Thermal conductivity due to collisions between electrons in a degenerate, relativistic, electron gas

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It is shown that in the degenerate, relativistic, electron gas of white dwarfs and neutron stars with a degeneracy parameter of $\lesssim 50$ and not very large ion charges ($Z \lesssim 10$) the collisions between electrons make an important contribution to the electronic thermal conductivity.

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The character of the thermal conductivity $\varkappa_{ee}$ due to collisions between degenerate electrons is different[1] at temperatures $T \ll T_p$ and $T_p \ll T \ll T_F$. Here $T_F = (\mu - mc^2)/k$ is the Fermi temperature, $T_p = \hbar\Omega_p/k$ (see Fig. 1 in Ref. 2), $\Omega_p = (4\pi n_e e^2 c^2/\mu)^{1/2}$ is the electron plasma frequency, μ is the Fermi energy, n_e is the electron concentration, and k is the Boltzmann constant. The point is that in the first case, in contrast to the second, the momenta transferred in the collisions ($\sim \hbar q_{TF}$, $q_{TF} = \sqrt{3}\Omega_p/v$) exceed the width of the thermal smearing out of the Fermi surface ($\sim pT/T_F$, where p and v are the Fermi momentum and velocity), as a consequence of which the collision frequency is strongly is strongly decreased owing to the Pauli principle.

Lampe[1] investigated the quantity $\varkappa_{ee}$ for a nonrelativistic gas (density $\rho < 10^6$ g/cm^3). He found that $\varkappa_{ee}$ makes a weighty contribution to the total electronic thermal conductivity $\varkappa$ for $T \gtrsim T_p$ and small ion charges (Z = 2). For the calculation he expanded the collision integral by polynomials of a certain type. Good accuracy (with an error of less than 15%) was already reached in a one-polynomial approximation with allowance for static screening of the interaction between electrons.

The thermal conductivity $\varkappa_{ee}$ for a relativistic gas at $T \ll T_p$ was found in Ref. 3. The variational method used there is equivalent to the one-polynomial approximation in Ref. 1; for $T \ll T_p$ an exact solution is also easy to construct,[4] but it differs from the variational solutions of Ref. 3 by no more than 10%. It is stated in Ref. 3 that in a relativistic gas, in contrast to a nonrelativistic one, collisions between electrons are important only at very low $T < \Theta$, where $\Theta = 0.45\hbar(4\pi n_i Z^2 e^2/m_i k^2)^{1/2}$ is the Debye temperature of an ionic crystal (see Fig. 1 in Ref. 2) and n_i and m_i are the concentration and mass of the ions. By comparing $\varkappa_{ee}$ from Ref. 3 with the thermal conductivity $\varkappa_{ei}$ due to other mechanisms of electron scattering (see Ref. 2, for example), it is easy to show that at $T \ll T_p$ the thermal conductivity $\varkappa_{ee}$ is much larger than $\varkappa_{ei}$ and therefore is unimportant: $\varkappa^{-1} = \varkappa_{ee}^{-1} + \varkappa_{ei}^{-1} \approx \varkappa_{ei}^{-1}$ (this is seen clearly from Fig. 1 of the present report and Fig. 3 in Ref. 2). In order to find out whether it is important at $T \gtrsim T_p$, we used the variational method and the static screening approximation of Ref. 3, and by analogy with Ref. 1 we obtained a relativistic expression for $\varkappa_{ee}$ at $T \ll T_F$:

$$\frac{1}{\varkappa_{ee}} = \frac{108}{\pi^3}\left(\frac{e^2}{\hbar c}\right)^2 \frac{T}{\hbar c^2 q_{TF}^3} J(y), \quad J(y) = \int_0^\infty dz \frac{z^4 e^z}{(e^z-1)^2} D\left(\frac{z}{y}\right), \tag{1}$$

$$D(t) = \int_0^{\pi/2} d\theta \sin^2\theta \frac{(mc/p)^4 + 4(mc/p)^2 \sin^2\theta + 4\sin^4\theta}{[1 + t^2(\cos^{-2}\theta - v^2/c^2)]^{3/2}}, \quad y = \frac{\sqrt{3}\,T_p}{T}. \tag{2}$$

Here 2θ is the electron collision angle. The results of Ref. 1 are obtained from (1) for $v \ll c$ and the results of Ref. 3 are obtained for $T \ll T_p$. The integral in (2) can be taken, but it has a cumbersome form. As $v \to c$,

$$D(t) = \frac{4t^5 - 28t^3 - 81t + 15(6t^2+1)I(t)}{12(t^2-1)^4}, \quad I(t>1) = \frac{\ln[t + (t^2-1)^{1/2}]}{(t^2-1)^{1/2}},$$
$$I(t<1) = \frac{\arccos t}{(1-t^2)^{1/2}}, \tag{3}$$

$$J(y \gg 1) = \frac{\pi^5}{6} - \frac{960}{y}\zeta(5) + \frac{5\pi^7}{y^2} - \frac{128 \cdot 1440}{y^3}\zeta(7) + \dots, \quad J(y \ll 1) = \frac{y^3}{3}\Lambda_{ee},$$
$$\Lambda_{ee} = \ln\frac{2}{y}, \tag{4}$$

where $\zeta(s)$ is the Riemann zeta function. For values of y = 0.5, 1.5, 3, 7; and 10 a numerical calculation gives J(y) = 0.056, 0.53, 2.6, 8.8, and 13.5, respectively. These values, the asymptotic form (4), and the numerical equations

$$\varkappa_{ee} = 8.1 \cdot 10^{19} \rho_6/\mu_e J(y) T_6 \text{ erg/cm} \cdot \text{sec} \cdot \text{deg},$$
$$y = 576(\rho_6/\mu_e)^{1/3}/T_6, \quad T_6 = T/10^6\,°\text{K} \tag{5}$$

