Surface plasmon polaritons (SPP): an alternative to cavity QED

Strong coupling to excitons & Intermediary for quantum entanglement

- **A. Gonzalez-Tudela, D. Martin-Cano, P. A. Huidobro, E. Moreno, F.J. Garcia-Vidal, C.Tejedor**

  Universidad Autonoma de Madrid

UNIVERSIDAD AUTONOMA DE MADRID

- **L. Martin-Moreno**

  Instituto de Ciencia de Materiales de Aragon
  CSIC

icma Instituto de Ciencia de Materiales de Aragón

SPP

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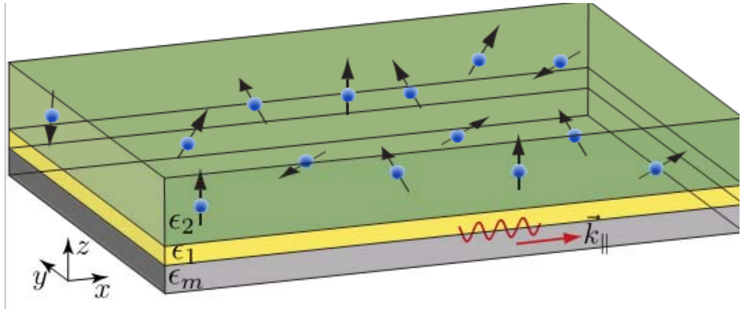

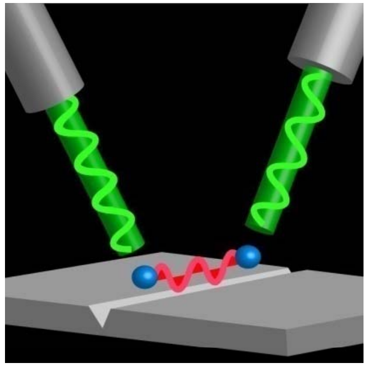

Outline

- Introduction to surface plasmon polaritons (SPP)
- Coupling of 1 quantum emitter (QE) to SPP
- Collective mode (excitons) strongly coupled to SPP
- Coupling & entanglement of 2 QE mediated by SPP
- Conclusion

Intro: Surface plasmon polaritons

- Dielectric response of a metal is governed by free electron plasma:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

ω_p : plasma frequency

γ : damping factor

Below its plasma frequency $\varepsilon(\omega)$ is negative...

$\Downarrow$

wavevector : $k = \frac{\omega\sqrt{\varepsilon}}{c}$ → purely imaginary → photonic insulator

What is a surface plasmon polariton ?

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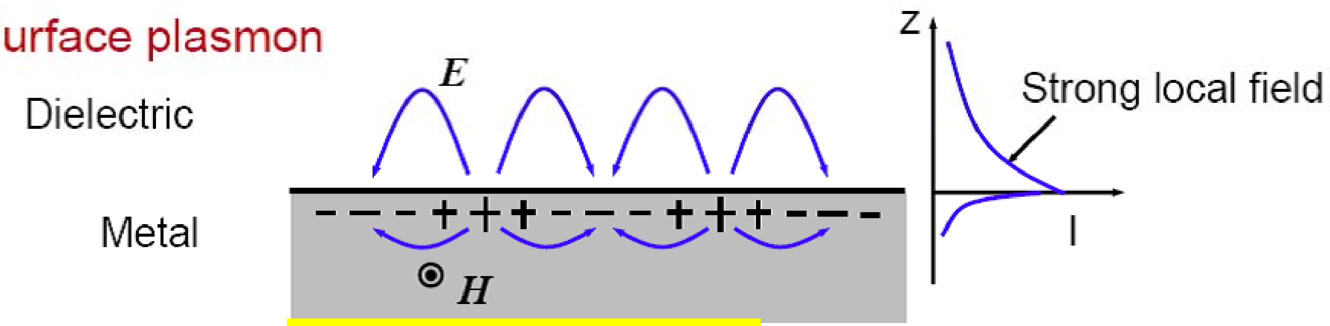
What is a surface plasmon polariton ?

Bulk plasmon

- Metals allow for EM wave propagation above the plasma frequency

They become transparent!

Surface plasmon



Note: SP is a TM wave!

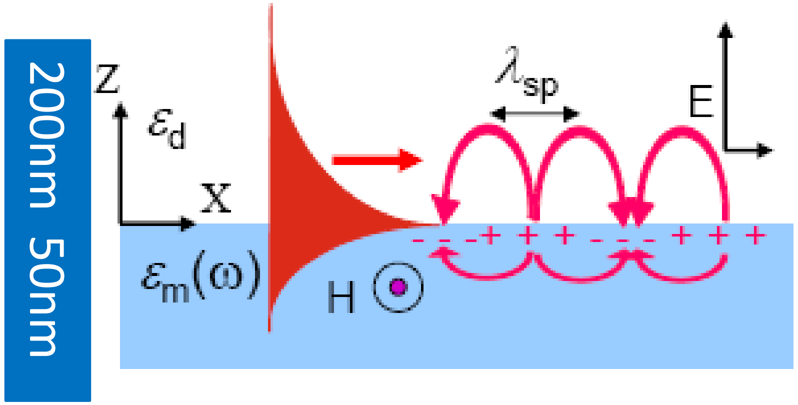
- Sometimes called a surface plasmon-polariton (strong coupling to EM field)

Intro: Surface plasmon polaritons

Electromagnetic radiation in dielectric ⊕ Localized Plasmons in a metal surface
⇓
SURFACE PLASMON POLARITONS

1. SPPs are primarily transverse in dielectrics but longitudinal in metals!

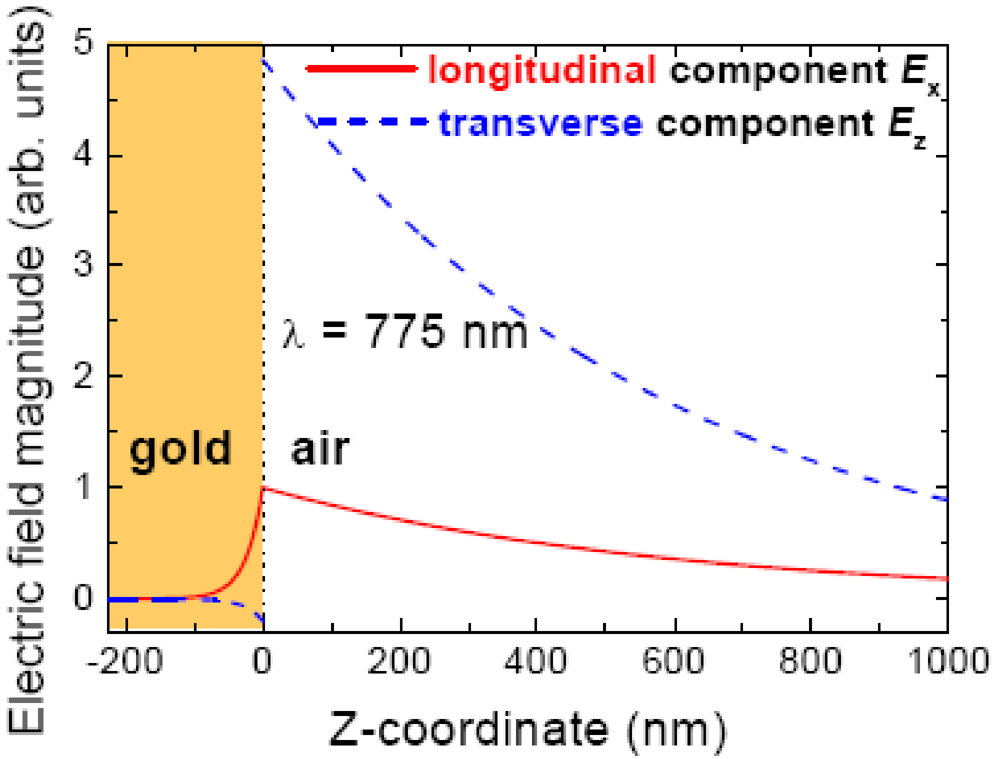
2. SPP properties are dictated by the boundary conditions for $E_{\parallel}$ and $E_{\perp}$!



$$E_z^d = i\sqrt{-\varepsilon_m / \varepsilon_d}\, E_x^0$$

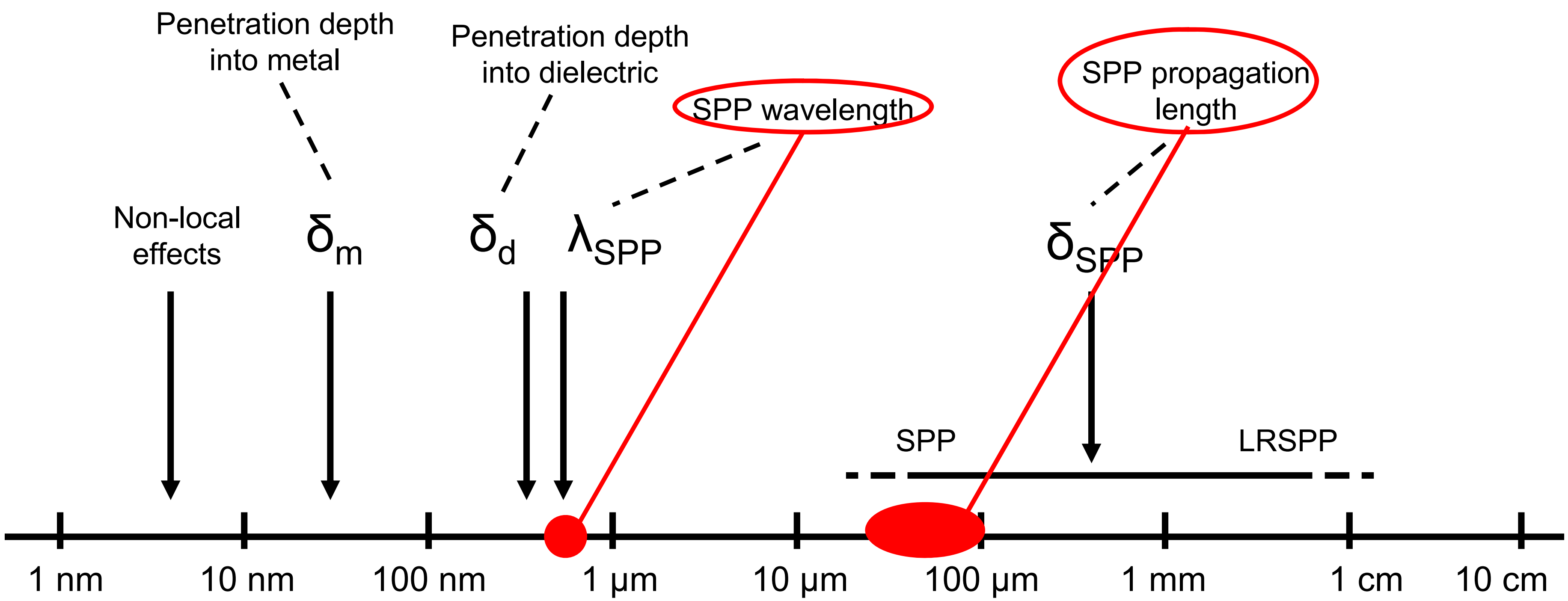
and

$$E_z^m = -i\sqrt{-\varepsilon_d / \varepsilon_m}\, E_x^0$$



Intro: Surface plasmon polaritons

SPP Length Scales span photonics and nano



Length scales span 7 orders of magnitude!

Intro: Surface plasmon polaritons

Plasmonics: Merging Photonics and Electronics at Nanoscale Dimensions

Ekmel Ozbay, Science, vol.311, pp.189-193 (13 Jan. 2006).

Some of the challenges that face plasmonics research in the coming years are

(i) demonstrate optical frequency subwavelength metallic wired circuits with a propagation loss that is comparable to conventional optical waveguides;

(ii) develop highly efficient plasmonic organic and inorganic LEDs with tunable radiation properties;

(iii) achieve active control of plasmonic signals by implementing electro-optic, all-optical, and piezoelectric modulation and gain mechanisms to plasmonic structures;

(iv) demonstrate 2D plasmonic optical components, including lenses and grating couplers, that can couple single mode fiber directly to plasmonic circuits;

(v) develop deep subwavelength plasmonic nanolithography over large surfaces;

(vi) develop highly sensitive plasmonic sensors that can couple to conventional waveguides;

(vii) demonstrate quantum information processing by mesoscopic plasmonics.

Intro: Surface plasmon polaritons

- **Interesting features of SPPs for photonic circuits**:

•Propagation length: 50-100 μm (Ag or Au) ⇔ lifetime ≤ 1 ps

•Two-dimensional character of EM-fields

•Optical and electrical signals carried without interference

Intro: Surface plasmon polaritons

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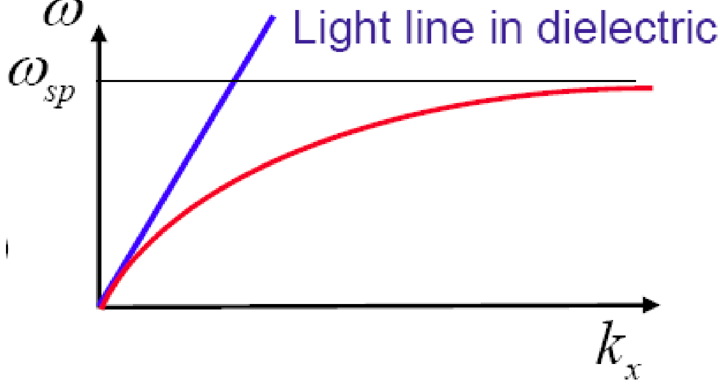
$k_{SPP}(\approx 10-100\,\mu m^{-1}) > \omega\sqrt{\varepsilon_d}/c \Rightarrow$ **Beyond the diffraction limit**

$$k_x = \frac{\omega}{c}\sqrt{\varepsilon_d}$$

$$\frac{\omega_p}{\sqrt{1+\varepsilon_d}} = \omega_{sp}$$



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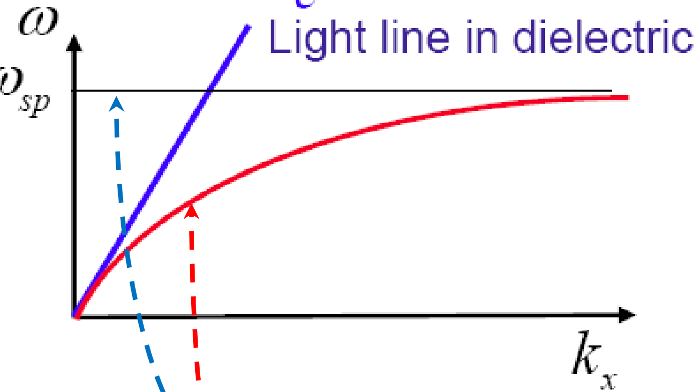


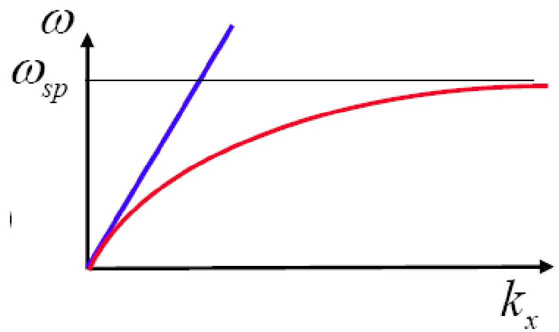
One problem: coupling in and out to SPPs

$$k_{\parallel} < k_{3D} = \frac{\omega}{c}\sqrt{\varepsilon_d} \Rightarrow$$

Photons can only make excitations **inside** the light cone

While SPP are **outside** the light cone

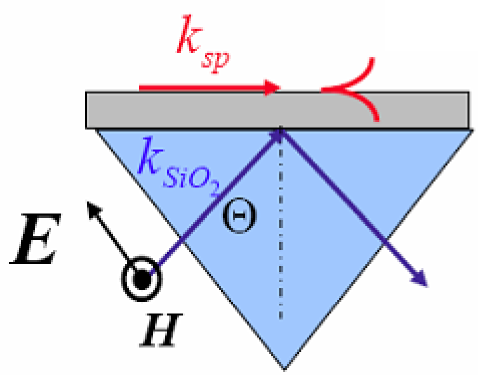
One problem: coupling light to SPPs



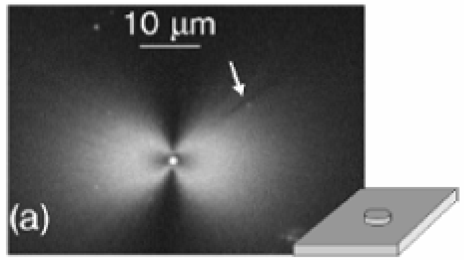
Coupling light to surface plasmon-polaritons

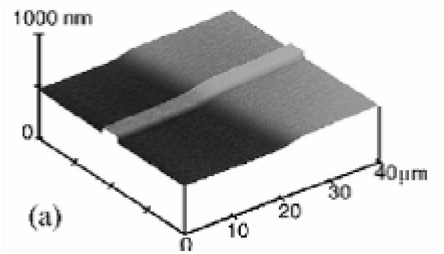
- Using high energy electrons (EELS)
- Kretchman geometry

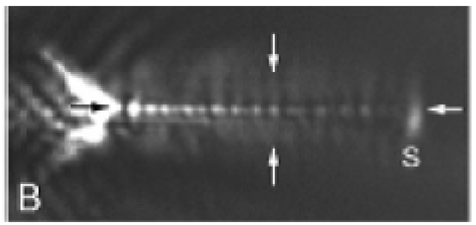
$$k_{//,SiO_2} = \sqrt{\varepsilon_d}\,\frac{\omega}{c}\sin\theta = k_{sp}$$

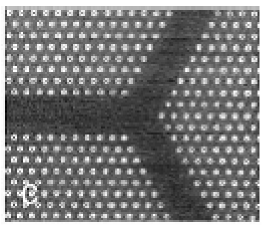


- Grating coupling
- Coupling using subwavelength features
- A diversity of guiding geometries



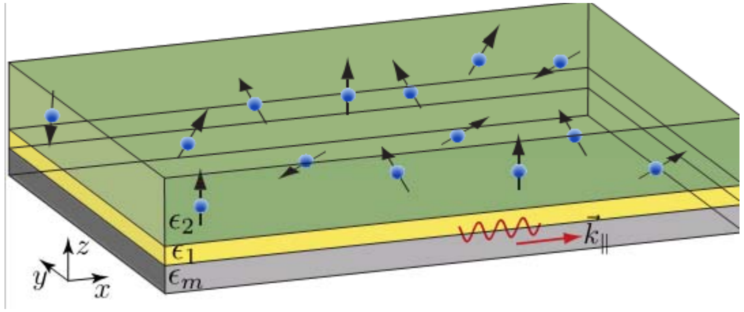




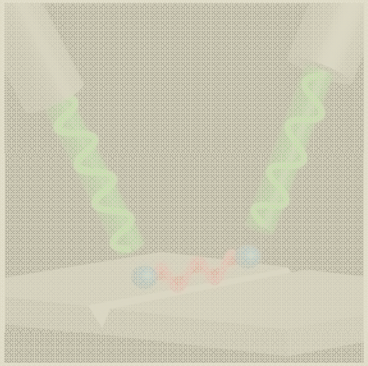


SPP

Strong coupling to excitons &



Intermediary for quantum entanglement



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Quantization of plasmons (without losses)

Quantization of an electric field

$$\vec{E}(\vec{r}) = \sum_{\vec{k}} \sqrt{\frac{\hbar\omega(k)}{2\epsilon_0 A}} \vec{u}_{\vec{k}}(z) e^{i(\vec{k}\cdot\vec{\rho}+k_z z)} a_{\vec{k}}$$

$$\vec{r} = (\vec{\rho}, z)$$

$$\vec{u}_{\vec{k}}(z) = \frac{1}{\sqrt{L(\vec{k})}} e^{-k_z z} \left(\hat{u}_{\vec{k}} - \frac{|\vec{k}|}{ik_z} \hat{u}_z \right)$$

