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Relic neutrinos distribution function at low coordinate momentums

V. Yurchenko, A. Ivanchik
yurchvlad@gmail.com

Ioffe Institute, St. Petersburg 194021, Russia

The effect of processes $p + e^- \to n + \nu_e$ and $\mu^+ \to e^+ + \tilde{\nu}_\mu + \nu_e$ on relic neutrino distribution function in the region of small coordinate momentums is studied. The Boltzmann kinetic equation on the electron neutrino distribution function is solved in this region at temperatures above 10 MeV taking into account generation of electron neutrinos due to the above processes in the assumption that at these temperatures neutrinos of larger momentums are thermalized instantaneously. The effect of thermalization of neutrinos due to their scattering on electrons and positrons on the distribution function in this region at temperatures lesser than 10 MeV is examined.

Keywords: relic neutrinos, kinetic equation, distribution function

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1. Relic neutrinos

Relic neutrinos are neutrinos appeared in the Universe shortly after the Big Bang and forming in the modern Universe a background similar to the cosmic microwave background (CMB) of relic photons. Relic neutrinos are described by the Fermi-Dirac distribution function with the temperature $T_\nu \approx 1.945$ K, that is somewhat lesser in comparison with the photon temperature $T_\gamma \approx 2.7255$ K.

Neutrinos are weakly interacting particles and therefore relic neutrinos stop interacting between themselves and with other particles at about one second after the Big Bang (see, e.g., [1, 2] for details). Around the same time the epoch of electron-positron annihilation begins and almost all entropy of electrons and positrons is transferred to photons resulting in increase of the photon temperature relative to the neutrino temperature. Nevertheless, a small part of entropy of electrons and positrons is still transferred to neutrinos due to residual processes of scattering of neutrinos and antineutrinos with each other and on electrons and positrons and it results in appearing of non-thermal distortion in neutrino distribution function which can be calculated by means of solving the Boltzmann equation (see, e.g., [3]):

$$Hx\frac{\partial f_\nu(y_\nu)}{\partial x} = I_{Coll}(y_\nu), \tag{1}$$

where H is the Hubble parameter, $x = m_0 \cdot a$ is a dimensionless scale factor of the Universe, m_0 is an arbitrary scale of mass, a is the scale factor of the Universe normalized by the condition $aT_\gamma = aT_\nu = 1$ at temperatures of photons and neutrinos striving for infinity; $y_\nu = a \cdot p_\nu$ is a dimensionless coordinate momentum of neutrino and p_ν is a physical momentum of neutrino. I_{Coll} is the collision integral describing all processes listed above.

2. Relic neutrinos of small momentums

The described distortion of relic neutrino distribution function is dependent on neutrino momentum. Namely, the quantity $(f_\nu - f_0)/f_0$ (where $f_0 = 1/(\exp\varepsilon/T + 1)$ is the equilibrium distribution function of neutrinos) increases quadratically with increasing neutrino momentum. Also this distortion differs only slightly from zero in the region of small momentums. And the smaller neutrino momentum, the lesser the distortion. One can find a range of coordinate momentums where the distortion is arbitrary small. It means, that at all temperatures there are neutrino particles in the Universe that are not thermalized. The simple idea comes, that if there existed some processes generating neutrinos in this range of coordinate momentums, these neutrinos would not subsequently thermalize. At some temperature the lowest momentum of neutrinos which thermalize can be estimated from the relation

$$p' \approx \sqrt{\frac{H}{G_F^2 \gamma_e^2 T_\gamma^3}}, \qquad \gamma_e \approx \frac{T_\gamma}{m_e}, \tag{2}$$

where G_F is the Fermi coupling constant. In the present work we considered the effect of the processes $p + e^- \to n + \nu_e$ and $\mu^+ \to e^+ + \tilde{\nu}_\mu + \nu_e$ that can generate electron neutrinos of arbitrary small momentums on the electron neutrino distribution function in the range of momentums $< p'$ at temperatures from 10 MeV to the temperature defined by eq. (2), given the value of p'.

3. Generation of neutrinos of small momentums

The two processes are considered in this paper that can generate neutrinos of arbitrary small momentum. The first one is electron capture on proton $p + e^- \to n + \nu_e$. It is threshold process with the required total energy of