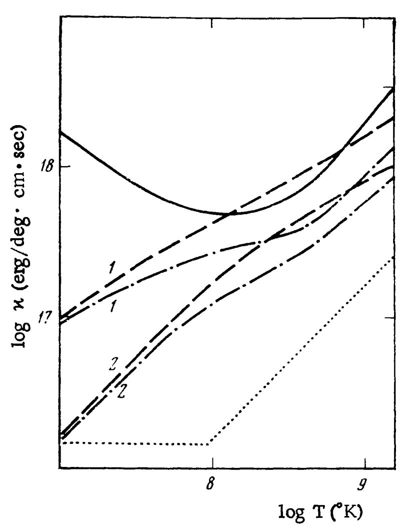

FIG. 1.

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are sufficient to find $\varkappa_{ee}$ for $T \ll T_F$ and $\rho_6 = \rho/10^6$ g $\cdot$ cm^{-3} > 1 ($\mu_e = A/Z$ and A is the mass number of the ions). The dependence $\varkappa_{ee}(T)$ found is similar to the dependence of Ref. 1 for $\rho_6 \ll 1$. In particular, as for $\rho_6 \ll 1$, the second asymptotic form of (4) works well up to values of $y \approx 1$, but the first one works only for $y > 20$. But the results of Ref. 3, corresponding to the first term of the expansion (4) for $y \gg 1$, are actually correct only for $T < T_p/60$.

As an example, in Fig. 1 we plot the dependence $\varkappa_{ee}(T)$ (solid line) for $\rho_6 = 20$ and $\mu_e = 2$. Here $T_F = 8.2 \cdot 10^{9}$°K, $T_p = 6.4 \cdot 10^{8}$°K, and $\Theta = 7.6 \cdot 10^{6}$°K. The thermal conductivities $\varkappa_{ei}$ and $\varkappa$ for Z = 2 (curves 1) and Z = 6 (curves 2) are plotted by dashed lines and dash–dot lines for comparison, using the data of Ref. 2, while $\varkappa_{ei} \approx \varkappa$ for Z = 26 is plotted by dots. The bend in the curve for Z = 26 corresponds to the ionic crystallization point ($T = T_M = Z^2e^2(4\pi n_i/3)^{1/3}/150$ k): scattering on ions was allowed for at $T > T_M$ and scattering on phonons at $T < T_M$. For Z = 2 and 6 we have $T_M \approx 10^6$ and $6.4 \cdot 10^{6}$°K, so that $\Theta > T_M$ and crystallization does not occur at $T < T_M$. Using the results of Ref. 2, we find that for $\rho_6 \gg 1$ and $T > T_M$ we have the ratio $\varkappa_{ee}/\varkappa_{ei} \approx Z\Lambda_{ei} \times y^2/10J(y)$, where $\Lambda_{ei} \approx 1$ is the Coulomb logarithm for electron–ion collisions. The minimum value of $\varkappa_{ee}/\varkappa_{ei} \approx Z\Lambda_{ei}/3$ is reached at $T = T_* = 0.6T_p$, i.e., at a degree of degeneracy $T_F/T_* \approx 31$. Therefore, collisions between electrons are unimportant at large $Z \approx 30$. But even for Z = 8 and $T \approx T_*$ the thermal conductivity $\varkappa_{ee}$ is $\sim 2.5\varkappa_{ei}$ and markedly improves the thermal insulating properties of the relativistic degenerate gas. We note that $(\varkappa_{ee}/\varkappa_{ei})_{min} \approx Z\Lambda_{ei}/1.6(\rho_6/\mu_e)^{1/6}$ and $T_* \approx 0.8T_p$ for $\rho_6 \ll 1$, according to Refs. 1 and 2. Then, in contrast to the case of $\rho_6 \gg 1$, with a decrease in ρ the role of $\varkappa_{ee}$ in the "favorable" temperature interval of $T \gtrless T_p$ decreases and the interval itself shifts toward a lower degeneracy, $T_F/T_* = 10 \cdot (\rho_6/\mu_e)^{1/6}$; therefore, $\varkappa_{ee}$ actually can play a role only for Z = 2.

A "favorable" temperature $T \gtrless T_p$ occurs in neutron stars and white dwarfs which have not cooled too much. According to Refs. 5 and 6, for example, for a neutron star of mass $\sim M_\odot$ and a radius of ~10 km such a temperature is maintained in the layer of $\rho_6 \approx 1$ so long as the surface temperature of the star exceeds $(1\text{-}3) \cdot 10^{6}$°K. If $Z \lesssim 10$ in this case then electron–electron collisions markedly retard the cooling of the star by the mechanism of heat conduction. It is still more important to allow for these collisions when studying rather deep ($\rho_6 \gtrsim 1$) thermonuclear burning of material (accreting, for example) with $Z \lesssim 10$: they worsen the heat conduction and hence create more favorable conditions for outburst (explosive) burning.

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