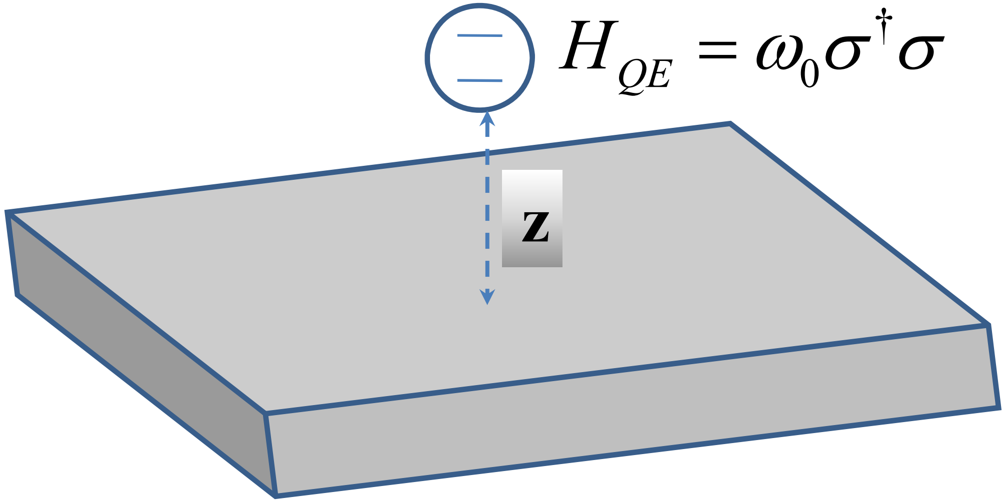
with

$$L(\omega) = \frac{\pi}{2} \frac{\epsilon_m(\omega) - \epsilon_d}{\sqrt{\epsilon_d \epsilon_m(\omega)}\, |\vec{k}(\omega)|} \left[\epsilon_m(\omega) + \epsilon_d \left(1 + \omega \frac{d\epsilon_m(\omega)}{d\omega} \right) \right]$$

effective length to normalize the energy of each mode,

$$H_{EM} = \sum_{\vec{k}} \omega(k) a_{\vec{k}}^{\dagger} a_{\vec{k}}$$

Interaction of 1 quantum emitter (QE) with SPP



$$H_{QE} = \omega_0 \sigma^\dagger \sigma$$

Interaction with a dipole

$$U = \int d\vec{r}\, \vec{\mu}(\vec{r}) \cdot \vec{E}(\vec{r})$$

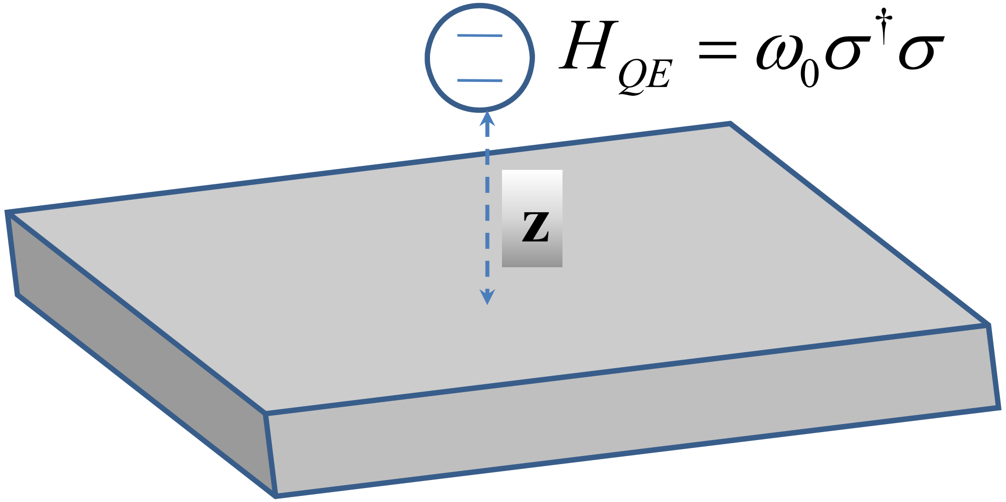
$$H_{int}(t) = \sum_{\vec{k}} \frac{g_{\vec{\mu}}(\vec{k};z)}{\sqrt{A}} \left(a_{\vec{k}} e^{i(\vec{k}\cdot\vec{r}-\omega(k)t)} + a_{\vec{k}}^\dagger e^{-i(\vec{k}\cdot\vec{r}-\omega(k)t)} \right) \left(\sigma^\dagger e^{i\omega_0 t} + \sigma e^{-i\omega_0 t} \right)$$

$$g_{\vec{\mu}}(\vec{k};z) = E_{\vec{k}} \vec{\mu} \cdot \vec{u}_{\vec{k}}(z) = \sqrt{\frac{\omega(\vec{k})}{2\epsilon_0 L(\vec{k})}} e^{-k_z z} \vec{\mu} \cdot \left(\hat{u}_{\vec{k}} - \frac{|\vec{k}|}{k_z} \hat{u}_z \right)$$

$$\mu^2 = 3\pi \varepsilon_0 c^3 \gamma_0 / \omega_0^3$$

decay rate of bare QE

Interaction of 1 quantum emitter (QE) with SPP



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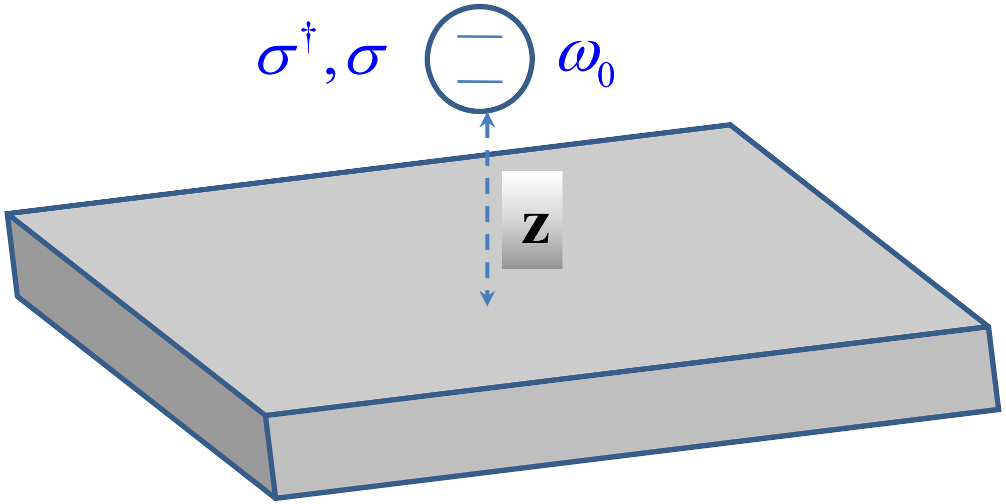
decay rate of bare QE

In RWA

$$H_{int} \approx \sum_{\vec{k}} \frac{g_{\vec{\mu}}(\vec{k};z)}{\sqrt{A}} \left(a_{\vec{k}} \sigma^\dagger e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{\vec{k}} \sigma e^{-i\vec{k}\cdot\vec{r}} \right)$$

Interaction of 1 quantum emitter (QE) with SPP

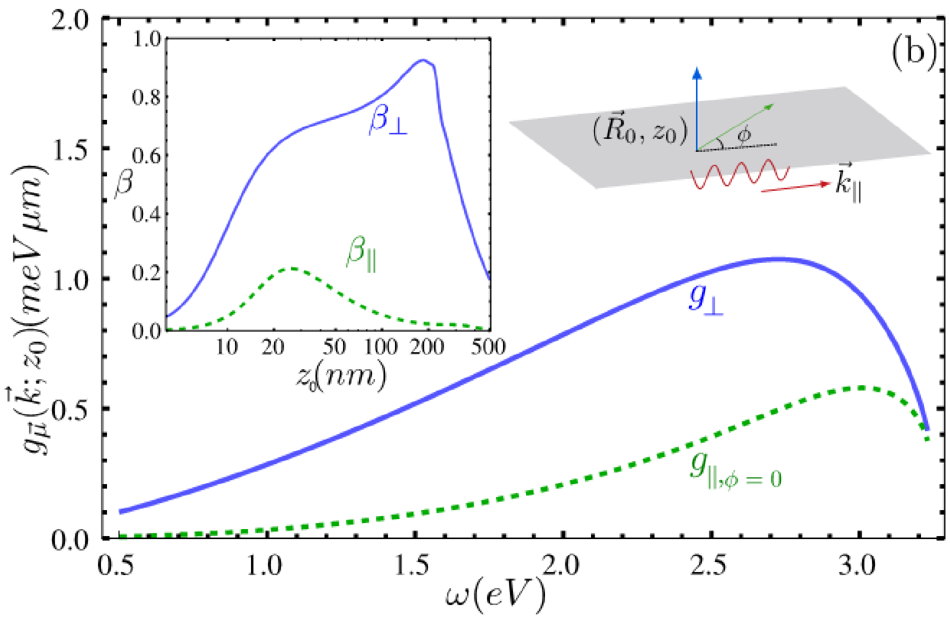


One QE with ω_0 only couples to a bright SPP $\equiv$ symmetric linear comb. (J_0 Bessel funct.) of all the modes with

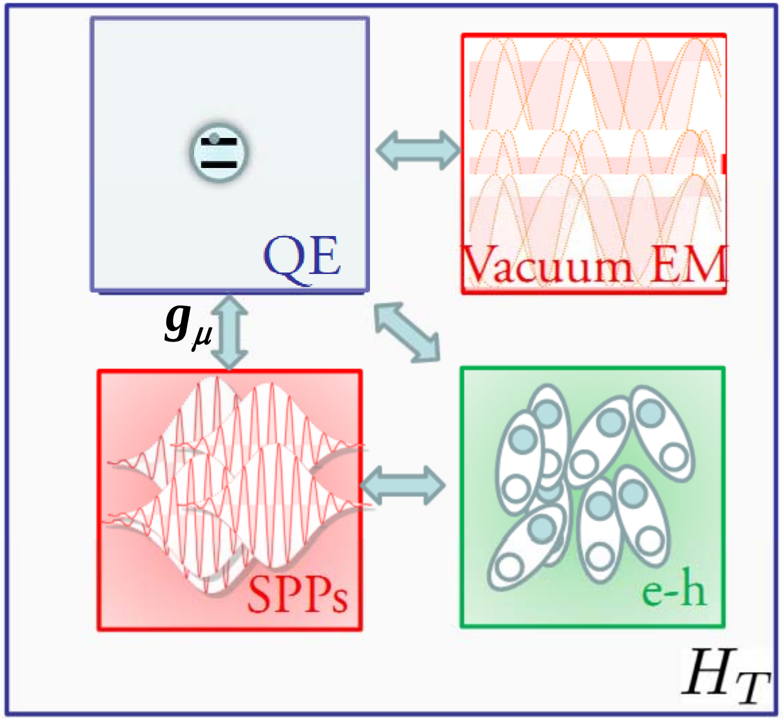
$$\left|\vec{k}\right|; \omega_0 = \omega_{SPP}\left(\left|\vec{k}\right|\right)$$

The higher coupling does not coincide with the higher β-factor

$$\beta = \frac{radiation\ to\ plasmons}{total\ radiation} = \frac{\gamma_{pl}}{\gamma}$$



Scheme of the quantum dynamics of an open system



Solving $i\frac{\partial\left|\Phi_{TOT}\right\rangle}{\partial t}=H_{TOT}\left|\Phi_{TOT}\right\rangle$

is complicated and unnecessary

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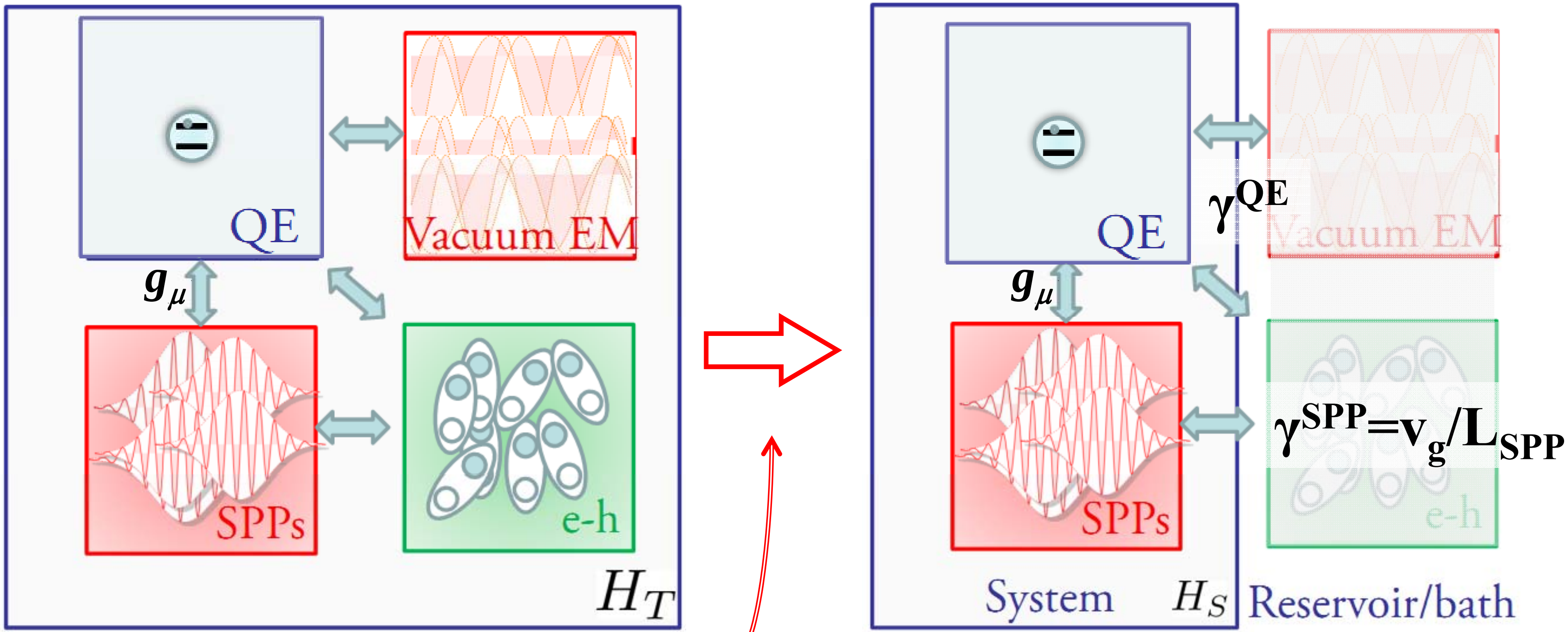

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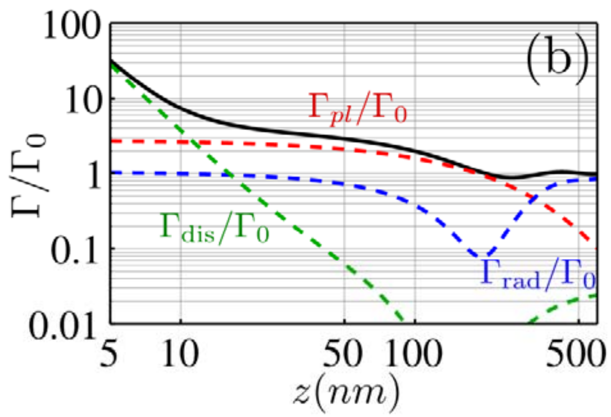
Tracing out the reservoir's degrees of freedom



Dynamics of the QE population: Weisskopf-Wigner

$$\dot{c}_\sigma(t) = -\int_0^t K_{\vec{\mu}}(t-\tau;z)c_\sigma(\tau)d\tau - \gamma_\sigma c_\sigma(t)/2$$

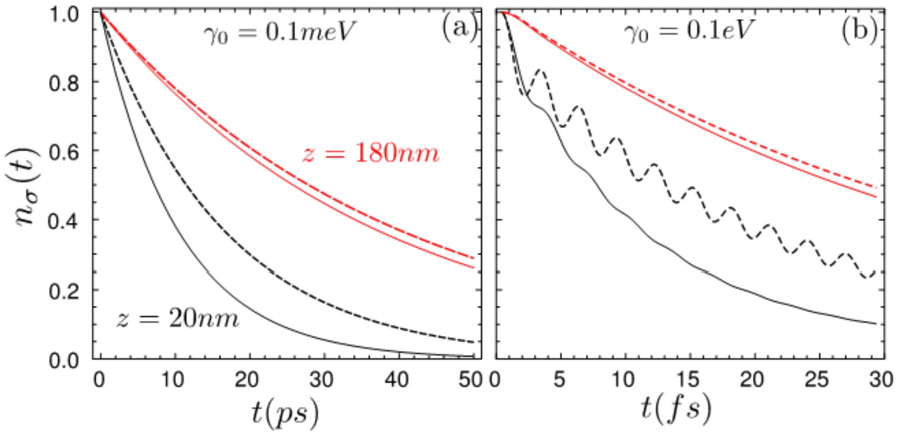
$$K_{\vec{\mu}}(\tau;z) = \sum_{\vec{k}} |g_{\vec{\mu}}(\vec{k};z)|^2 \, e^{i[\omega_0-\omega(\vec{k})]\tau} = \int_0^{\omega_c} d\omega J_{SPP}(\omega;z)e^{i(\omega_0-\omega)\tau}$$

With dissipation ———
Without dissipation - - - -

$$J_{SPP}(\omega;z) = \frac{1}{\pi\epsilon_0}\vec{\mu}[\frac{\omega^2}{c^2}\mathrm{Im}[\hat{G}_{\mathrm{SPP}}(\vec{r}_0,\vec{r}_0,\omega)]]\vec{\mu}$$

$$J^T_{\vec{\mu}}(\omega;z) = \quad ...\hat{G}(\vec{r}_0,\vec{r}_0,\omega)...$$

$$\gamma_\sigma(z_0) = 2\pi[J^T_{\vec{\mu}}(\omega_0;z_0) - J_{SPP}(\omega_0;z_0)]$$



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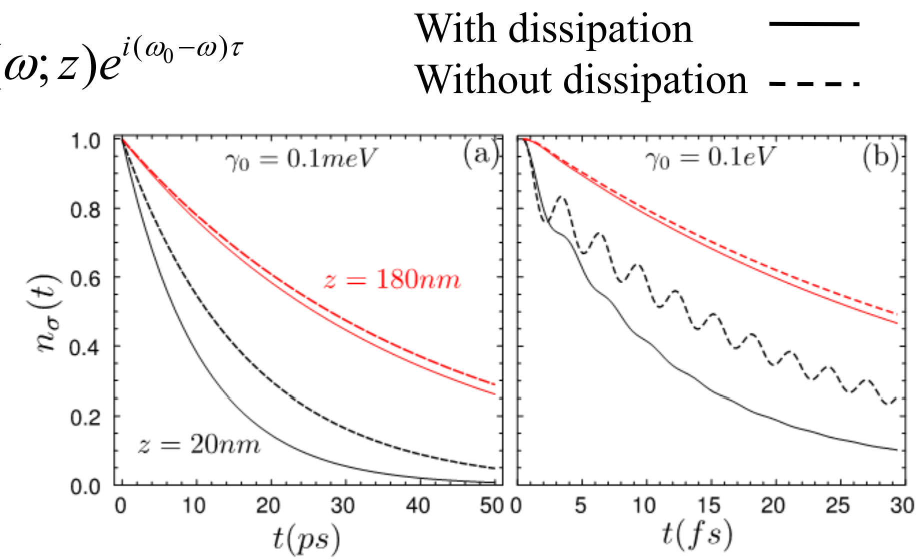
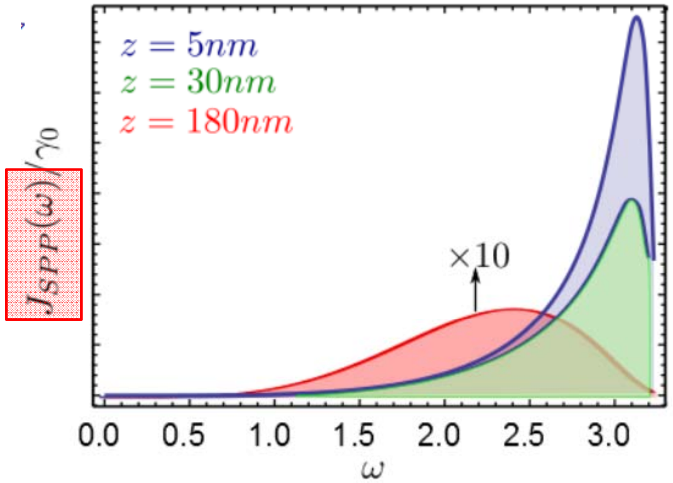
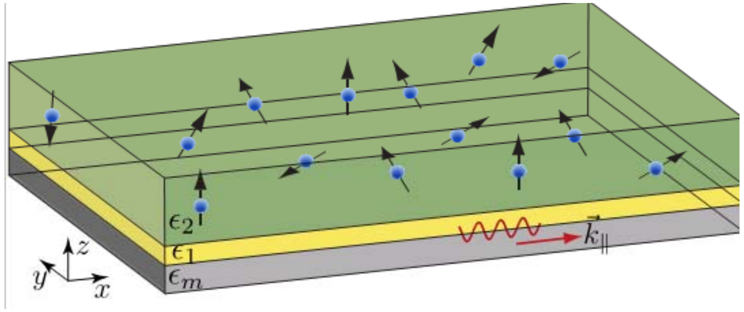


- **For a single QE, the SPP spectral density J_{SPP} governs the dynamics !**
- **Non lorentzian shape of $J_{SPP} \Rightarrow$ different dynamics that a pseudomode (cavity QED) !**
- **Height/width ratio of J_{SPP} determines the possible reversibility !**
- **$\gamma_0 << \omega_p \Rightarrow$ plasmon leave so fast that there is no time for reabsorption $\Leftrightarrow$ weak coupling**

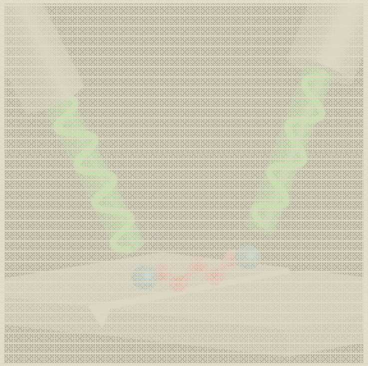


SPP

Strong coupling to excitons &



Intermediary for quantum entanglement



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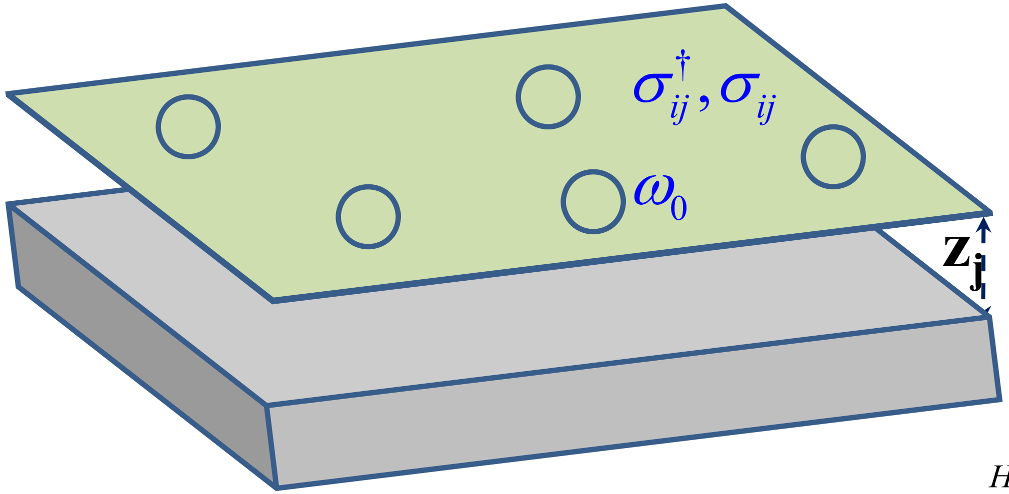
Exciton collective mode of emitters in a plane



More complicated system: Dynamics described by master eq. for density matrix & quantum regression th.

$$H_{0^N} + H_{pl} = \sum_{i=1}^{N_s} \omega_0 \; \sigma^{\dagger}{}_{i,j}\sigma_{i,j} + \sum_{\vec{k}} \; \omega(\mathrm{k}) a^{\dagger}{}_{\vec{k}} a_{\vec{k}}$$

$$H^{N}_{int} = \sum_{\vec{k}} \sum_{i=1}^{N_S} \frac{g_{\vec{\mu}}(\vec{k}; z_j)}{\sqrt{A}} (a^{\dagger}{}_{\vec{k}} \sigma_{i,j} e^{i\vec{k}\cdot\vec{r}_i} + a^{\dagger}{}_{\vec{k}} \sigma_{i,j} e^{-i\vec{k}\cdot\vec{r}_i})$$

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No fitting !!!

