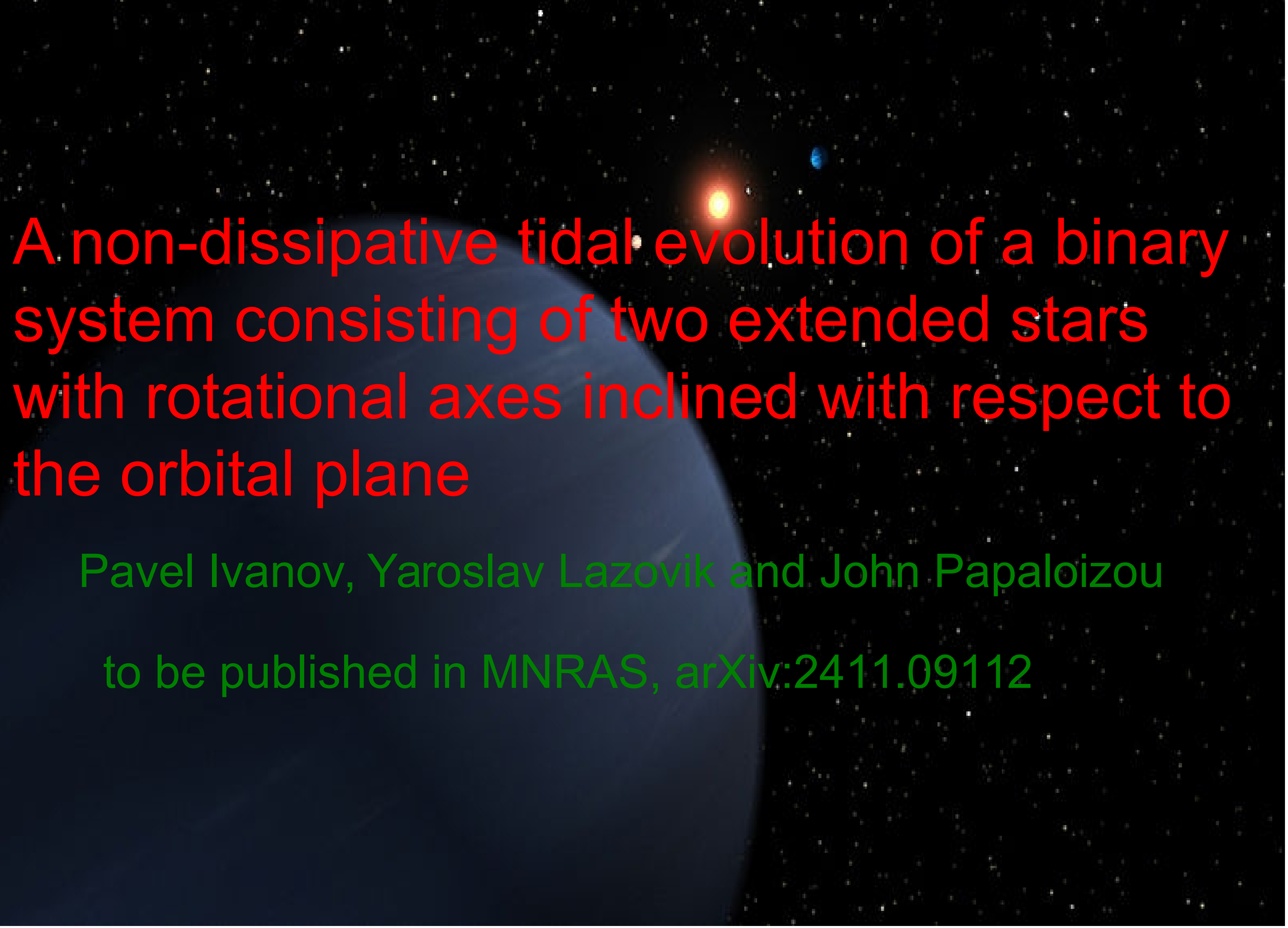

A non-dissipative tidal evolution of a binary system consisting of two extended stars with rotational axes inclined with respect to the orbital plane

