be confused with their dependence on energy. Only their low-energy limits are analysed here).

The analysis reveals no statistically significant variation of α, m_p or m_n. At the 2σ (95%-significance) level, the following upper bounds are available for the epoch corresponding to the cosmological redshifts $z \sim 2-3$: $\Delta\alpha/\alpha < 1.5 \times 10^{-3}$, $\Delta m_p/m_p < 2 \times 10^{-4}$, and $\Delta m/m < 3 \times 10^{-4}$, where Δx is a possible deviation of a quantity x from its present value, $m = m_p + m_n$, and the nucleon masses are in units of m_e.

In addition to the study of the z-dependence of α averaged over the celestial sphere, we have also checked a possible spatial anisotropy of fine-splitting values at large z. Within a relative statistical error $3\sigma < 1\%$ the values of α turned out to be the same in various quadrants of the celestial sphere, which corresponds to their equality in causally disconnected areas. However, at the 2σ level a tentative anisotropy of estimated relative fine-splitting values is found in directions, that approximately coincide with the direction of the cosmic microwave background anisotropy.

Some particular theoretical models of α variation are considered. For the power law $\alpha \propto t^n$ a restriction $|n| < 8 \times 10^{-4}$ is obtained, at variance to predictions of some of the models. Also the hypothesis of Teller, $\alpha^{-1} \propto \ln(t/\tau)$, is ruled out by the present results. Thus, the revealed constraints serve as effective criteria for selection of those theoretical models which predict variations of α, m_p or m_n with the cosmological time.

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