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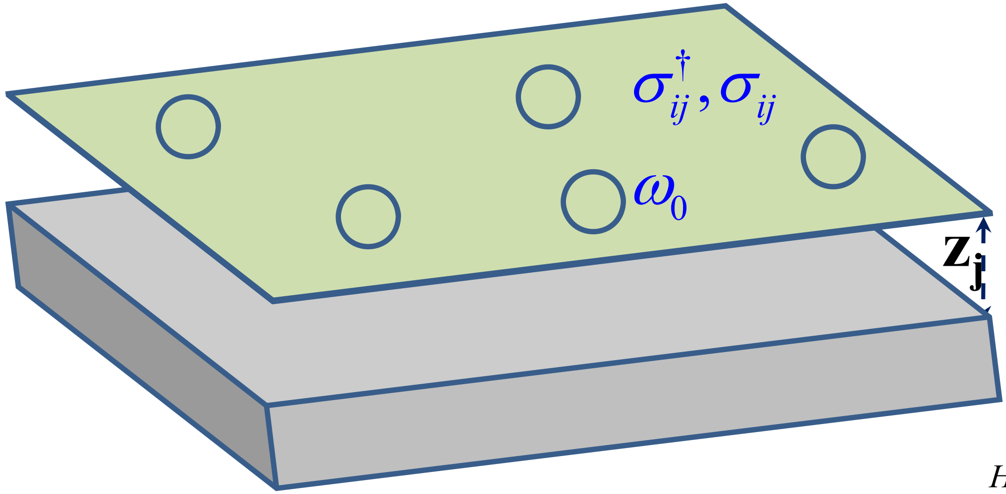

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No fitting !!!

Collective mode

Holstein-Primakoff transf.

(low excitation $\Rightarrow$ no saturation)

$\Downarrow$

$$D_j^{\dagger}(\vec{q}) = \frac{1}{\sqrt{N_s}} \sum_{i=1}^{N_s} \sigma_{i,j}^{\dagger} e^{i\vec{q}\cdot\vec{r}_i}$$

$$\sigma^{\dagger}{}_{i,j} = \frac{1}{\sqrt{N_s}} \sum_{\vec{q}} D_j^{\dagger}(\vec{q}) e^{-i\vec{q}\cdot\vec{r}_i}$$

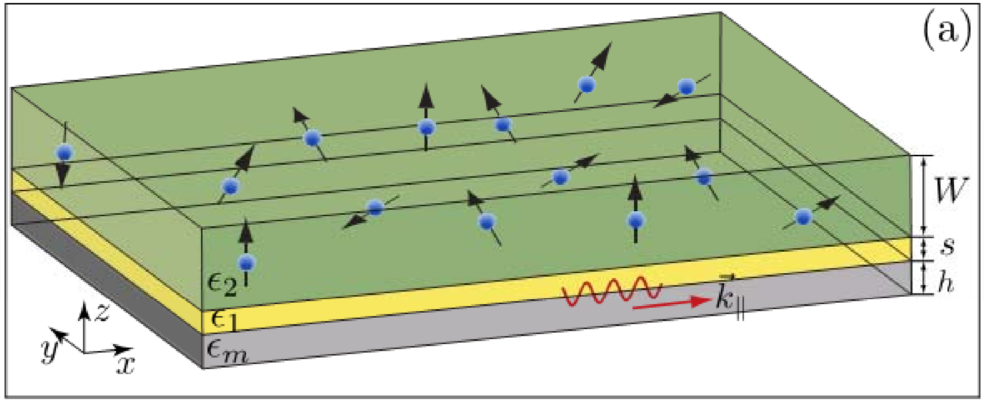
$$H_{int} = \sum_{\vec{k},\vec{q}} \sum_{i=1}^{N_s} \frac{g_{\vec{\mu}}(\vec{k}; z_j)\sqrt{n_s}}{N_s} \left(S(\vec{k}-\vec{q})\cdot\vec{r}_i a_{\vec{k}} D_j^{\dagger}(\vec{q}) + S^{*}(\vec{k}-\vec{q})\cdot\vec{r}_i a^{\dagger}{}_{\vec{k}} D_j(\vec{q}) \right)$$

$$S(\vec{k}) = \frac{1}{N_s} \sum_{i=1}^{N_s} e^{i\vec{k}\cdot\vec{r}_i}$$

Structure factor

Experimental evidence of strong coupling of SPP & excitons

QE are not just in a plane



Ensembles of organic molecules

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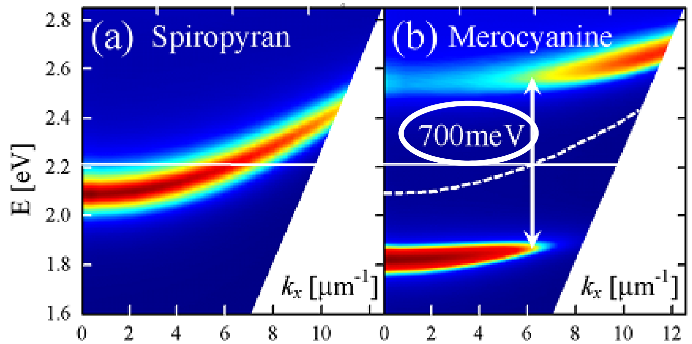

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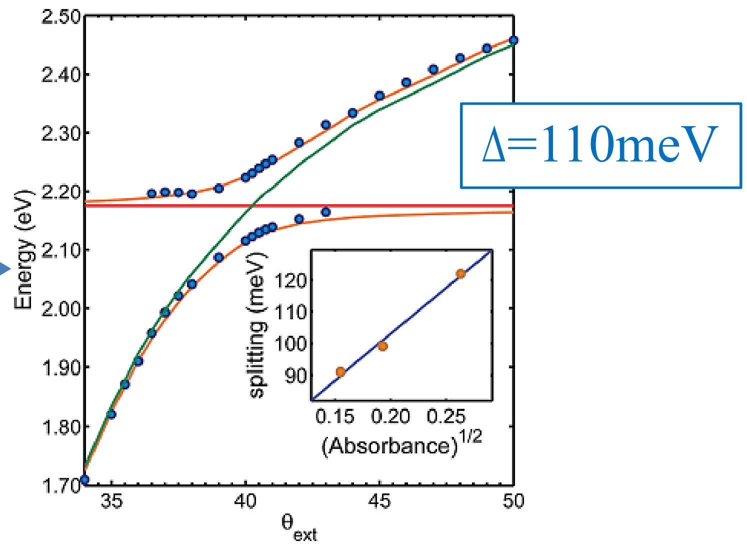
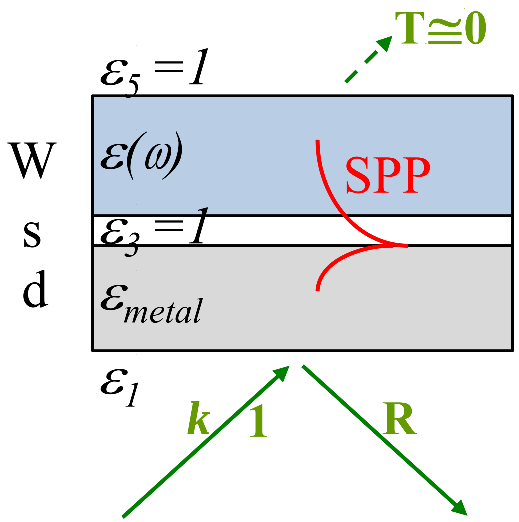


(Semi)-classical description

Polarizability of 1 emitter

$$\alpha(\omega) = \frac{f_0 e^2 / m}{\omega_0^2 - \omega^2 i\omega\gamma} \; ; \; \varepsilon(\omega) = \frac{1 + (2/3) N\alpha(\omega)}{1 - (1/3) N\alpha(\omega)}$$

Eff. dielectric funct. of emitters



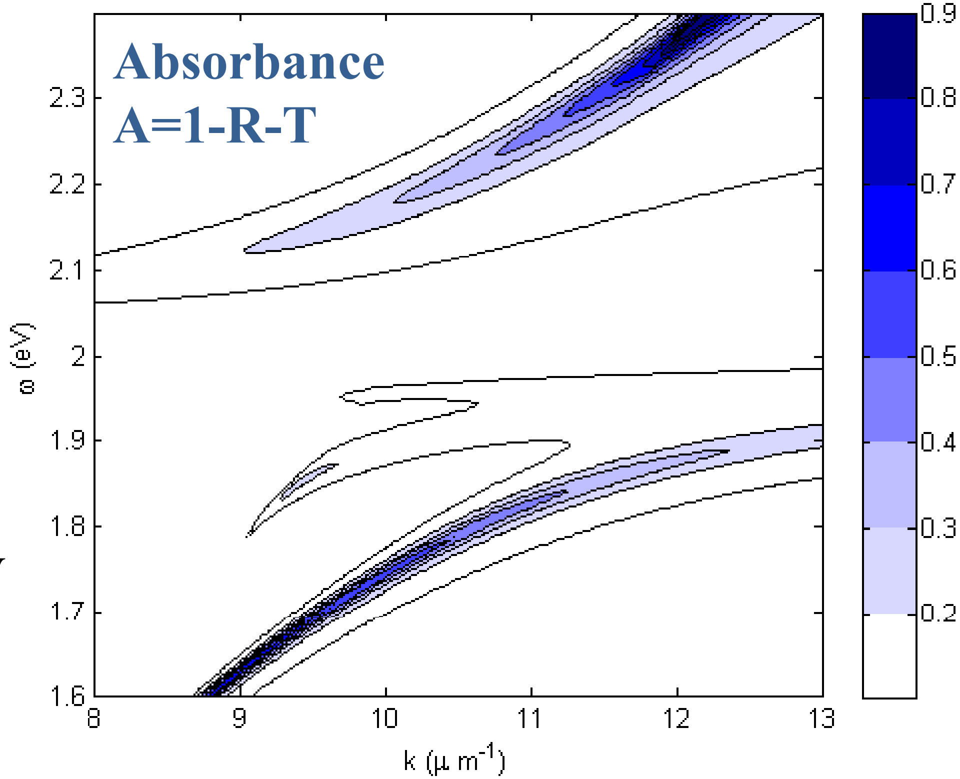
Oscillator strength from microsocpic info.

$$f_0 = \frac{2m\omega_0}{3e^2\hbar}|\vec{\mu}|^2 = \frac{2\pi m}{e^2}\frac{\varepsilon_0 c^3}{\omega_0^2}\gamma_0$$

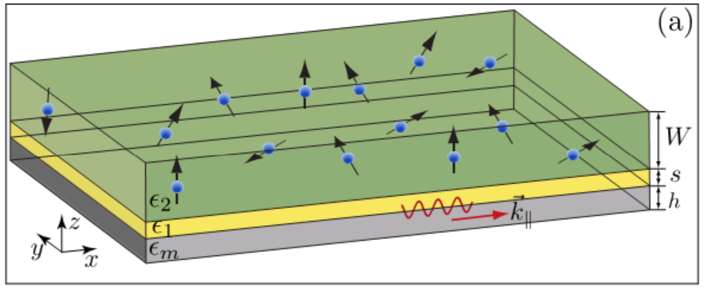
ε_1=3; d=50nm; s=1nm; W=500nm

ω_0=2eV; N=10^6 μm^{-3}

$\gamma = \gamma_0 + \gamma_{deph}$; γ_0=0.1meV; γ_{deph}=40meV



Excitonic collective mode in the volume of width W



Coupling depends on distance z_j

Average of random orientations

$g_{\vec{\mu}}(\vec{k};z_j) \Rightarrow$ **More complicated collective mode**

For many QE with disorder $S(\vec{k}-\vec{q}) \approx \delta_{\vec{k},0}$ momentum is conserved

$$H_{\text{int}} = \sum_{\vec{k}} \sum_{j=1}^{N_L} g_{\vec{\mu}}(\vec{k};z_j)\sqrt{n_s}\left(a_{\vec{k}} D_j^\dagger(\vec{k}) + a^\dagger_{\vec{k}} D_j(\vec{k})\right)$$

$$D^\dagger(\vec{k}) = \frac{1}{g_{\vec{\mu}}^N(\vec{k})} \sum_{j=1}^{N_L} g_{\vec{\mu}}(\vec{k};z_j) D_j^\dagger(\vec{k}) \quad ; \quad [D_i(\vec{k}), D_j^\dagger(\vec{k})] = \delta_{ij}$$

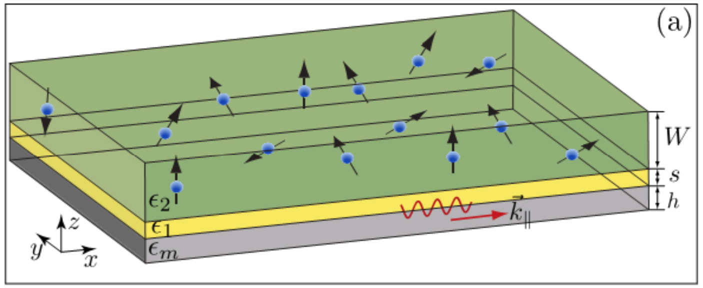
$$g_{\vec{\mu}}^N(\vec{k}) = \sqrt{\sum_{j=1}^{N_L} |g_{\vec{\mu}}(\vec{k},z_j)|^2} \rightarrow \sqrt{n\int_s^{s+\mathcal{W}} \mathrm{d}z |\, g_{\vec{\mu}}(\vec{k},z)|^2}$$

No fitting !

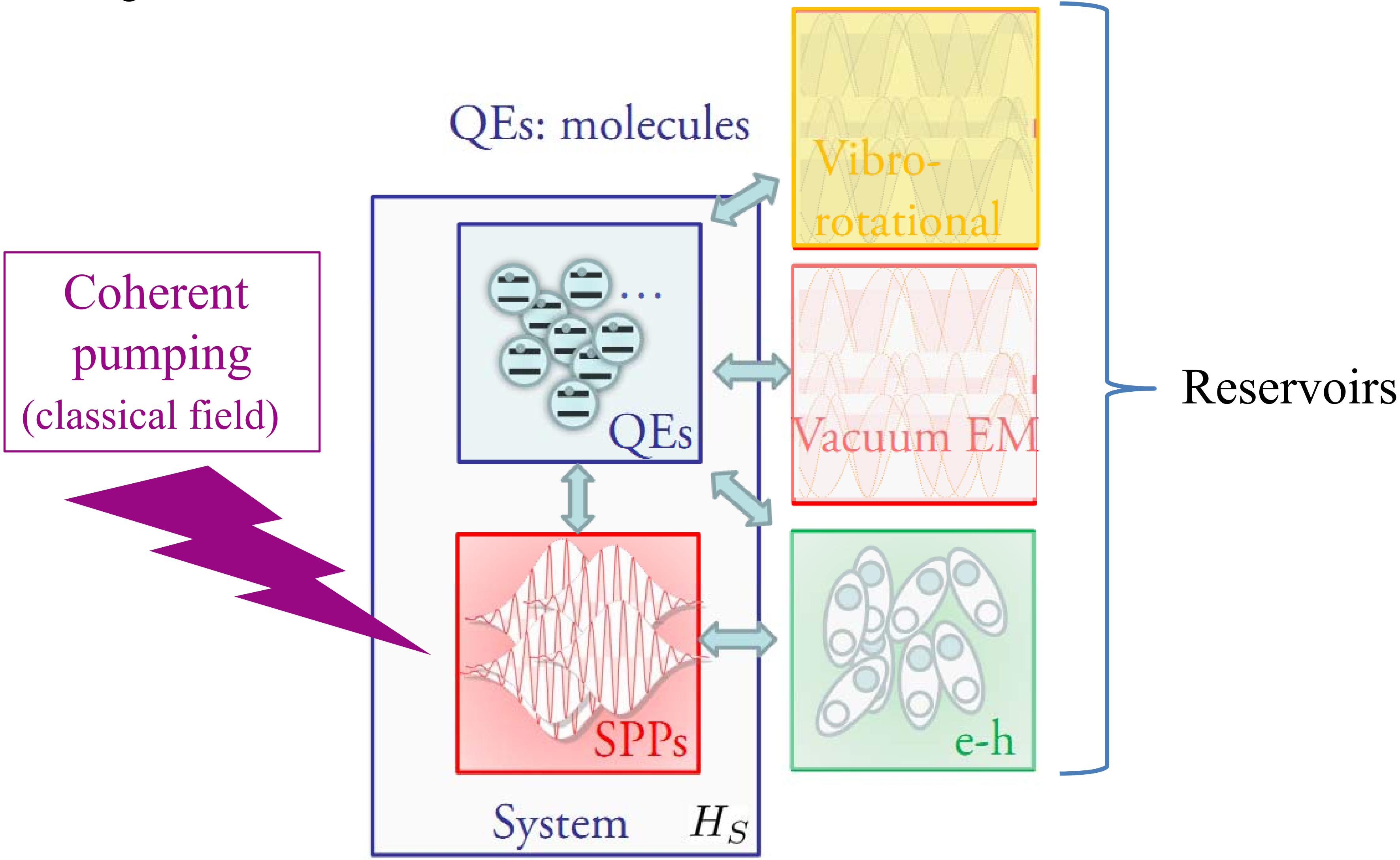
$$H_{\vec{k}}^N = \omega_0 D^\dagger(\vec{k}) D(\vec{k}) + \omega_{\vec{k}} a^\dagger(\vec{k}) a(\vec{k}) + g_{\vec{\mu}}^N(\vec{k})(a(\vec{k}) D^\dagger(\vec{k}) + a^\dagger(\vec{k}) D(\vec{k}))$$

Decay of the collective mode $\gamma_{D_{\vec{k}}} = \dfrac{n}{|g_{\vec{\mu}}^N(\vec{k})|^2} \displaystyle\int_s^{s+\mathcal{W}} \mathrm{d}z\, \gamma_\sigma(z)\, |\, g_{\vec{\mu}}^N(\vec{k},z)|^2$

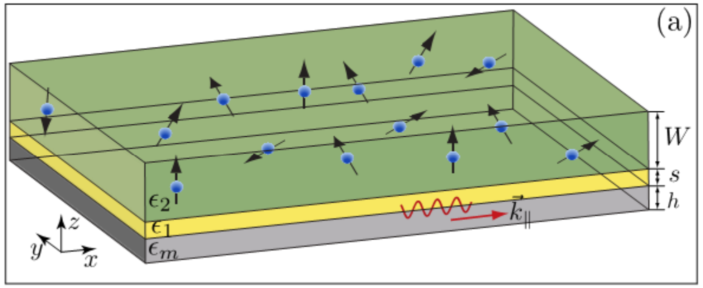
Dynamics under coherent pumping of a SPP with k-vector