Pavel Ivanov, Yaroslav Lazovik and John Papaloizou

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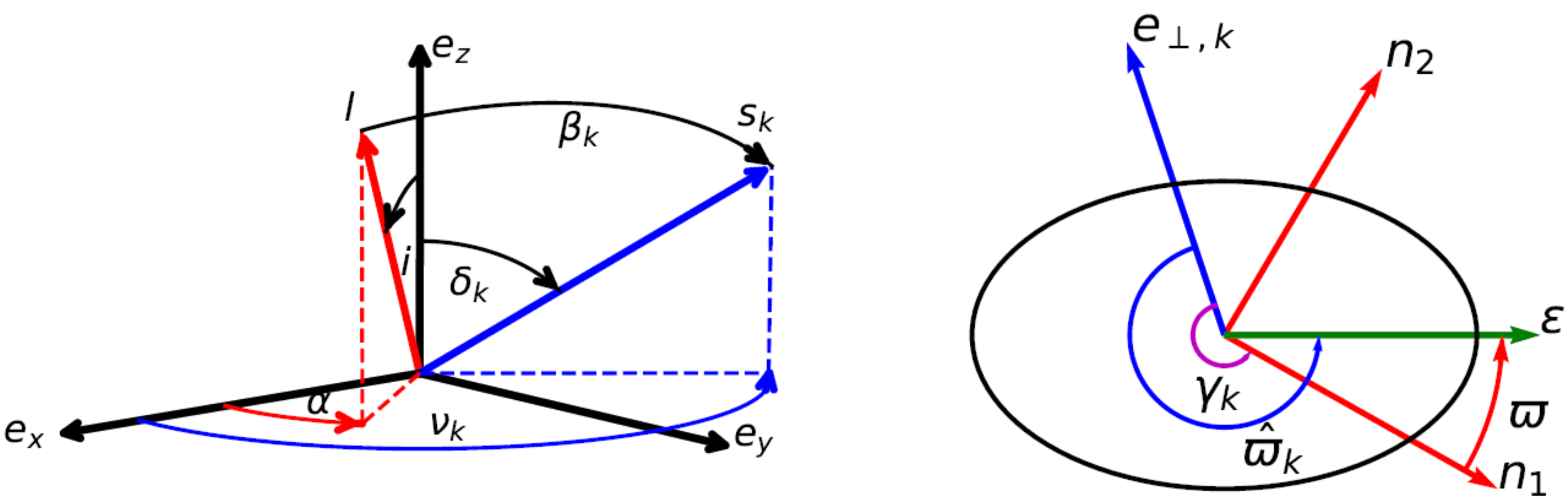
Coordinate systems used below



Our primary coordinate system is associated with the direction of total angular momentum, J. The unit vector e_z is aligned with this direction, while the vectors e_x and e_y are in the perpendicular plane. The direction of orbital angular momentum is characterised by inclination angle i and rotational angle α, while directions of stellar spins are characterised by inclination angles δ_k and rotational angles v_k. The angles β_k determine the relative inclination of the spins and the orbital angular momentum.

General dynamical equations follow from the law of conservation of angular momentum and definitions of the inclination angles

$$\mathbf{J} = \mathbf{L} + \sum_{k=1,2} \mathbf{S}_k$$

$$\mathbf{S}_k = S_k(\cos\delta_k \mathbf{e}_z + \sin\delta_k(\cos\nu_k \mathbf{e}_x + \sin\nu_k \mathbf{e}_y)) \quad \text{for} \quad k = 1, 2$$

$$\mathbf{L} = L(\cos i \mathbf{e}_z + \sin i(\cos\alpha \mathbf{e}_x + \sin\alpha \mathbf{e}_y))$$

$$\frac{d\mathbf{S}_k}{dt} = \mathbf{T}_k, \quad \text{with} \quad \mathbf{T}_k = T_{\|,k}\mathbf{s}_{\|,k} + T_{\perp,k}\mathbf{s}_{\perp,k}$$

$$\frac{d\mathbf{L}}{dt} = -\sum_{k=1,2} \mathbf{T}_k.$$

$$\mathbf{s}_{\|,k} = \frac{(\mathbf{l} - \cos\beta_k \mathbf{s}_k)}{\sin\beta_k}, \quad \mathbf{s}_{\perp,k} = \frac{\mathbf{s}_k \times \mathbf{l}}{\sin\beta_k}, \quad \text{and} \quad \mathbf{s}_k, \quad \text{for} \quad k = 1, 2$$

The torques

$T_{\perp,k}$ is mainly determined by the stellar flattening, the corresponding contribution has a well known form (e.g. Barker, O'Connel, 1975)