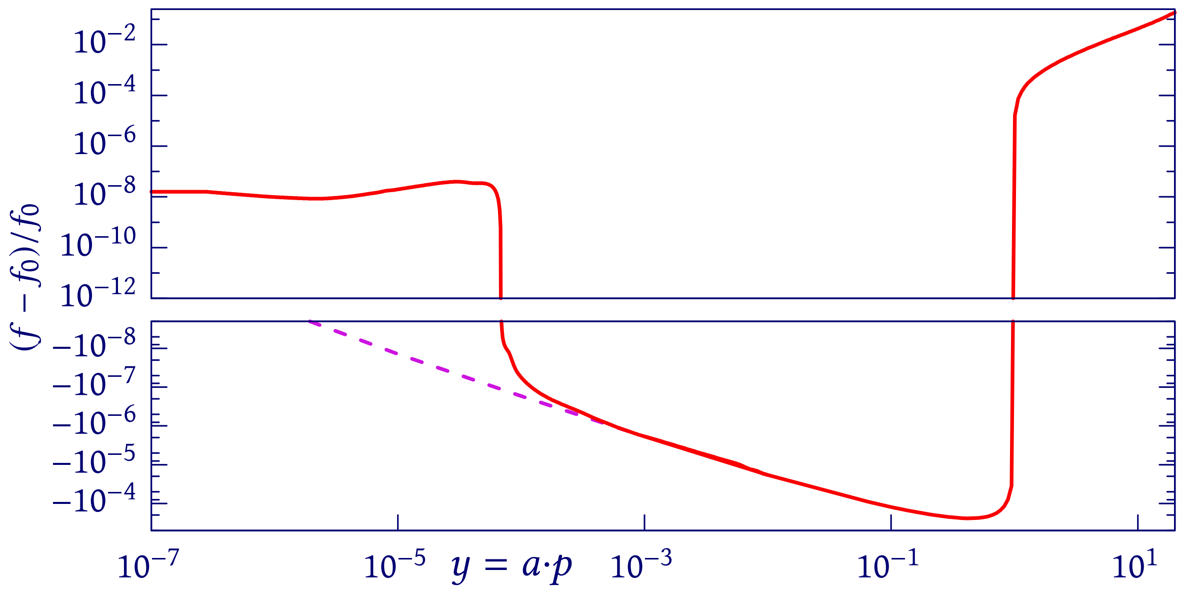

Figure 1: The resulting distortion of electron neutrino distribution due to processes generating neutrinos taking into account thermalization at temperatures < 10 MeV. The effect of thermalization dominates at momentums $> 10^{-3}$. The effect of generation of neutrinos due to processes under consideration dominates at momentums $< 10^{-4}$. The distortion without the processes generating neutrinos at momentums $\lesssim 10^{-3}$ is shown with dashed magenta line.

collision equal $m_n - m_p - m_e \equiv \Delta - m_e \approx 0.782$ MeV. The second one is muon or antimuon decay for the time $\tau_\mu \approx 2.2{\cdot}10^{-6}$ to form electron antineutrino or neutrino respectively that possesses continuous spectrum.

The effect of these processes on neutrino distribution in the range of low momentums can be found by means of solving eq. (1) with appropriate rhs.:

$$I^{p+e}_{Coll} = \frac{2\pi^2}{\tau_n \lambda_0} n_p(x) \sqrt{(\Delta + p_\nu)^2 - m_e^2} (\Delta + p_\nu) f_e(\Delta + p_\nu)(1 - f_\nu(\varepsilon_e)) \tag{3}$$

for the process of electron capture and

$$I^{\mu}_{Coll} = \frac{1}{\tau_\mu} \frac{n_\mu}{n_{\nu_e}} w^2 (1-w)(1 - f_\nu(\varepsilon_e)), \qquad w = 2\frac{p_\nu}{m_\mu} \tag{4}$$

for the process of muon/antimuon decay. In these expressions τ_n is neutron lifetime, λ_0 is some normalization constant of dimension [MeV^5], n_p is the concentration of protons in the matter of the Universe, $f_{e,\nu}$ are distribution functions of electrons and neutrinos; n_μ and n_{ν_e} are concentrations of muons and electron neutrinos respectively, m_μ is the muon mass. In these formulas p_ν should be expressed through y_ν. The electron neutrino spectrum from muon decay is taken in [4].

The resulting distortion of electron neutrino distribution function due to net effect of these processes in the considered range is obtained as

$$\Delta f_\nu \Big|_{y_\nu} = \int_{x_0}^{x_1} \frac{\mathrm{d}x}{Hx} (I^{p+e}_{Coll}(y_\nu) + I^{\mu}_{Coll}(y_\nu)), \tag{5}$$

with x_0 corresponding the maximum temperature in the Universe when neutrinos of momentum y_ν were non-thermalizing and x_1 corresponding $T_\gamma = 10$ MeV.

After this distortion was calculated the effect of thermalization processes on the obtained distorted distribution function at temperatures < 10 MeV was examined. The final result for the quantity $(f_\nu - f_0)/f_0$ is presented in Fig. 1.

4. Results

In the present paper the distortion of electron relic neutrinos distribution function due to processes $p + e^- \to n + \nu_e$ and $\mu^+ \to e^+ + \tilde{\nu}_\mu + \nu_e$ in the range of low coordinate momentums was calculated. At temperatures < 10 MeV the distorted distribution function obtained was summarized with the effect of neutrino thermalization. It turned out, that in the range of coordinate momentums $\lesssim 10^{-4}$ an positive addition to neutrino distribution function appears due to considered processes, while without these processes this addition would be negative.

Thus, a numerical scheme was tried out that allows to test the effect of various processes generating neutrinos in the range of low momentums on relic neutrino distribution function with subsequent study of thermalization process on obtained distribution function.

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