Average of random orientations



Dynamics under coherent pumping of a SPP with k-vector



$$H^N_{\vec{k}} = \omega_0 D^\dagger(\vec{k})D(\vec{k}) + \omega_{\vec{k}} a^\dagger(\vec{k})a(\vec{k}) + g^N_{\vec{\mu}}(\vec{k})(a(\vec{k})D^\dagger(\vec{k}) + a^\dagger(\vec{k})D(\vec{k}))$$

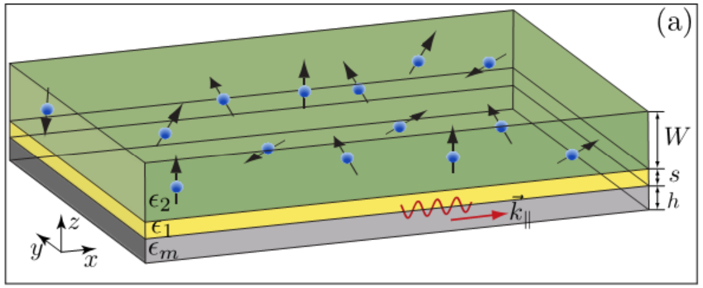
$$H^L_{\vec{k}}(t) = \Omega_{\vec{k}}(a_{\vec{k}} e^{i\omega_L t} + a^\dagger_{\vec{k}} e^{-i\omega_L t})$$

$$\dot{\rho}_{\vec{k}} = i[\rho_{\vec{k}}, H^N_{\vec{k}} + H^L_{\vec{k}}] + \frac{\gamma_{D_{\vec{k}}}}{2}\mathcal{L}_{D_{\vec{k}}} + \frac{\gamma_{a_{\vec{k}}}}{2}\mathcal{L}_{a_{\vec{k}}} + \frac{\gamma_\phi}{2}\mathcal{L}_{D^\dagger_{\vec{k}} D_{\vec{k}}}$$

Exciton decay — Plasmon decay — Pure dephasing (vibro-rotation)

$$\mathcal{L}_c = (2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$$

Dynamics under coherent pumping of a SPP with k-vector



$$H_{\vec{k}}^{N} = \omega_0 D^{\dagger}(\vec{k})D(\vec{k}) + \omega_{\vec{k}} a^{\dagger}(\vec{k})a(\vec{k}) + g_{\vec{\mu}}^{N}(\vec{k})(a(\vec{k})D^{\dagger}(\vec{k}) + a^{\dagger}(\vec{k})D(\vec{k}))$$

$$H_{\vec{k}}^{L}(t) = \Omega_{\vec{k}}(a_{\vec{k}} e^{i\omega_L t} + a_{\vec{k}}^{\dagger} e^{-i\omega_L t})$$

$$\dot{\rho}_{\vec{k}} = i[\rho_{\vec{k}}, H_{\vec{k}}^{N} + H_{\vec{k}}^{L}] + \frac{\gamma_{D_{\vec{k}}}}{2}\mathcal{L}_{D_{\vec{k}}} + \frac{\gamma_{a_{\vec{k}}}}{2}\mathcal{L}_{a_{\vec{k}}} + \frac{\gamma_{\phi}}{2}\mathcal{L}_{D_{\vec{k}}^{\dagger}D_{\vec{k}}}$$

Exciton decay — Plasmon decay — Pure dephasing (vibro-rotation)

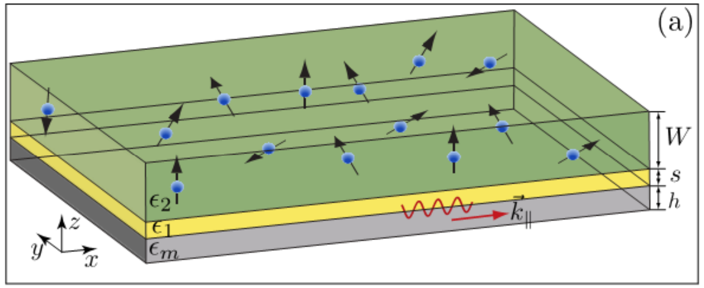
$$\mathcal{L}_c = (2c\rho c^{\dagger} - c^{\dagger}c\rho - \rho c^{\dagger}c)$$

At the crossing (k_0) between exciton and SPP,

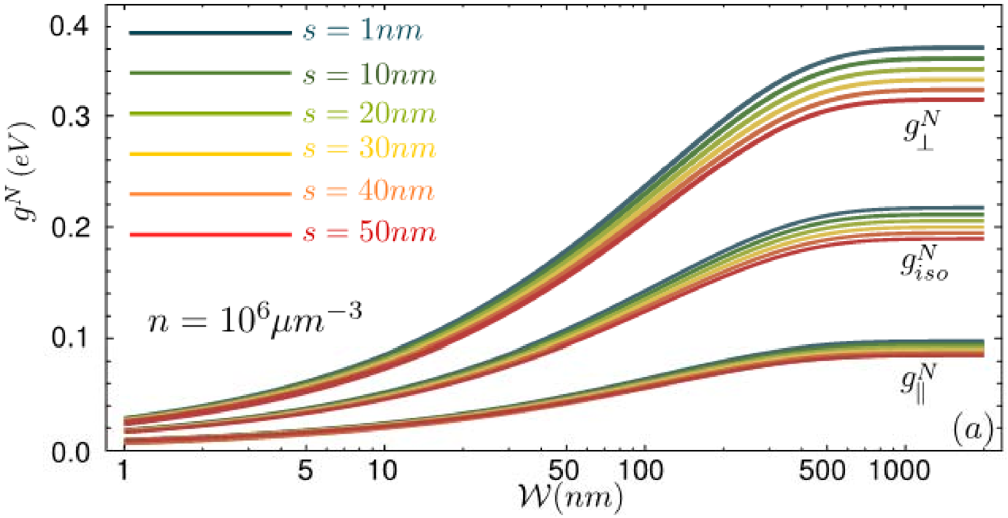
Rabi splitting is analytical

$$R = \sqrt{[g_{\vec{\mu}}^{N}(\vec{k}_0)]^2 - (\gamma_{D_{\vec{k}_0}} + \gamma_{\phi} - \gamma_{a_{\vec{k}_0}})^2/4} \quad with\ [g_{\vec{\mu}}^{N}(\vec{k}_0)]^2 \propto n$$

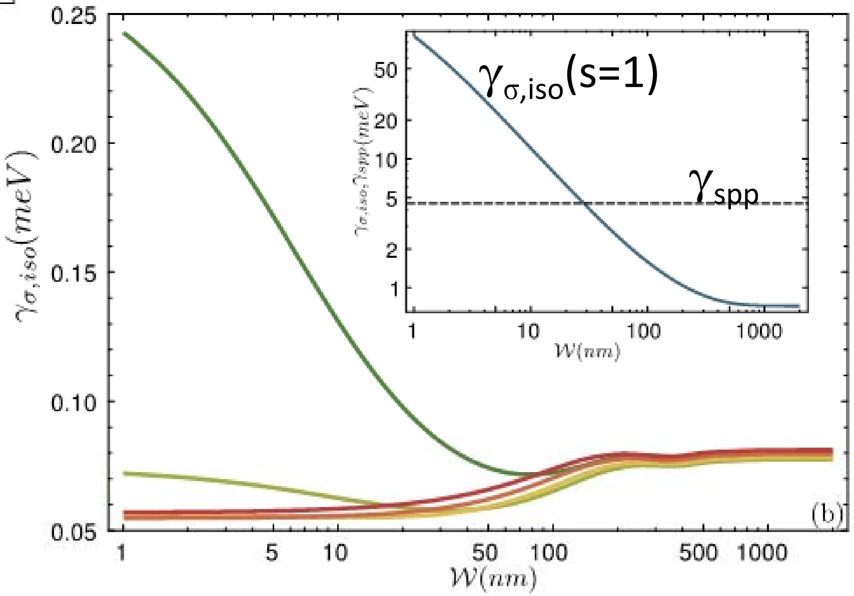
Strong coupling between SPP & excitons



$$\gamma_0 = 0.1meV$$



- g^N **depends on W** with saturation due to SPP z-decay
- g^N **practically independ. on s**



$$\Omega_{\vec{k}} = 0.1 g^N$$

$$n = 10^6\ \mu m^{-3}$$

$$\omega_0 = 2eV$$

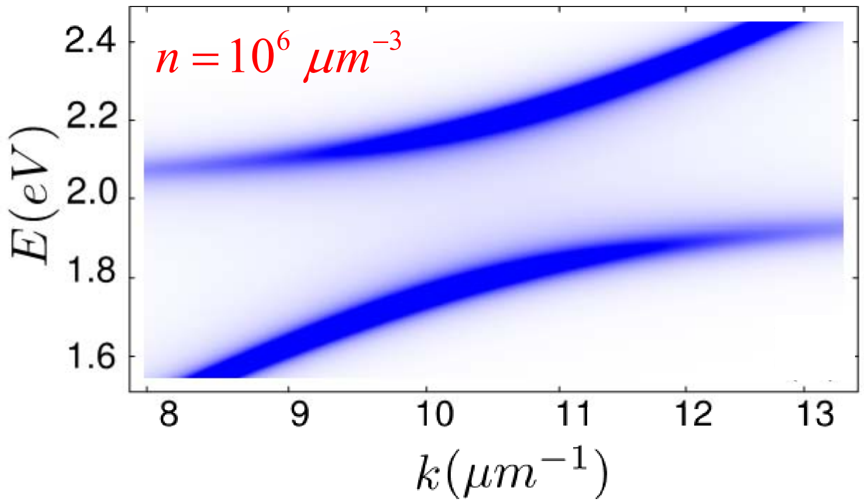
Strong coupling between SPP & excitons

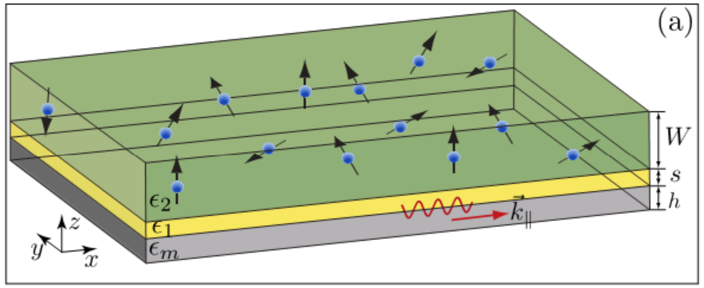
$$s = 1\,nm\,;\, W = 500\,nm$$

$$\omega_0 = 2eV\;;\;\; \Omega_{\vec{k}} = 0.1g^N\;(40meV)$$

$$\gamma_0 = 0.1meV\,;\, \gamma_\phi = 0.1g^N\,(RT)$$

Polariton populations $\propto$ absorption spect.





Strong coupling between SPP & excitons

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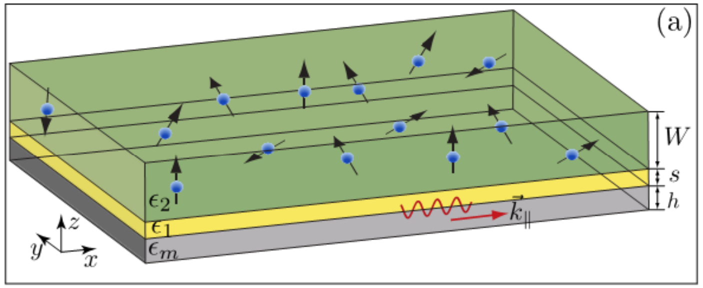
$\omega_0 = 2eV\ ;\ \Omega_{\vec{k}} = 0.1g^N\ (40meV)$

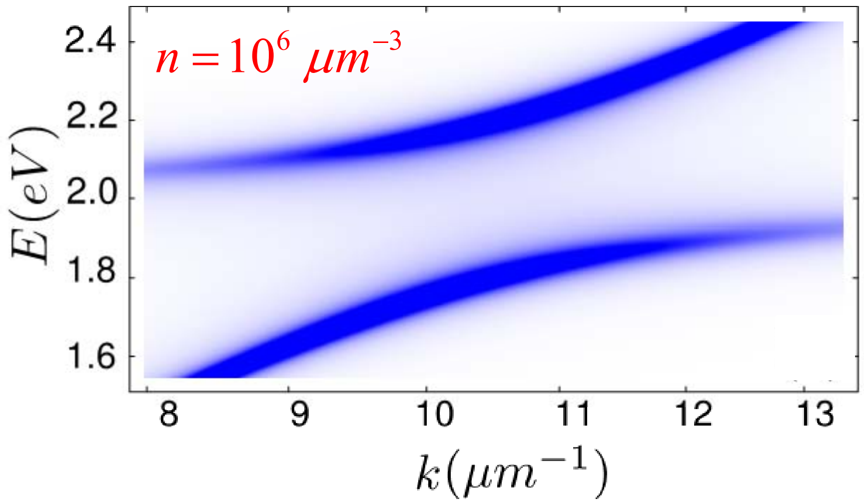
$\gamma_0 = 0.1meV\,;\,\gamma_\phi = 0.1g^N\,(RT)$

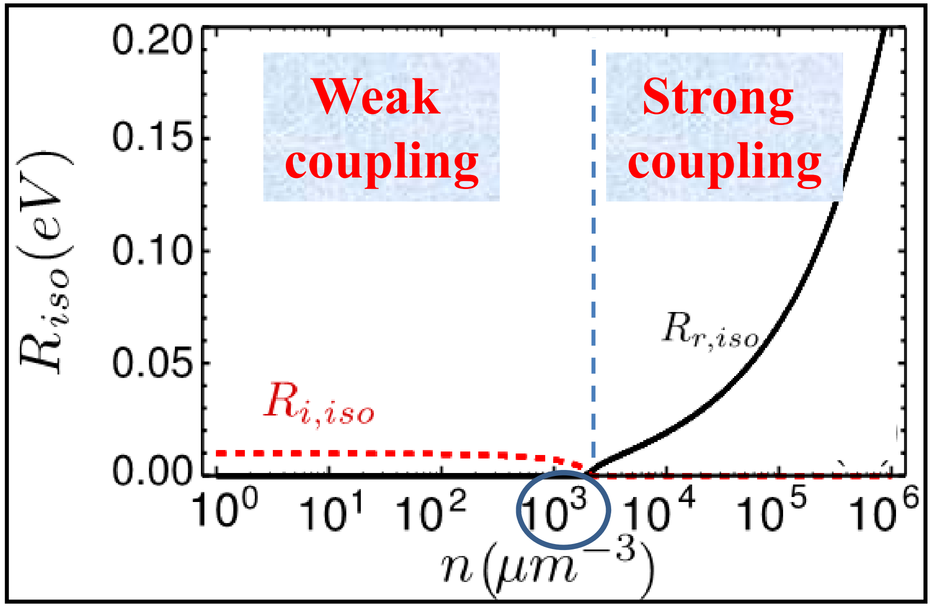


Rabi splitting (at k_0)

$R = \sqrt{[g_{\vec{\mu}}^N(\vec{k}_0)]^2 - (\gamma_{D_{\vec{k}_0}} + \gamma_\phi - \gamma_{a_{\vec{k}_0}})^2/4}\,;[g_{\vec{\mu}}^N(\vec{k}_0)]^2 \propto n$

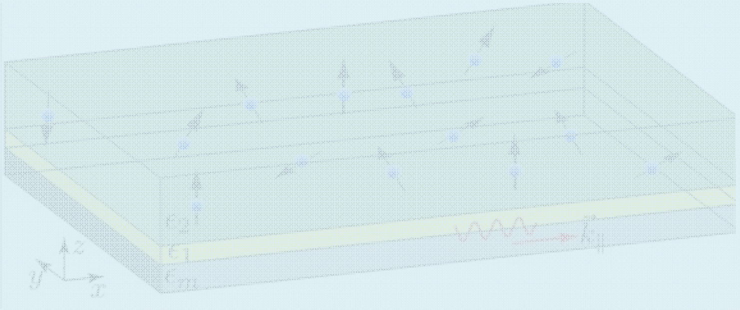
Polariton populations $\propto$ absorption spect.





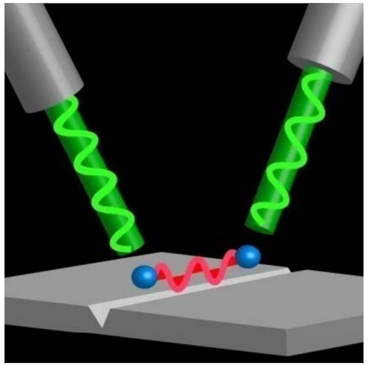
At RT, the incoherent processes (γ_ϕ) determine a critical density for observing strong coupling

Strong coupling to excitons &



SPP

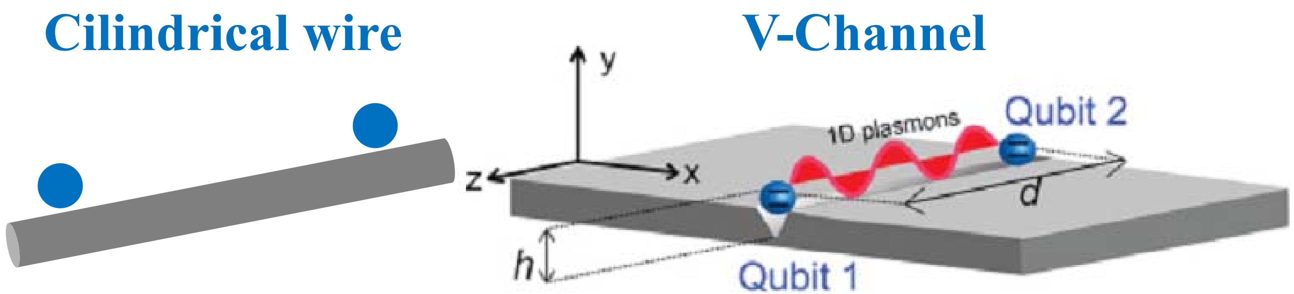
Intermediary for quantum entanglement



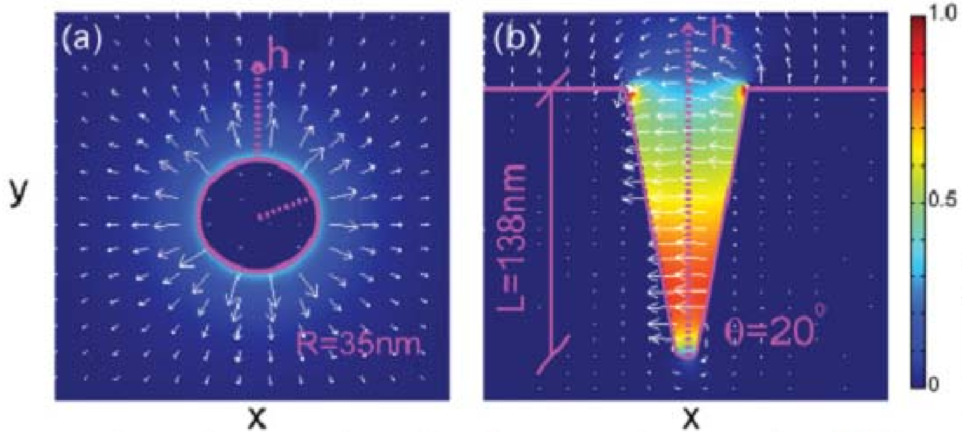
Outline

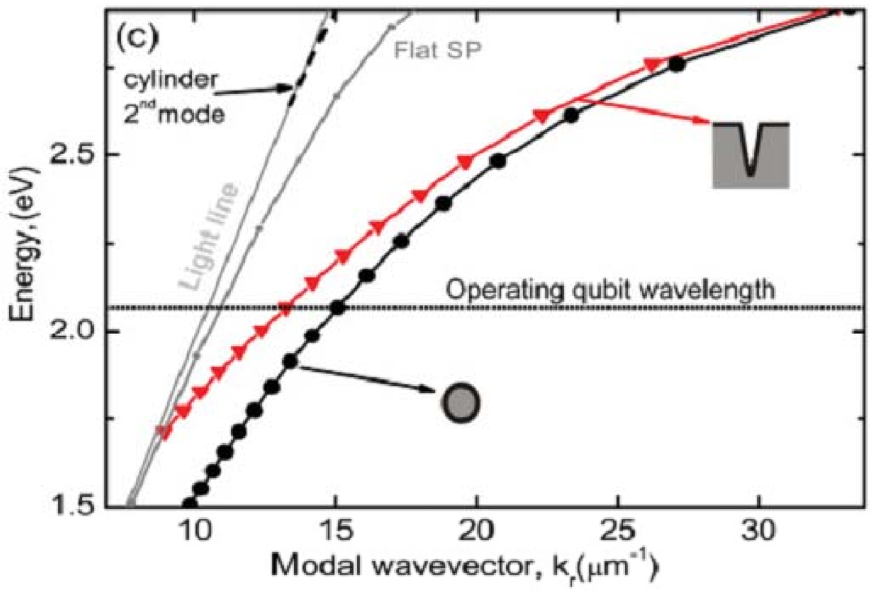
- Introduction to surface plasmon polaritons (SPP)
- Coupling of 1 quantum emitter (QE) to SPP
- Collective mode (excitons) strongly coupled to SPP
- Coupling & entanglement of 2 QE mediated by SPP
- Conclusion