$T_{\parallel,k}$ has been derived in Ivanov & Papaloizou 2021, it can be represented in the form

$$T_{\parallel,k} = -C_k \sin\delta_k \sin(2\hat{\varpi}_k) \qquad \hat{\varpi}_k = \varpi + \gamma_k$$

$$\cos\gamma_k = \frac{\cos\delta_k - \cos\beta_k \cos i}{\sin\delta_k \sin i} = -\cos i \cos(\alpha - \nu_k) + \frac{\sin i \cos\delta_k}{\sin\delta_k}, \quad \text{and}$$

$$\sin\gamma_k = -\frac{\sin\delta_k \sin(\alpha - \nu_k)}{\sin\beta_k}.$$

$$\frac{d\varpi}{dt} = \dot{\varpi}_{\rm T} + \dot{\varpi}_{\rm E} + \dot{\varpi}_{\rm R} + \dot{\varpi}_{\rm NI}.$$

In general, equations are rather cumbersome

$$\frac{dL}{dt} = -\sum T_{\parallel}^{k} \sin \beta_k .$$

$$\frac{di}{dt} = \frac{1}{L \sin i} \sum_k \frac{1}{\sin \beta_k} \left(T_{\parallel}^{k} \cos \beta_k (\cos \beta_k \cos i - \cos \delta_k) + T_{\perp}^{k} \sin i \sin \delta_k \sin(\alpha - \nu_k) \right)$$

$$\frac{d\alpha}{dt} = -\frac{1}{L \sin i} \left(\sum_k \frac{1}{\sin \beta_k} (T_{\parallel}^{k} \cos \beta_k \sin \delta_k \sin(\alpha - \nu_k) + T_{\perp}^{k} (\sin i \cos \delta_k - \cos i \sin \delta_k \cos(\alpha - \nu_k)) \right)$$

$$\frac{d\delta_k}{dt} = -\frac{T_{\parallel}^{k}}{S_k \sin \beta_k \sin \delta_k} (\cos i - \cos \beta_k \cos \delta_k) - \frac{T_{\perp}^{k}}{S_k \sin \beta_k} \sin i \sin(\alpha - \nu_k), \quad \text{and} \tag{28}$$

$$\frac{d\nu_k}{dt} = \frac{T_{\parallel}^{k}}{S_k \sin \delta_k \sin \beta_k} \sin i \sin(\alpha - \nu_k) + \frac{T_{\perp}^{k}}{S_k \sin \delta_k \sin \beta_k} (\sin i \cos \delta_k \cos(\alpha - \nu_k) - \cos i \sin \delta_k).$$

but, only three of them are independent.

The case S << L

In this case equations take a simpler form

$$\frac{d\delta_k}{dt} = -\frac{T_{\parallel,k}}{S_k} + \frac{iT_{\perp,k}}{S_k \sin\delta_k}\sin(\nu_k - \alpha)$$

$$\frac{d\nu_k}{dt} = -\frac{T_{\perp,k}}{S_k \sin\delta_k};$$

When variations of deltas are small, we can set

$$\delta_k = \delta_k^0 + \Delta_k$$

In this case r.h.s of the dynamical equations do not depend on the dynamical variables. In particular, we have

$$\nu_k = \omega_k t + \nu_k^0, \quad \omega_k = -\frac{T_{\perp,k}}{S_k \sin\delta_k^0}.$$

The evolution near the critical curve $\dot{\varpi}_{\rm tot} = 0$

We arrange the indices 1 and 2 in such a way that

$\xi = S_2 \sin\delta_2 / (S_1 \sin\delta_1)$ < 1

In this case, when the inclination angles of the system are not very close to 90 degrees, only

$\Delta = \Delta_1$ is expected to vary significantly due to the presence of the parallel torque

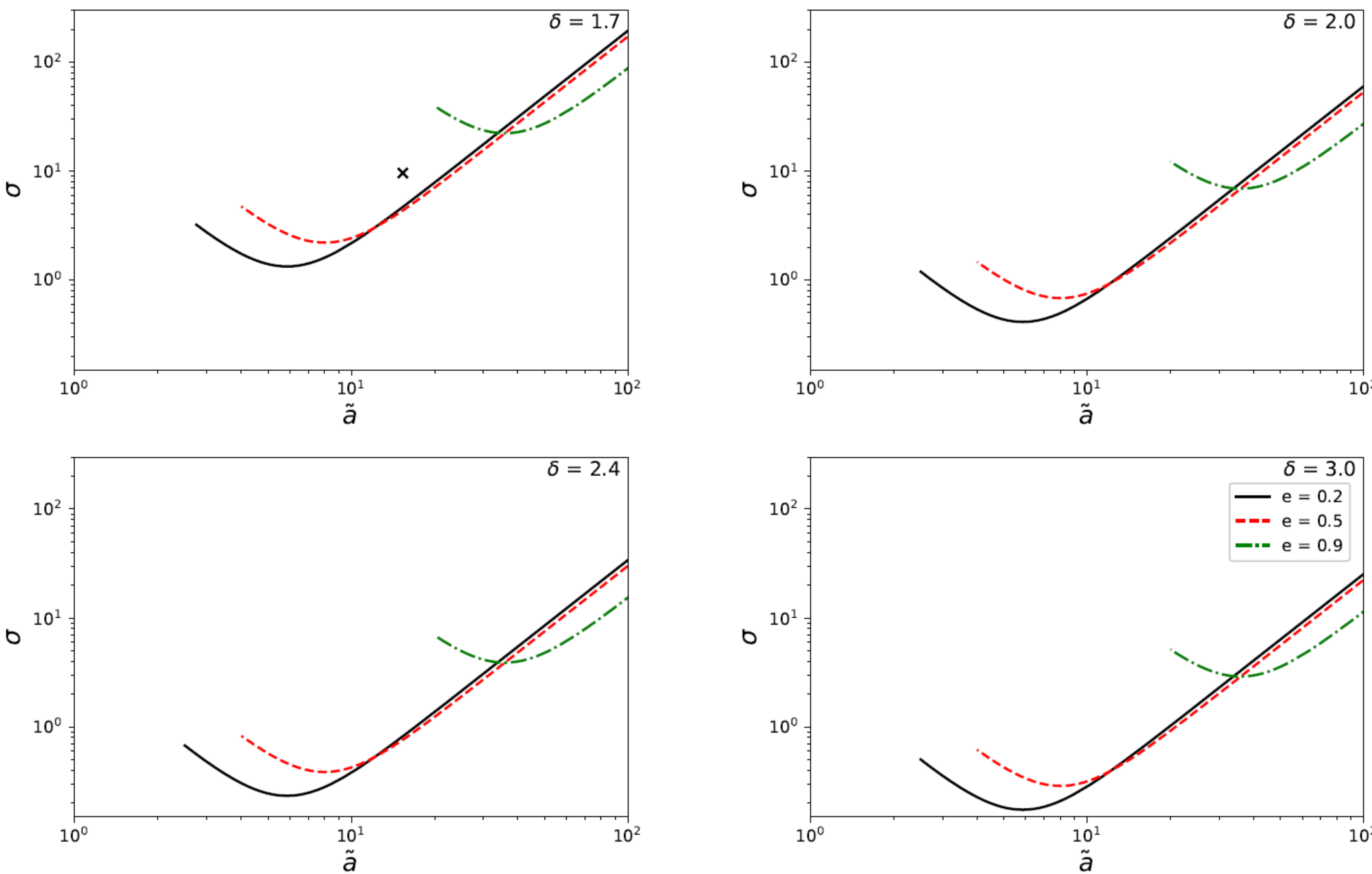
the dynamics is reduced to the equation of simple pendulum

$$\frac{d^2\theta}{dt^2} = -\Omega_{\parallel}^2 \sin\theta + \frac{d\dot{\varpi}_{fast}}{dt}\,.$$