QE-QE coupling mediated by plasmonic waveguides



Waveguide to reinforce QE-QE effects



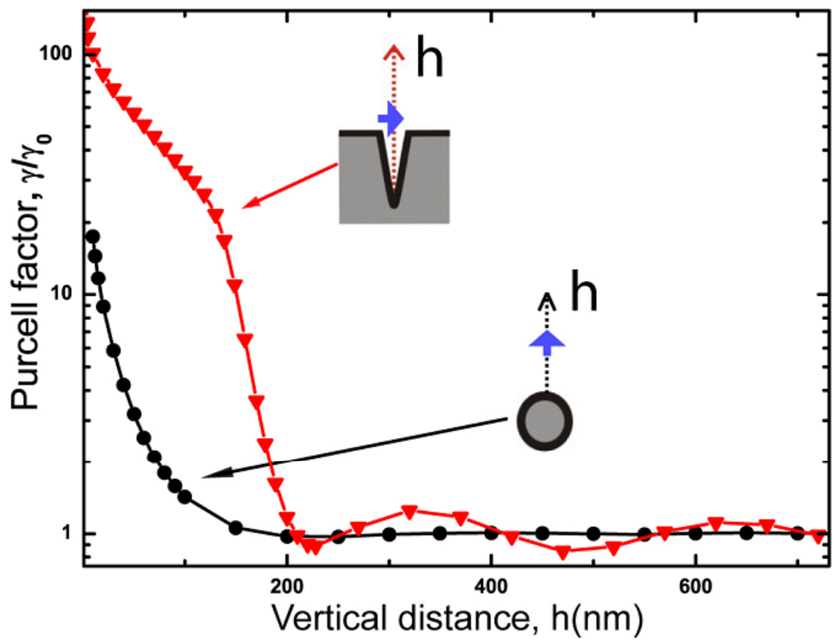


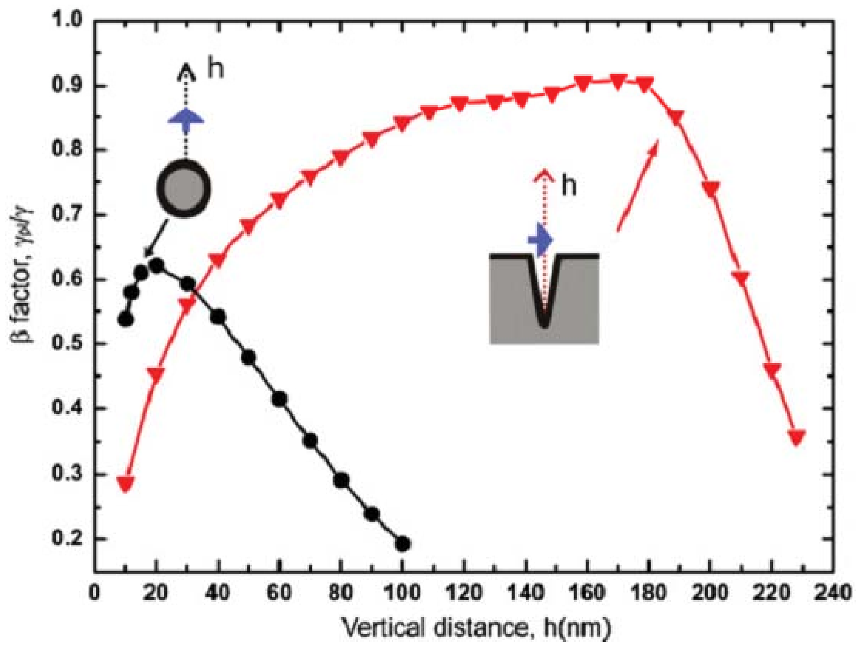
Fields are stronger in the channel than in the cilinder

1QE: β and Purcell factors

Metallic nanostructures increase the emission from a QE (Purcell)
but,
Is it always a coherent emission of SPP's**???** (**β**)

$$Purcell\ \ factor = \frac{total\ radiation}{QE\ radiation\ to\ vacuum} = \frac{\gamma}{\gamma_0};\ \beta - factor = \frac{radiation\ to\ plasmons}{total\ radiation} = \frac{\gamma_{pl}}{\gamma}$$

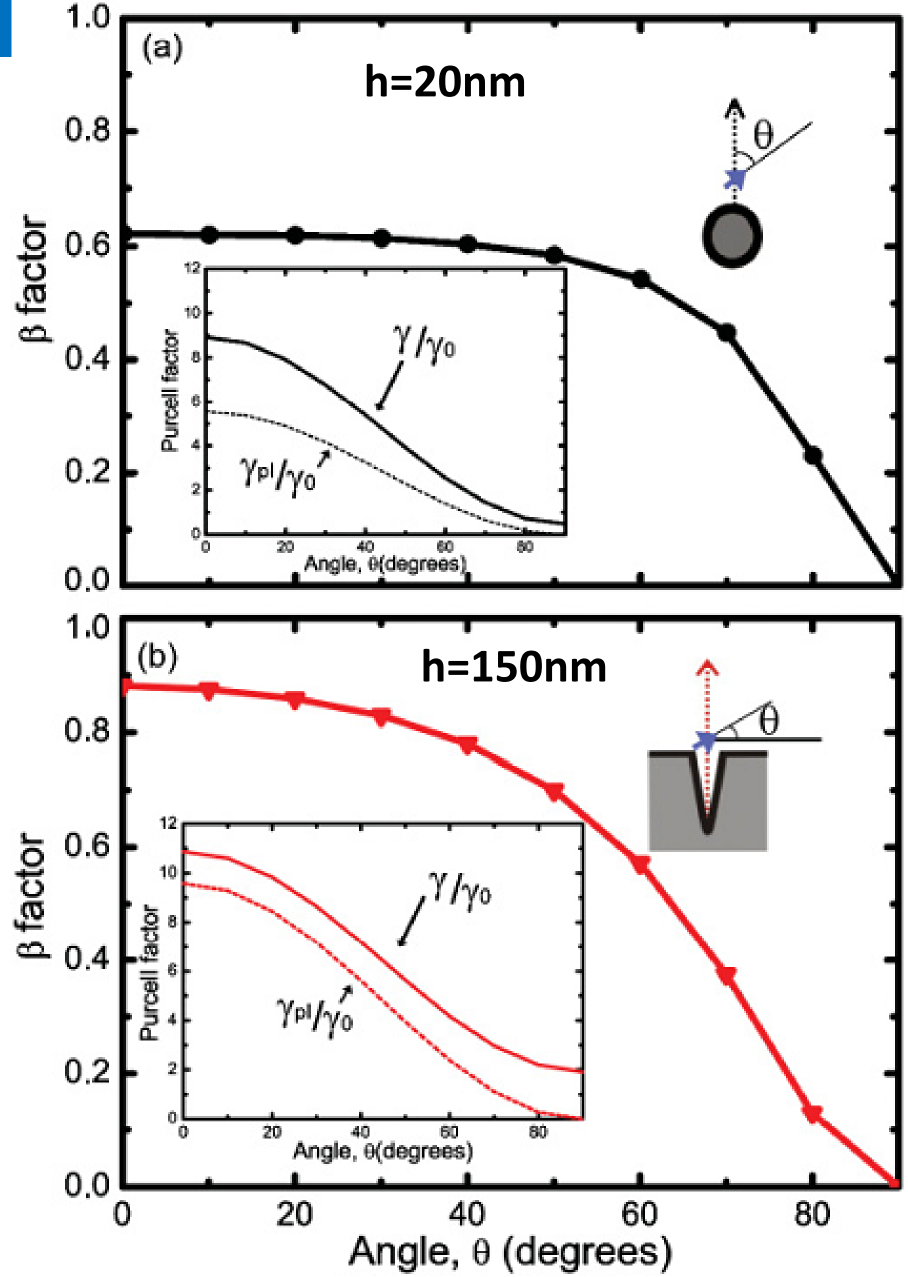




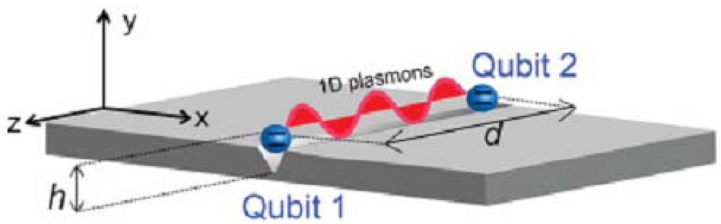
The channel is more convenient than the cilinder

β and Purcell factors

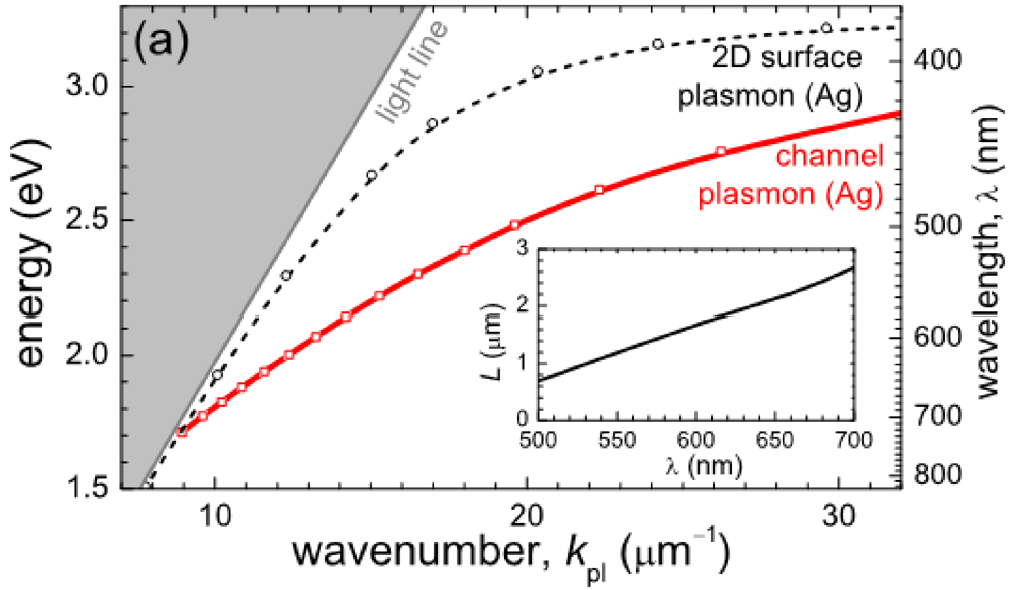
β- factor is very stable in a broad range of dipole orientations while Purcell factor decreases significantly when the dipole is not properly oriented

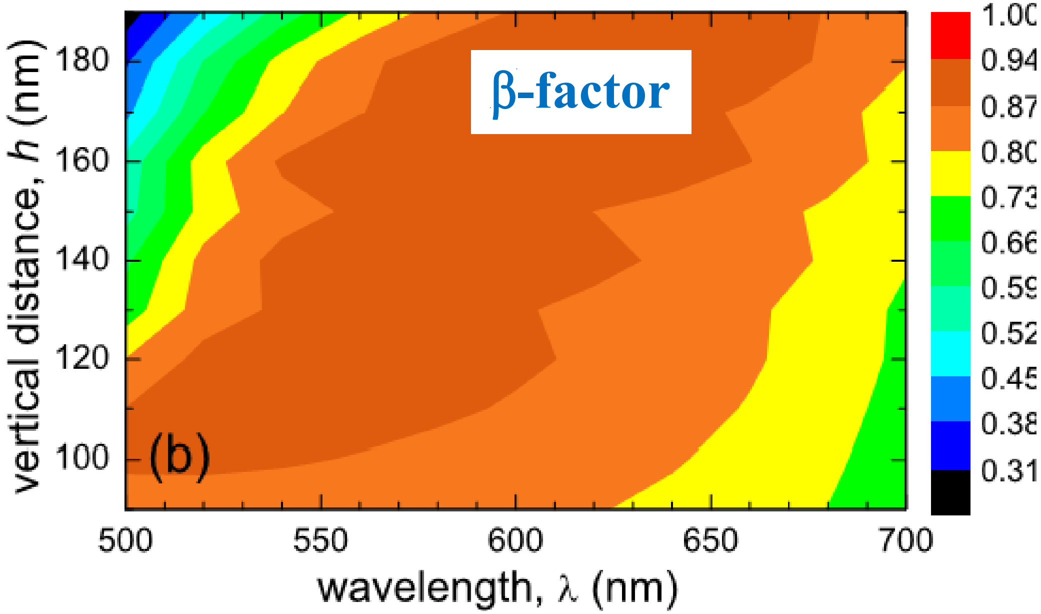


Dispersion & β factor for V-channel



h=140nm
V-angle= 20 degrees





Two QE's dynamics

All the degrees of freedom (SPP, dissipation, radiation) can be traced out producing effective coherent & incoherent interactions between the two QE's that can be computed from the classical Green's function:

$$\hat{H} = \int d^3\mathbf{r} \int_0^\infty d\omega \hbar\omega a^\dagger(\mathbf{r},\omega) a(\mathbf{r},\boldsymbol{\omega}) + \sum_{i=1,2} \hbar\omega_0 \hat{\sigma}_i^\dagger \hat{\sigma}_i - \sum_{i=1,2} \int_0^\infty d\omega [\hat{\mathbf{d}}_i \hat{\mathbf{E}}(\mathbf{r}_i,\omega) + h.c.]$$

$$\hat{\mathbf{E}}(\mathbf{r},\omega) = i\sqrt{\frac{\hbar}{\pi\epsilon_0}} \frac{\omega^2}{c^2} \int d^3\mathbf{r}' \sqrt{\varepsilon_i(\mathbf{r}',\omega)}\, \mathbf{G}(\mathbf{r},\mathbf{r}',\boldsymbol{\omega}) a(\mathbf{r}',\omega)$$

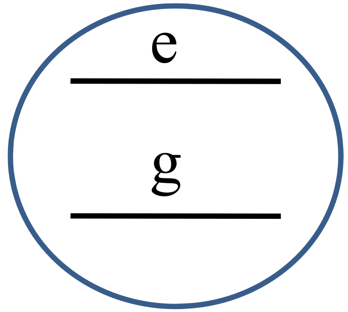
$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s + \hat{H}_L, \hat{\rho}] - \frac{1}{2}\sum_{i,j} \gamma_{ij} (\hat{\rho}\hat{\boldsymbol{\sigma}}_i^\dagger \hat{\boldsymbol{\sigma}}_j + \hat{\boldsymbol{\sigma}}_i^\dagger \hat{\boldsymbol{\sigma}}_j \hat{\rho} - 2\hat{\boldsymbol{\sigma}}_j \hat{\rho} \hat{\boldsymbol{\sigma}}_i^\dagger)$$

$$\hat{H}_s = \sum_i \hbar(\omega_0 + \delta_i)\hat{\sigma}_i^\dagger \hat{\sigma}_i + \sum_{i \neq j} \hbar g_{ij} \hat{\sigma}_i^\dagger \hat{\sigma}_j \qquad \hat{H}_L = -\frac{1}{2}\sum_i \hbar\Omega_i \hat{\sigma}_i^\dagger e^{i\omega_L t} + h.c.$$

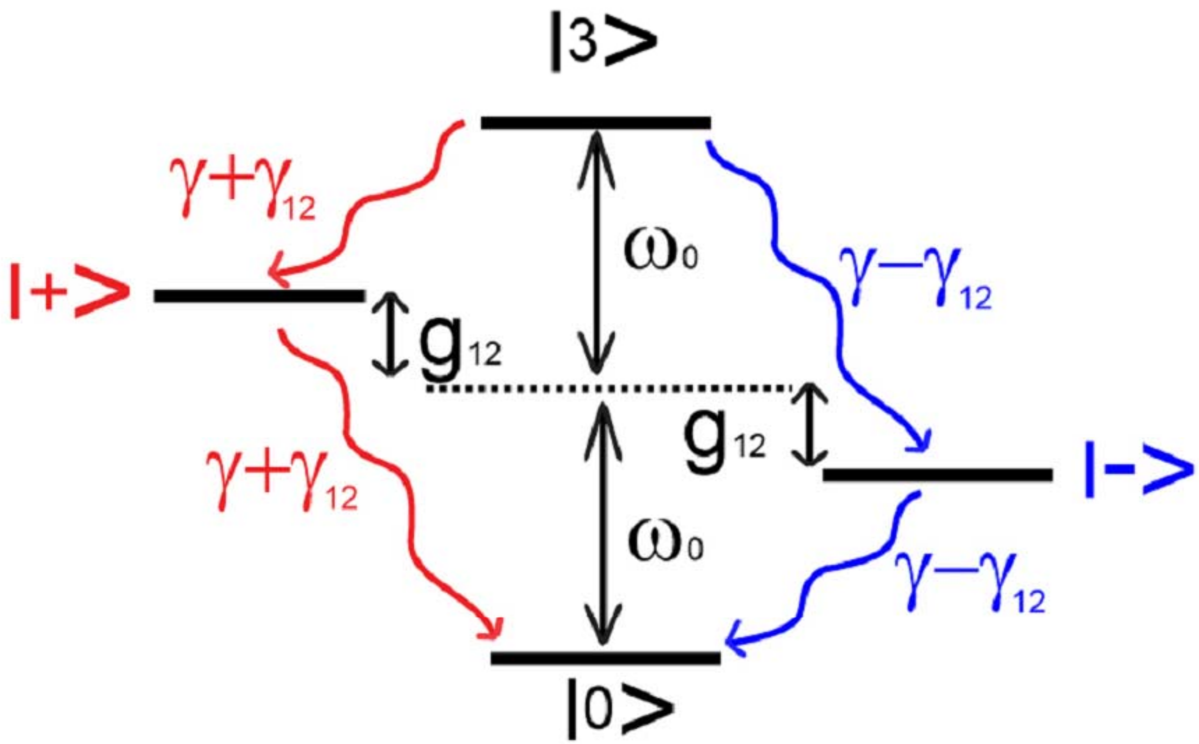
$$g_{ij} = \frac{\omega_0^2}{\hbar\varepsilon_0 c^2} \mathbf{d}_i^* Re\mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega_0)\mathbf{d}_j \qquad \gamma_{ij} = \frac{2\omega_0^2}{\hbar\varepsilon_0 c^2} \mathbf{d}_i^* Im\mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega_0)\mathbf{d}_j$$

Scheme of levels

QE



$$|3\rangle = |e_1 e_2\rangle \quad |0\rangle = |g_1 g_2\rangle \quad |\pm\rangle = \frac{1}{\sqrt{2}}(|g_1 e_2\rangle \pm |e_1 g_2\rangle)$$



Modulation of γ_{12} would allow to swicth on/off red and blue paths

Two QE's dynamics

It is possible to identify the effects of SPP & dissipation

SPP Green's function

$$\mathbf{G}_{SPP}(\mathbf{r},\mathbf{r}') = \frac{i\,\mathbf{E}^{t}(\mathbf{r}^{t})\otimes\mathbf{E}^{t}(\mathbf{r}'^{t})}{2\omega\mu_0\int_{S_\infty} dS\,\mathbf{u}_z(\mathbf{E}^{t}\times\mathbf{H}^{*t})}\, e^{ik(z-z')}$$

⇓

Effective interactions

Coherent

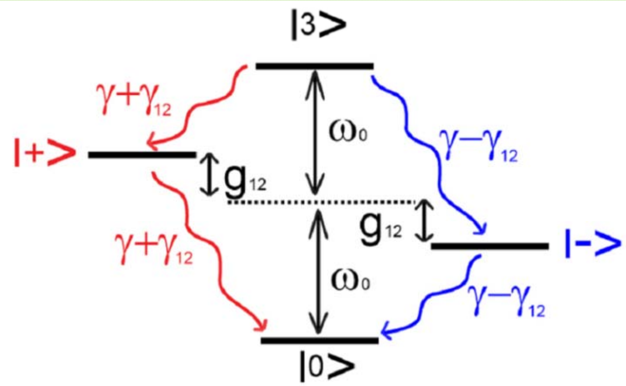
$$g_{ij} \simeq g_{ij,\mathrm{pl}} = \frac{\gamma}{2}\beta e^{-d/2\ell}\sin(k_r d)$$

Incoherent

$$\gamma_{ij} \simeq \gamma_{ij,\mathrm{pl}} = \gamma\beta e^{-d/2\ell}\cos(k_r d)$$

π/2 shift allows switching on/off

- **Coherent versus incoherent interactions**
- **Control of different decay paths**



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⇓

Effective interactions

Coherent

Incoherent

$$g_{ij} \simeq g_{ij,\mathrm{pl}} = \frac{\gamma}{2}\beta e^{-d/2\ell}\sin(k_r d)$$

$$\gamma_{ij} \simeq \gamma_{ij,\mathrm{pl}} = \gamma\beta e^{-d/2\ell}\cos(k_r d)$$

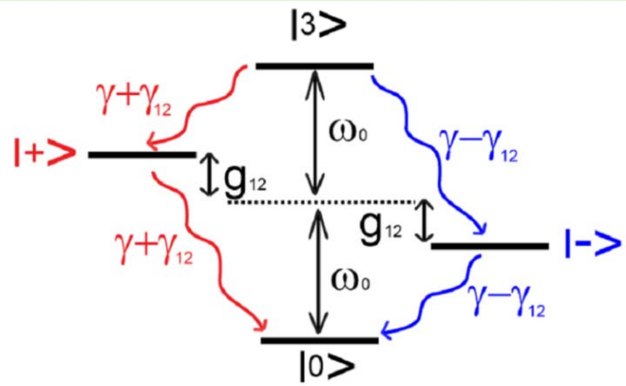
$$\frac{\partial\hat\rho}{\partial t} = -\frac{i}{\hbar}[\hat H_s + \hat H_L, \hat\rho] - \frac{1}{2}\sum_{i,j}\gamma_{ij}(\hat\rho\hat{\boldsymbol\sigma}_i^\dagger\hat{\boldsymbol\sigma}_j + \hat{\boldsymbol\sigma}_i^\dagger\hat{\boldsymbol\sigma}_j\hat\rho - 2\hat{\boldsymbol\sigma}_j\hat\rho\hat{\boldsymbol\sigma}_i^\dagger)$$

$$\hat H_s = \sum_i \hbar(\omega_0+\delta_i)\hat\sigma_i^\dagger\hat\sigma_i + \sum_{i\neq j}\hbar g_{ij}\hat\sigma_i^\dagger\hat\sigma_j$$

$$\hat H_L = -\frac{1}{2}\sum_i \hbar\Omega_i\hat\sigma_i^\dagger e^{i\omega_L t} + h.c.$$

π/2 shift allows switching on/off

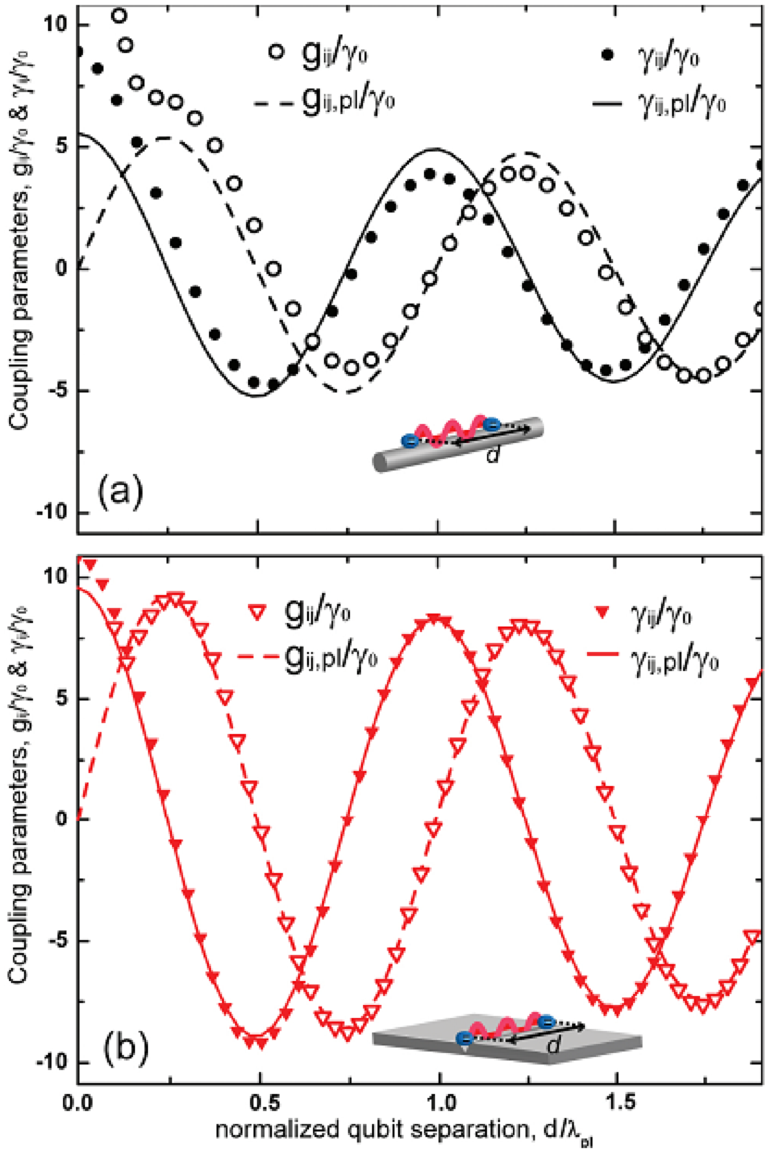
- **Coherent versus incoherent interactions**
- **Control of different decay paths**



For inequivalent dipoles

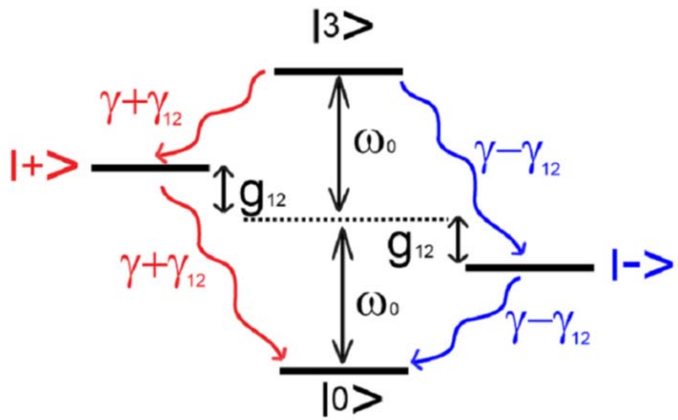
$$\gamma_{ij,\mathrm{pl}} = \sqrt{\gamma_{ii}\gamma_{jj}}\sqrt{\beta_i\beta_j}\,e^{-d/2\ell}\cos(k_r d)$$

Coherent (g_{ij}) & incoherent (γ_{ij}) effective couplings between QE's



Incoherent coupling much more important than the coherent one because it switchs on/off each decay path with respect to the other

$$|\gamma_{12}| \leq \gamma$$



Entanglement measure

VOLUME 80, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1998

Entanglement of Formation of an Arbitrary State of Two Qubits

William K. Wootters
Department of Physics, Williams College, Williamstown, Massachusetts 01267
(Received 12 September 1997)

Concurrence

Complex definition….

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$$

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

What we need to know:

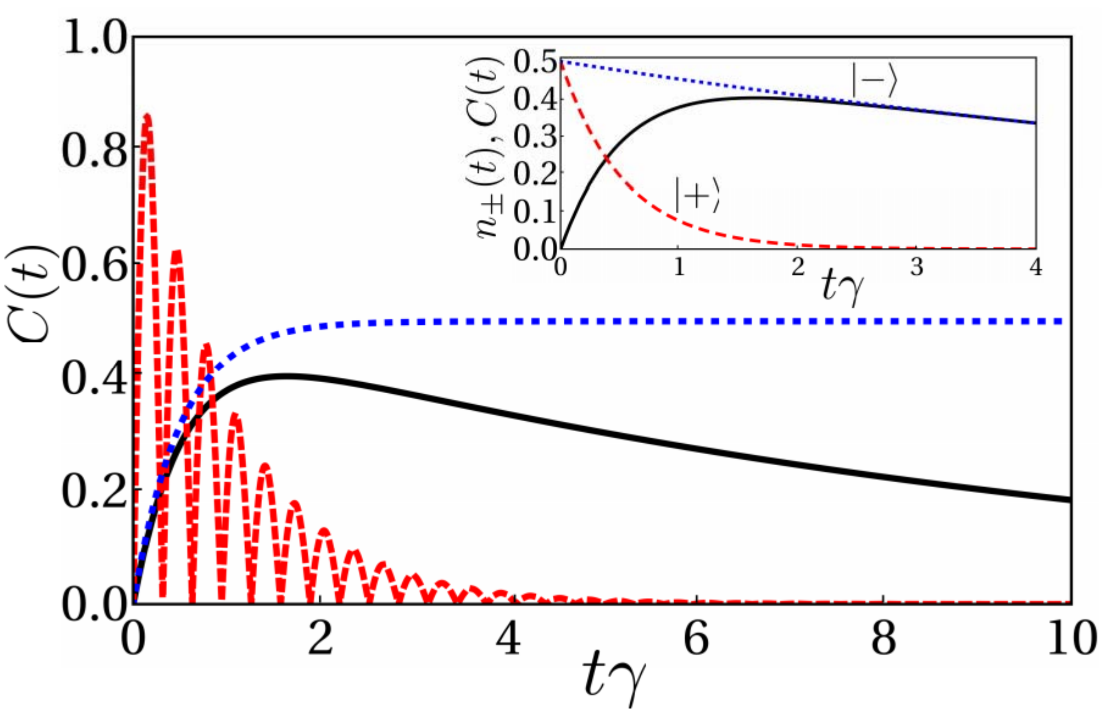
Separable states(i.e $|0\rangle$) => $C = 0$

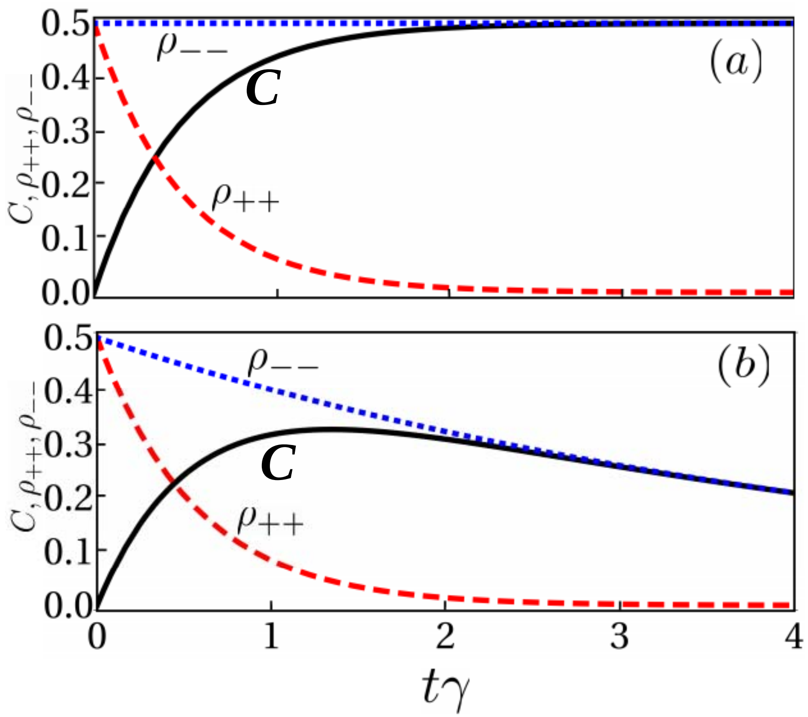
Entangled states(i. e.$|-\rangle$) => $C = 1$

Spontaneous decay of a single excitation

$|\psi(t=0)\rangle = |1\rangle = |e_1 g_2\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ $\Rightarrow$ **Concurrence becomes:**

$$C(t) = \sqrt{[\rho_{++}(t) - \rho_{--}(t)]^2 + 4Im[\rho_{+-}(t)]^2} = e^{-\gamma t}\sinh[\gamma\beta e^{-\lambda_{\rm pl}/(2\ell)} t]$$



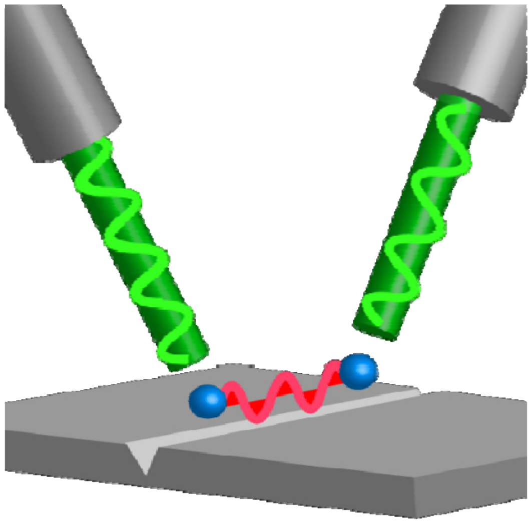


Stationary entanglement

(in the previous viewgraph) Spontaneous decay mediated by plasmons produces finite-time entanglement starting from an unentangled state

$$|\psi(t=0)\rangle = |1\rangle = |e_1 g_2\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

But one wants both to ***obtain*** and ***manipulate*** **stationary entanglement.**
This can be done by means of lasers**:**