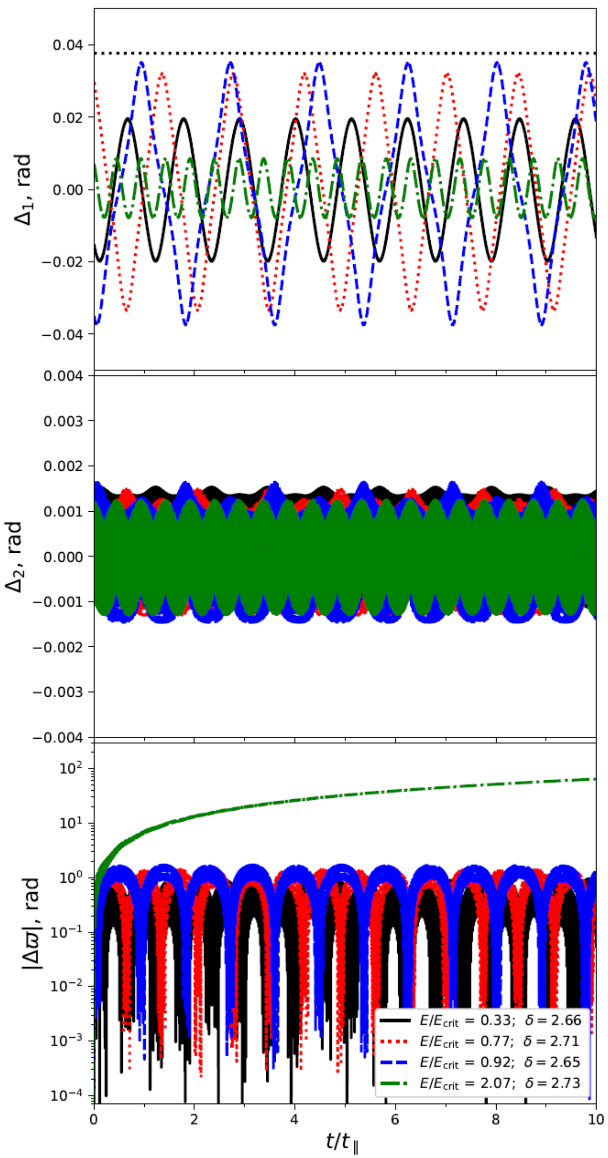
which has the integral of motion $E = \dfrac{\dot{\theta}^2}{2} + \Omega_{\parallel}^2 (1 - \cos\theta).$

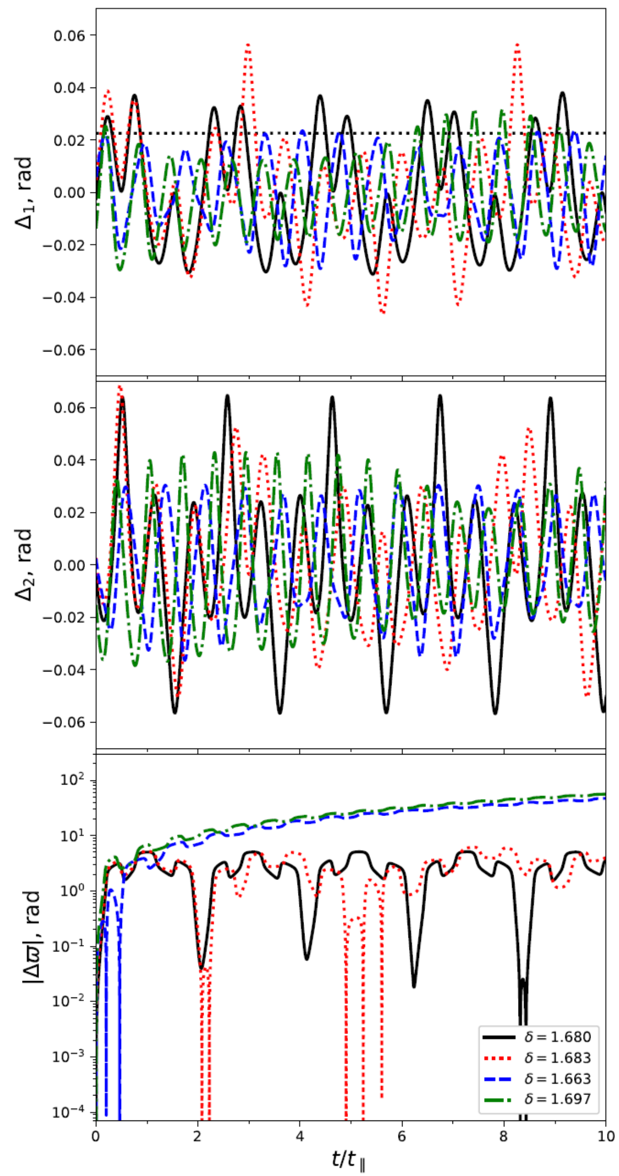
when the last term on r.h.s. is neglected.

Examples of the critical curves



Numerical calculations





$$t_\parallel = 7.6\times10^4 \text{ yr}$$

for DI Herc system

CONCLUSIONS

The joint evolution of stellar rotational axis and the apsidal line allows for a rather non-trivial dynamics in case of inclined systems of two distributed stars. When the orbital parameters are such that the system happens to be close to the critical curve there is a possibility to have librations of the apsidal line.

The results obtained would allow one to model the dynamical evolution of the inclined systems on timescale smaller than a tidal dissipation timescale. We provided a detailed prescription of determination of all variables needed for this purpose.