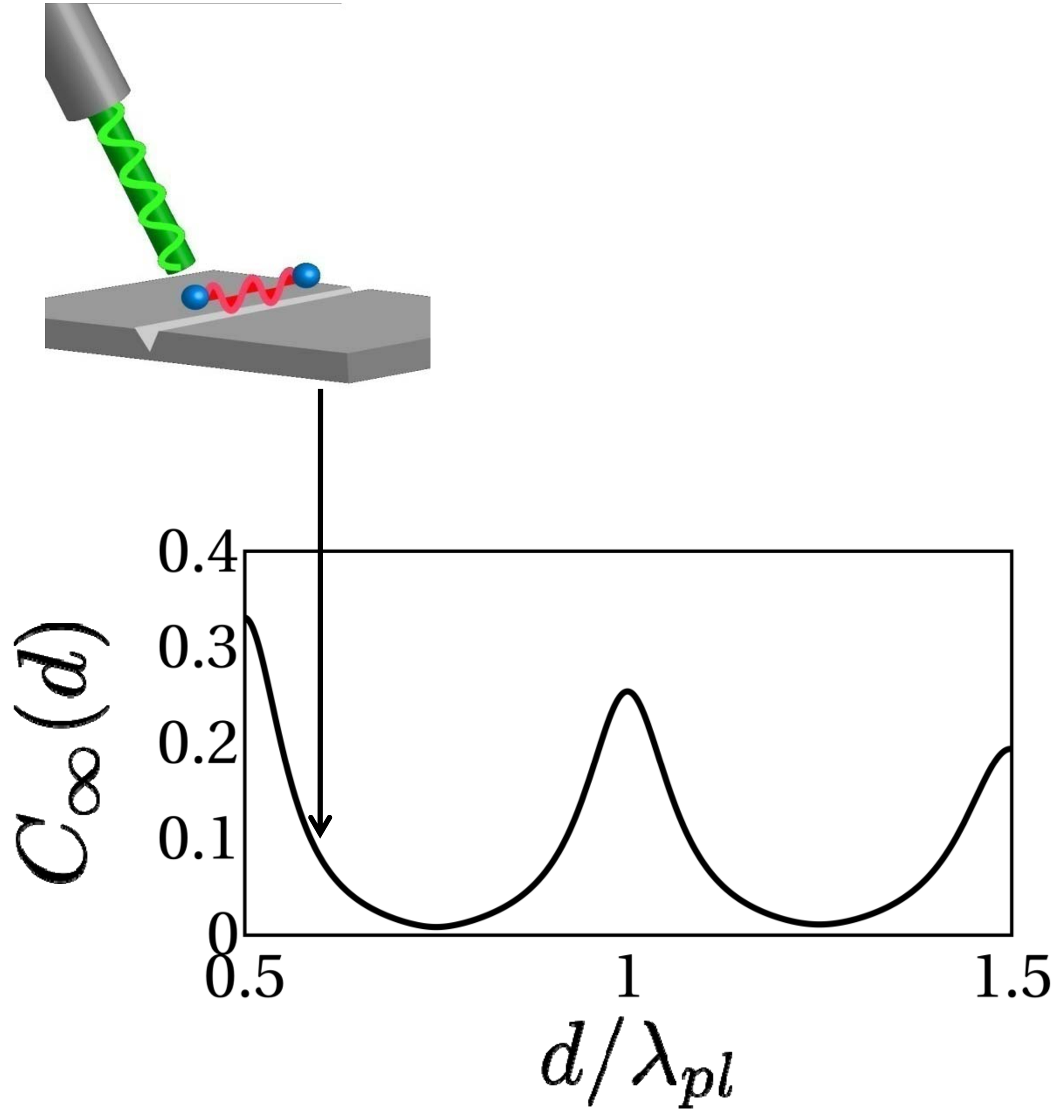


$$H_{las} = \sum_{i=1}^{2} \Omega_i(\sigma_i + \sigma_i^\dagger)$$

In the coherent part of the master equation

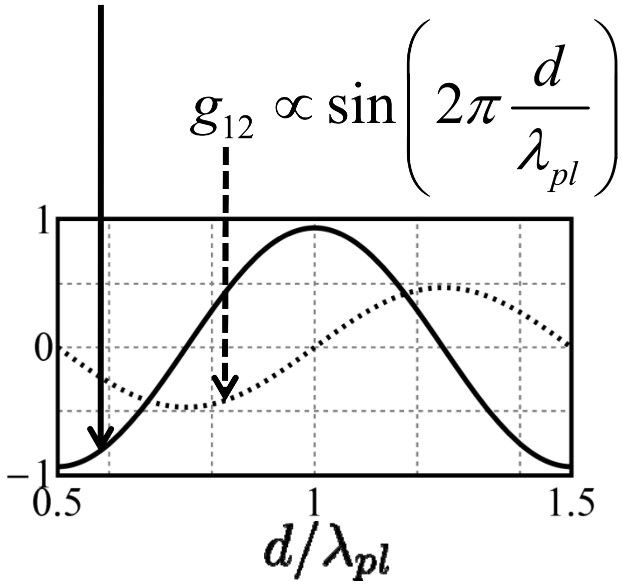
Stationary entanglement

$\Omega_1 \neq 0, \Omega_2 = 0$

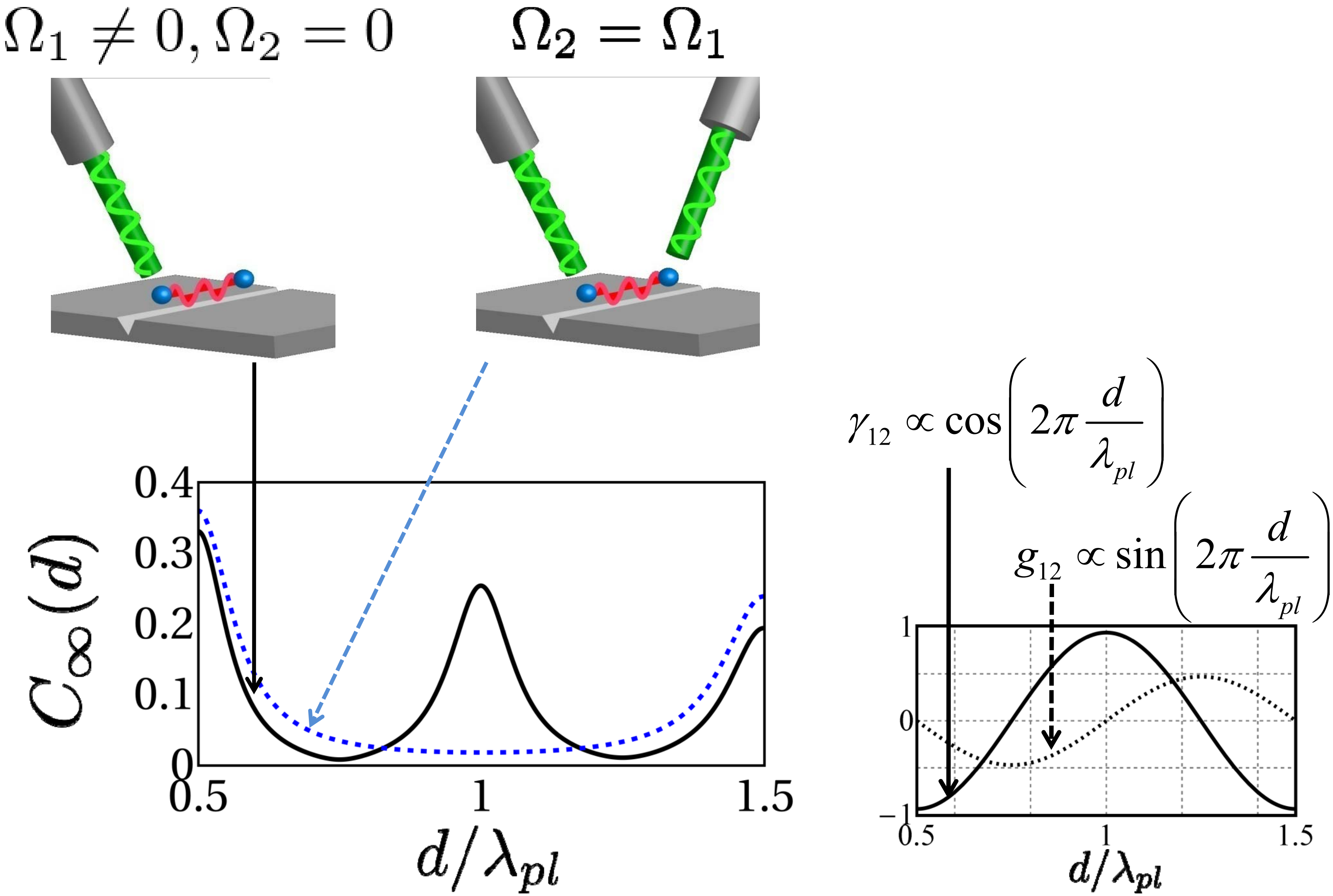


$\gamma_{12} \propto \cos\left(2\pi \frac{d}{\lambda_{pl}}\right)$

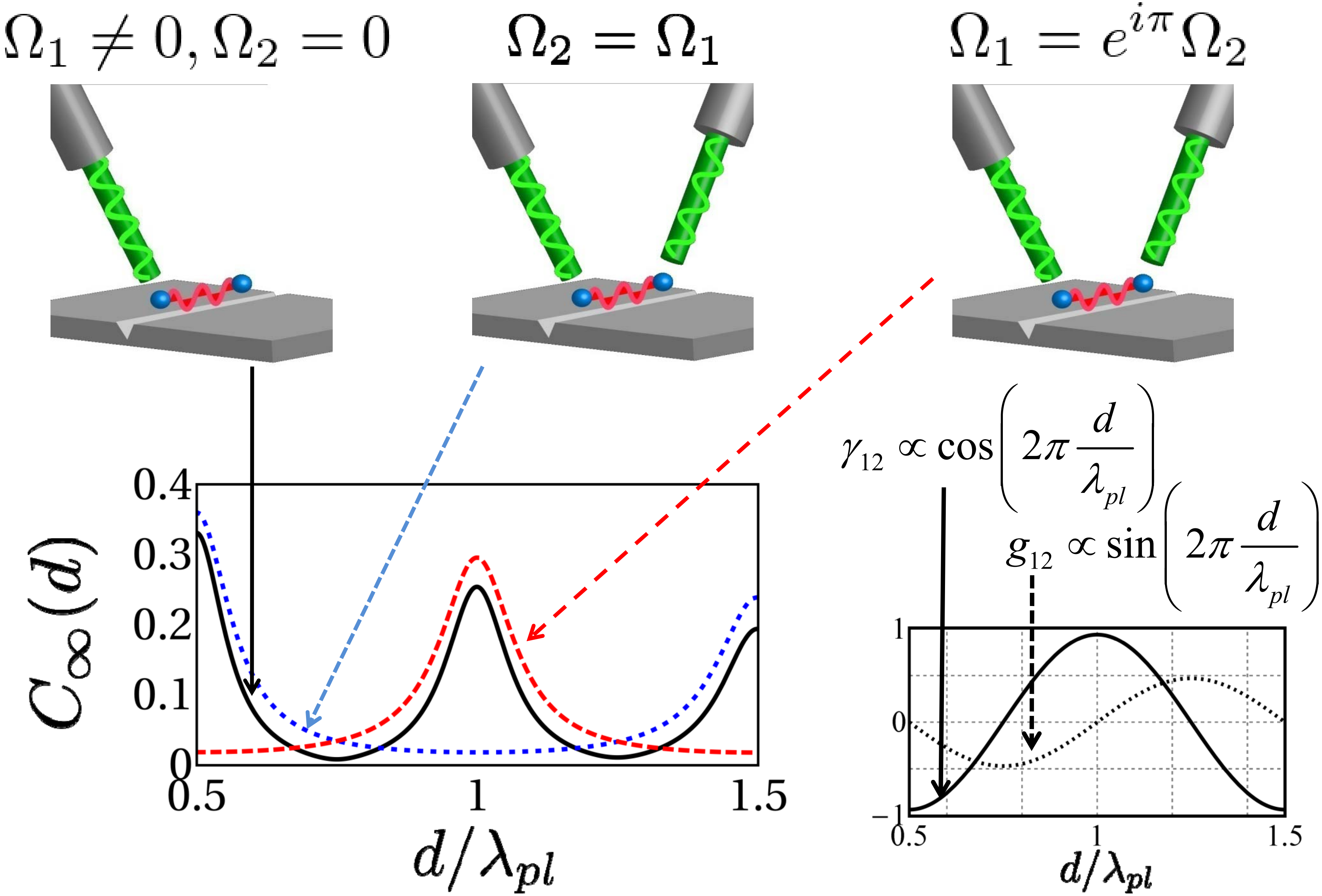
$g_{12} \propto \sin\left(2\pi \frac{d}{\lambda_{pl}}\right)$



Stationary entanglement



Stationary entanglement



How is stationary entanglement generated?

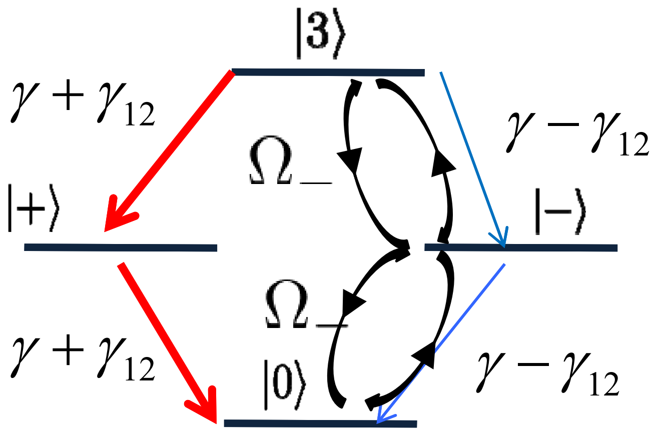
$$\gamma_{12} \propto \cos\left(2\pi \frac{d}{\lambda_{pl}}\right)$$

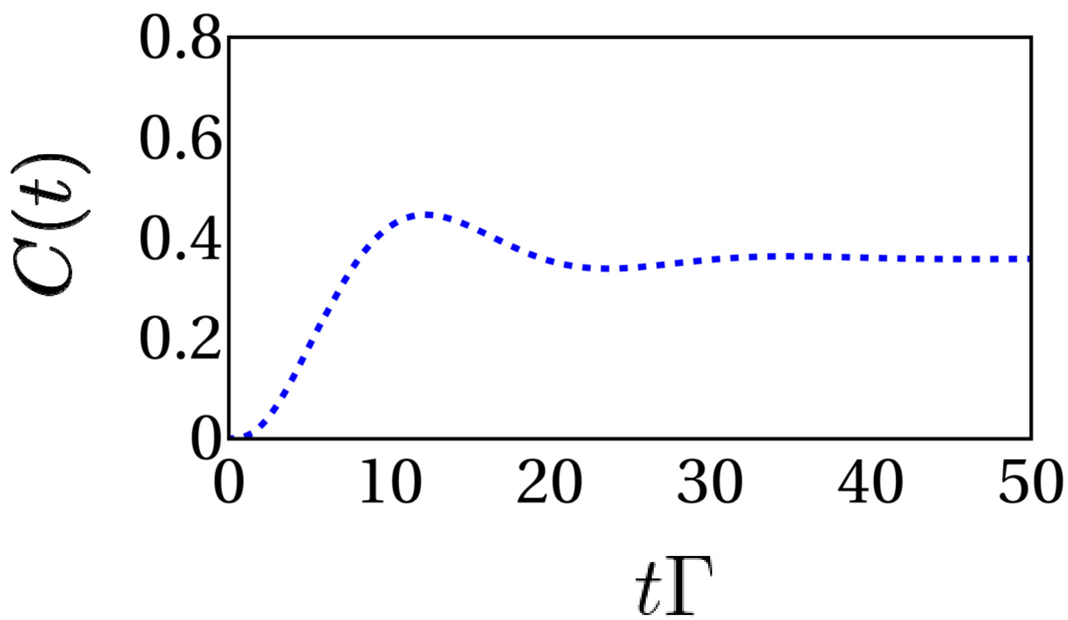
$$d \approx \lambda_{pl}$$

$$\Omega_1 = e^{i\pi}\Omega_2$$

$$\Omega_- = \frac{(\Omega_1 - \Omega_2)}{\sqrt{2}}$$

$$\begin{cases} H_{las}|0\rangle = \Omega_-|-\rangle \\ H_{las}|-\rangle = \Omega_-(|0\rangle + |3\rangle) \\ H_{las}|+\rangle = 0 \\ H_{las}|3\rangle = \Omega_-|-\rangle \end{cases}$$





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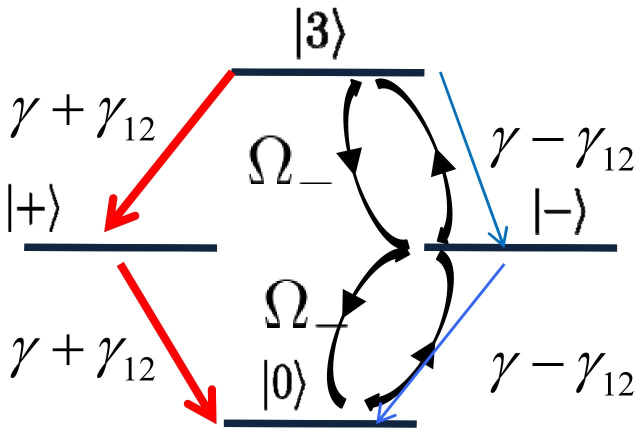

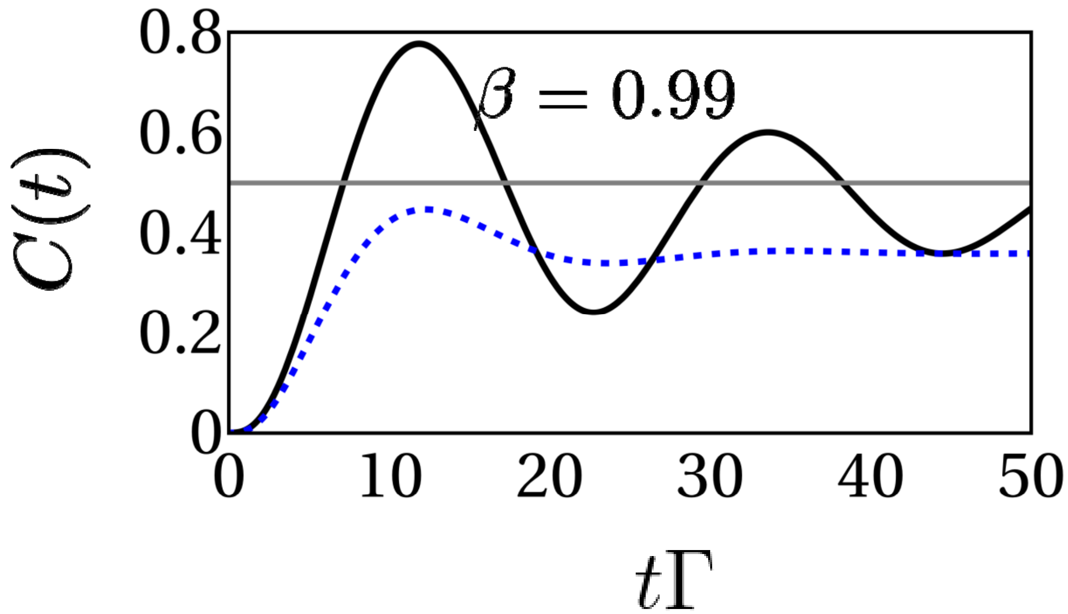

How is stationary entanglement generated?

$$\gamma_{12} \propto \cos\left(2\pi \frac{d}{\lambda_{pl}}\right)$$

$$d \approx \lambda_{pl}$$

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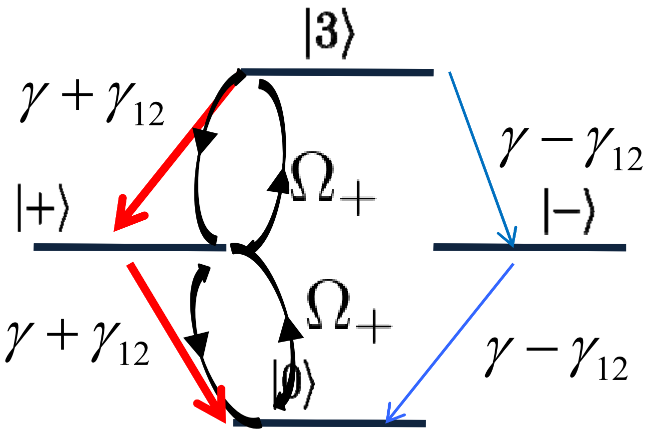
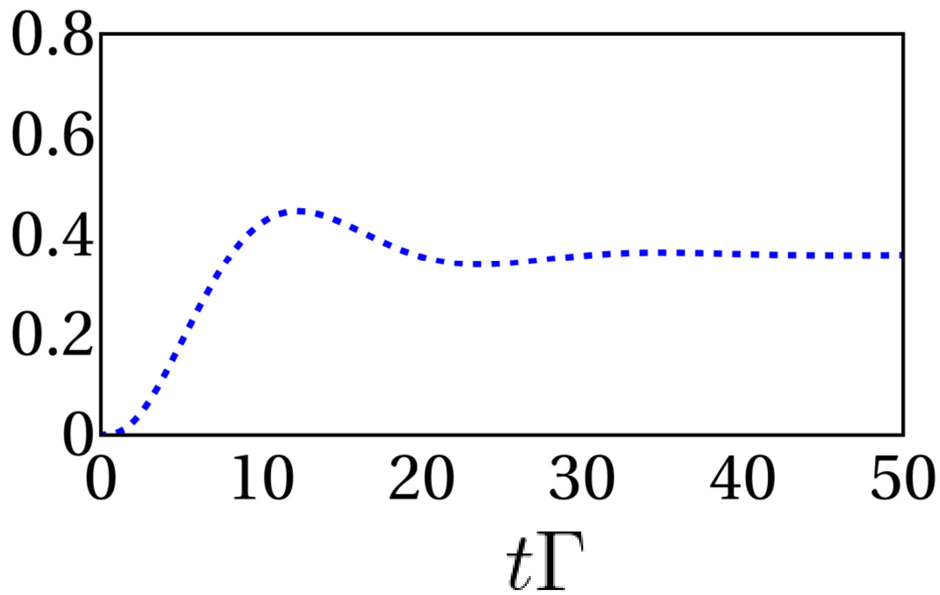
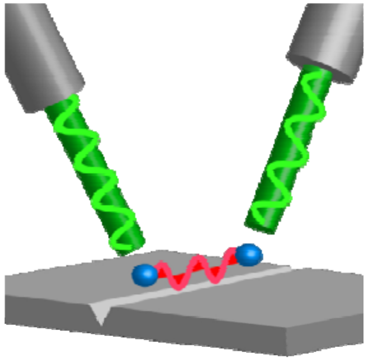
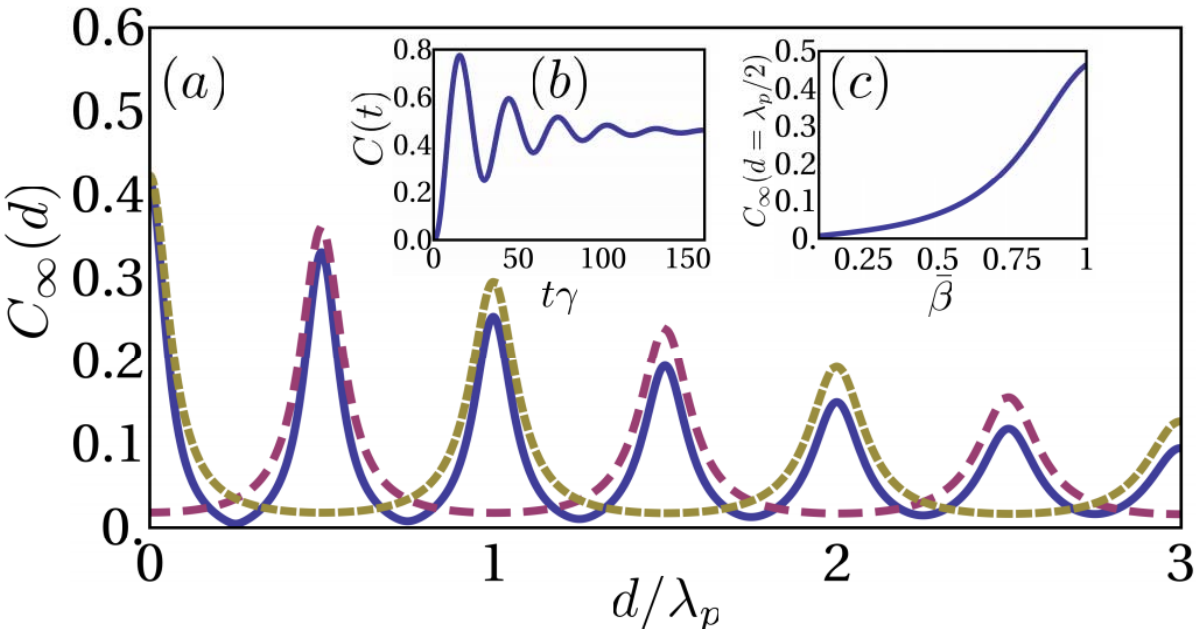
$$\begin{cases} H_{las}|0\rangle = \Omega_+|+\rangle \\ H_{las}|-\rangle = 0 \\ H_{las}|+\rangle = \Omega_+(|0\rangle + |3\rangle) \\ H_{las}|3\rangle = \Omega_+|+\rangle \end{cases}$$





Stationary state concurrence

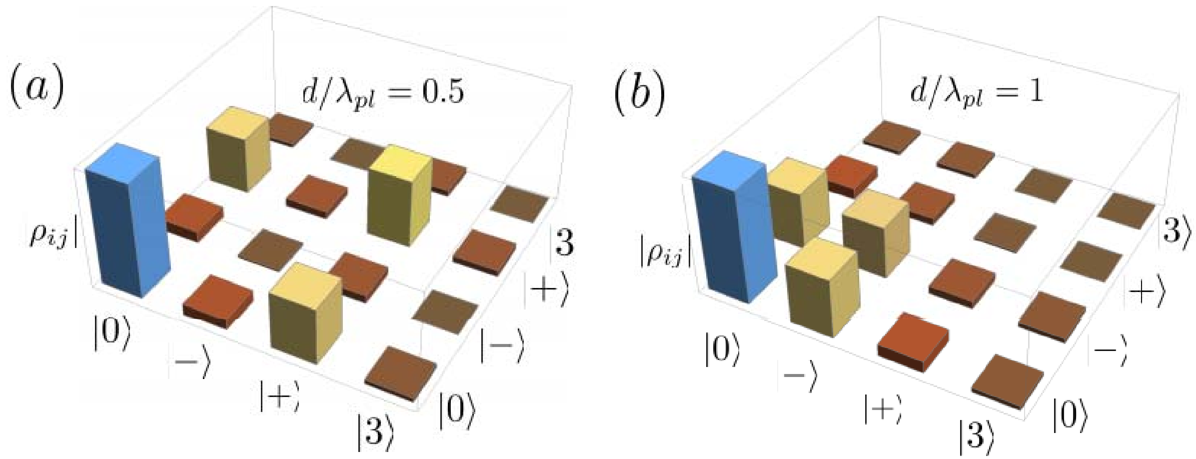




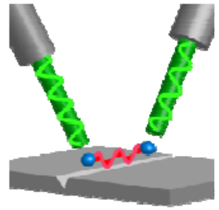
Stationary state tomography

Stationary density matrix

$\Omega_1 = 0.15\gamma,\ \Omega_2 = 0$



How to measure stationary concurrence: QE-QE correlation



Second order cross-coherence

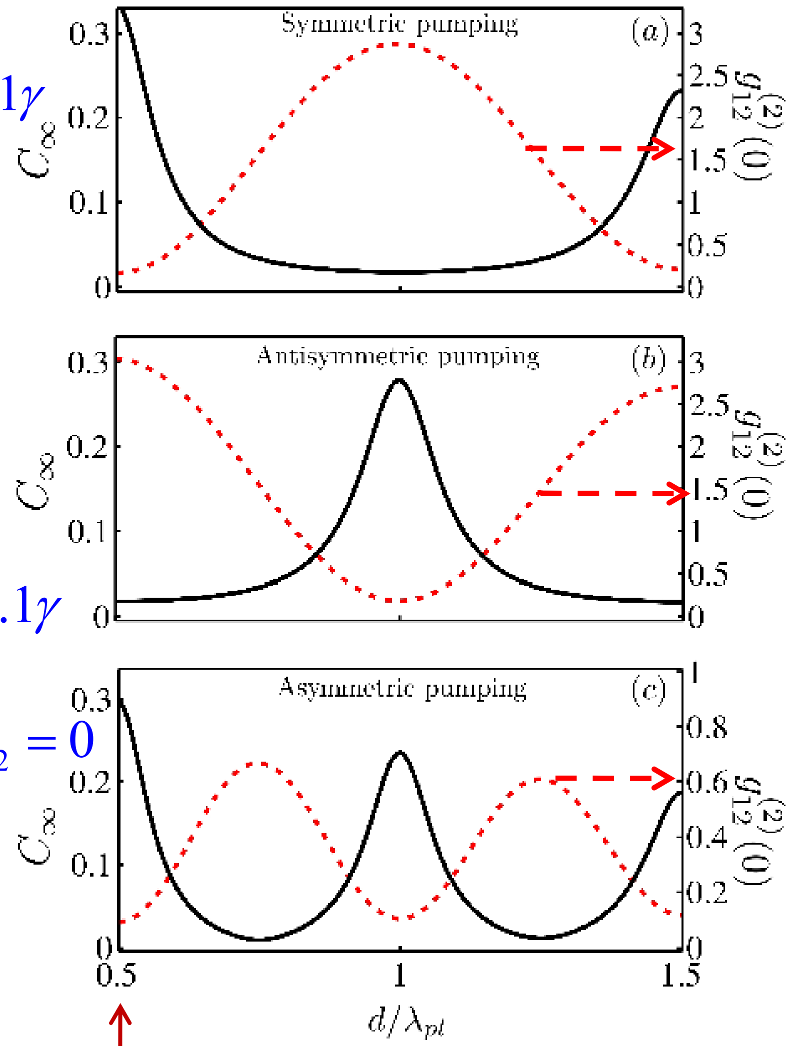
between the two QE's

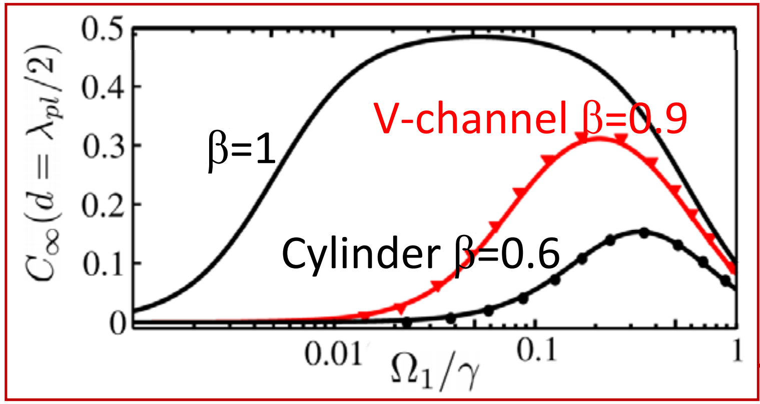
$$g_{12}^{(2)} = \frac{<\sigma_1^\dagger \sigma_2^\dagger \sigma_2 \sigma_1>}{<\sigma_1^\dagger \sigma_1><\sigma_2^\dagger \sigma_2>}$$

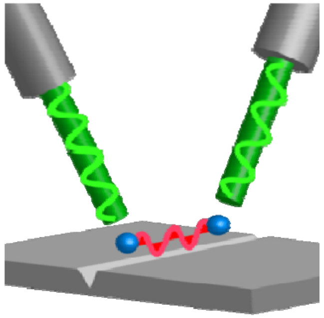
$\Omega_1 = \Omega_2 = 0.1\gamma$

$\Omega_1 = -\Omega_2 = 0.1\gamma$

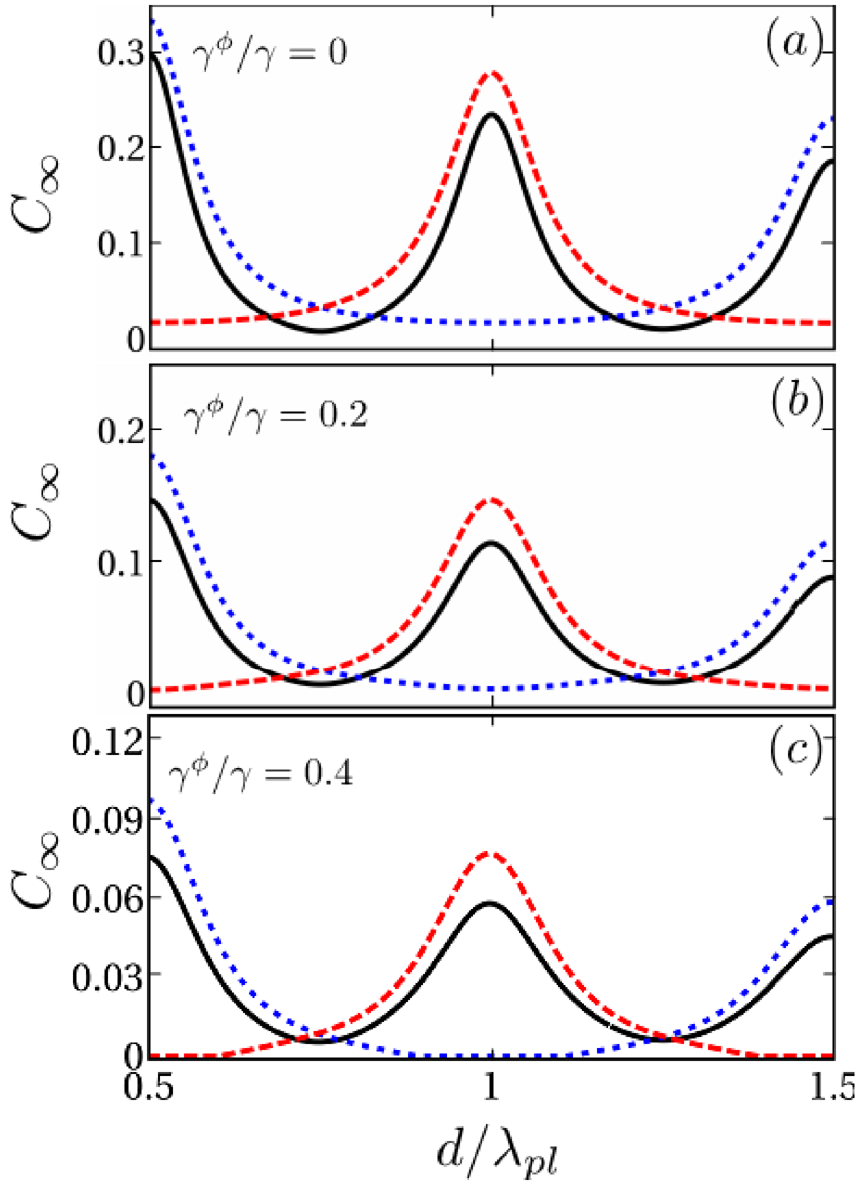
$\Omega_1 = 0.15\gamma, \Omega_2 = 0$







Effect of pure dephasing



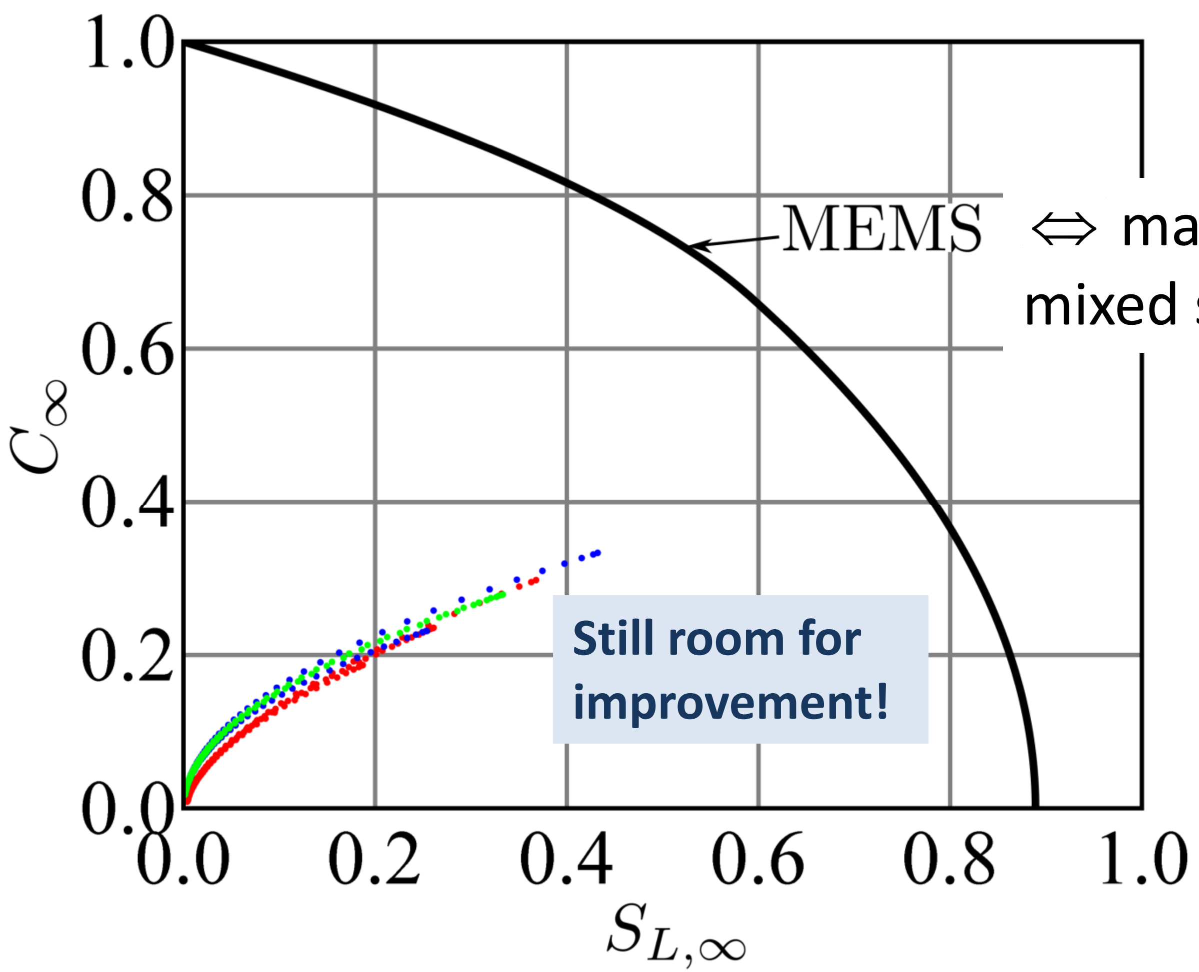
$$\mathcal{L}_{\text{deph}}[\hat{\rho}] = \frac{\gamma^{\phi}}{2} \sum_i \left[[\hat{\sigma}_i^{\dagger}\hat{\sigma}_i, \hat{\rho}], \hat{\sigma}_i^{\dagger}\hat{\sigma}_i \right]$$

Pure dephasing reduces, but not critically, both correlations & concurrence

Purity

Concurrence - Linear entropy diagram

$$S_L = \frac{4}{3}\left(1 - Tr\,\rho^2\right)$$

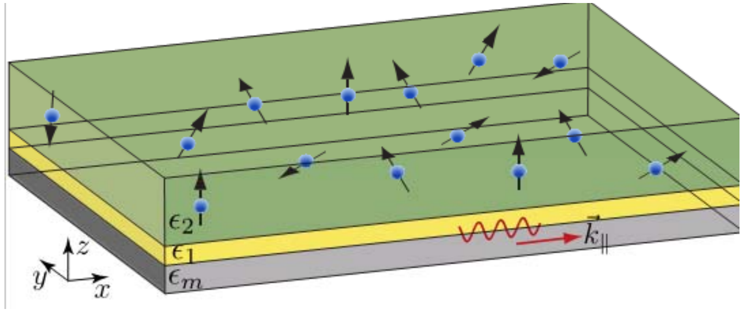


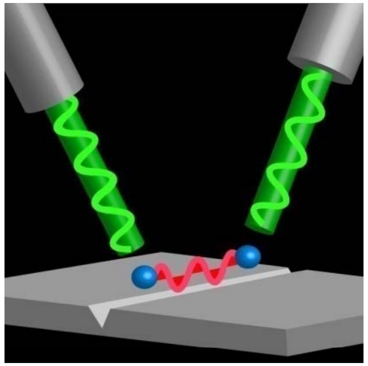
⇔ maximally entangled mixed states

β=0.94
L=2μm
d/λ_{SPP}=0.5-1.5
$\Omega_1=0.15\gamma$; $\Omega_2=0$
$\Omega_1=\Omega_2=0.1\gamma$
$\Omega_1=-\Omega_2=0.1\gamma$

SPP

Strong coupling to excitons & Intermediary for quantum entanglement





Outline

- Introduction to surface plasmon polaritons (SPP)
- Coupling of 1 quantum emitter (QE) to SPP
- Collective mode (excitons) strongly coupled to SPP
- Coupling & entanglement of 2 QE mediated by SPP
- **Conclusion**

Conclusion: Plasmon-polaritons share many properties & capabilities with cavity exciton-polaritons.

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SPP

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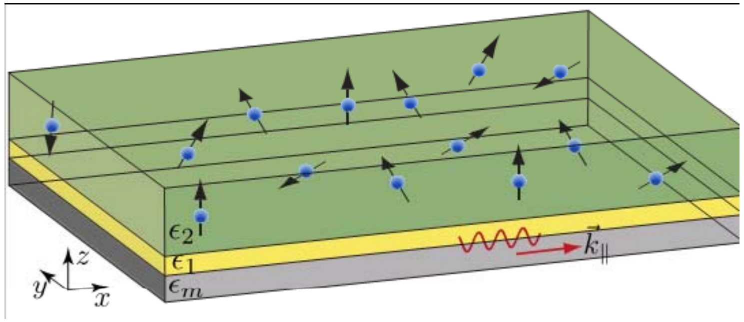
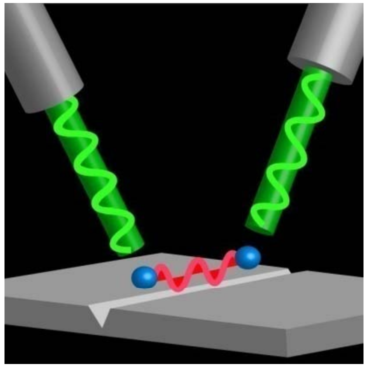




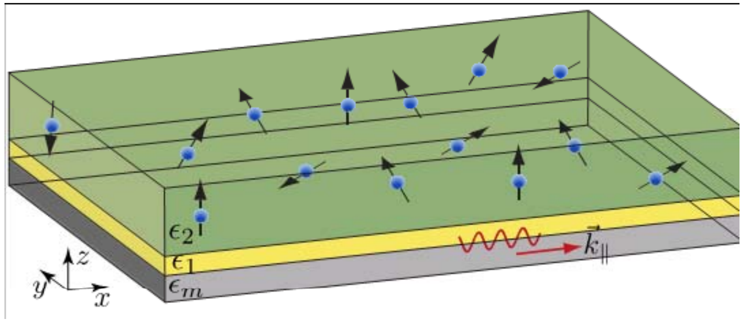
- A. Gonzalez-Tudela *et al*, arXiv:1205.3938

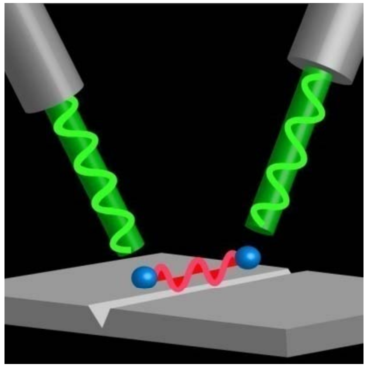
- A. Gonzalez-Tudela *et al*, Phys. Rev. Lett. **106** , 020501(2011)
- D. Martin-Cano, *et al, Phys. Rev. B* **84,** 235306 (2011)

Conclusion: **Plasmon-polaritons share many properties & capabilities with cavity exciton-polaritons.**

SPP

Strong coupling to excitons & Intermediary for quantum entanglement





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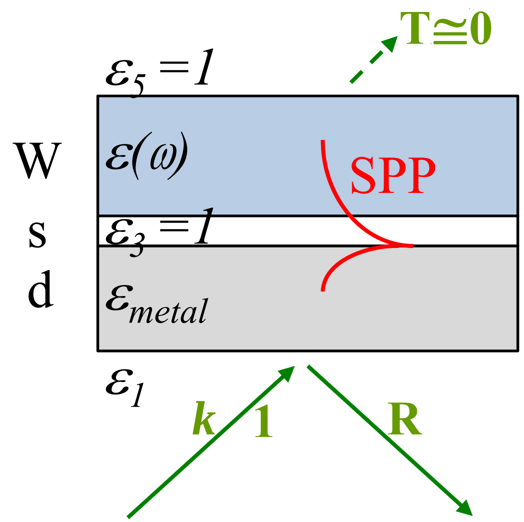
Thanks for your attention
Спасибо за ваше внимание

(Semi)-classical description

Polarizability of 1 emitter
$$\alpha(\omega) = \frac{f_0 e^2 / m}{\omega_0^2 - \omega^2 i\omega\gamma} \; ; \; \varepsilon(\omega) = \frac{1+(2/3)N\alpha(\omega)}{1-(1/3)N\alpha(\omega)}$$
Eff. dielectric funct. of emitters

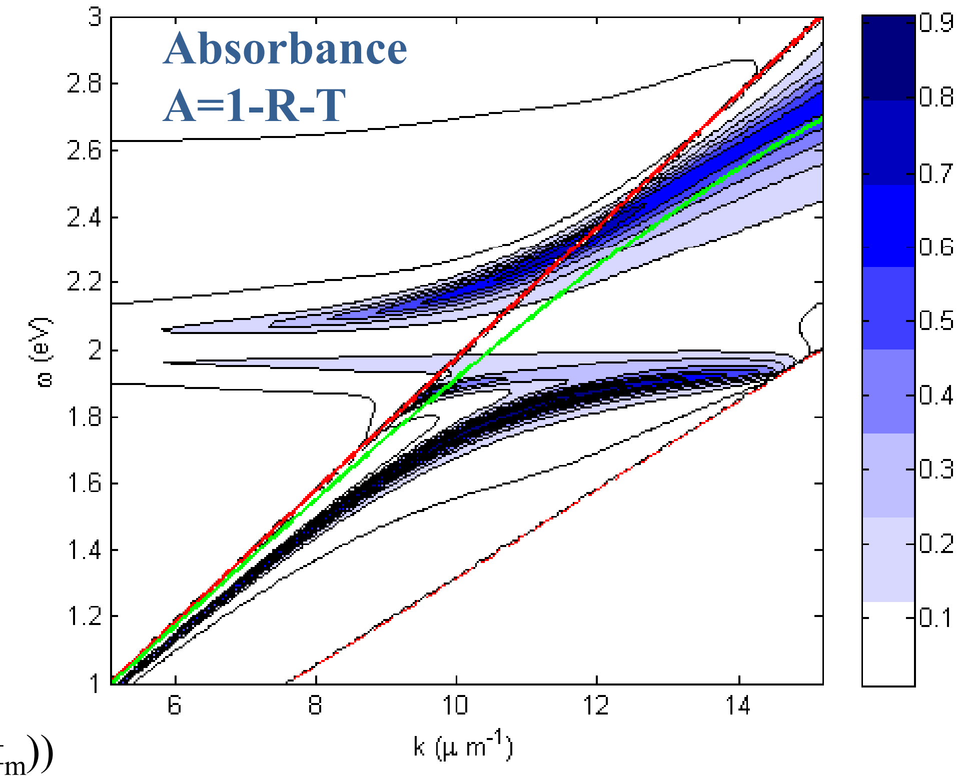


Oscillator strength from microsocpic info.
$$f_0 = \frac{2m\omega_0}{3e^2\hbar}|\vec{\mu}|^2 = \frac{2\pi m}{e^2}\frac{\varepsilon_0 c^3}{\omega_0^2}\gamma_0$$

ε_1=2.25; d=30nm; s=1nm; W=500nm

ω_0=2eV; N=10^6 μm^{-3}

$\gamma = \gamma_0 + \gamma_{deph}$; γ_0=0.1meV; γ_{deph}=40meV

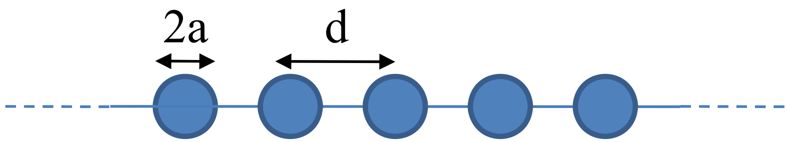


Red: light line ω = c k

Red thin: light line ω = c k / √(ϵ_1)

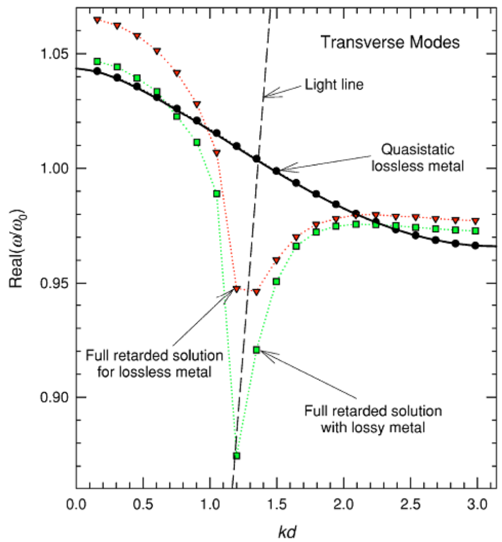
Green: SPP dispersion relation ω = c k/ √(ϵ_1 ϵ_m/(ϵ_1+ ϵ_m))

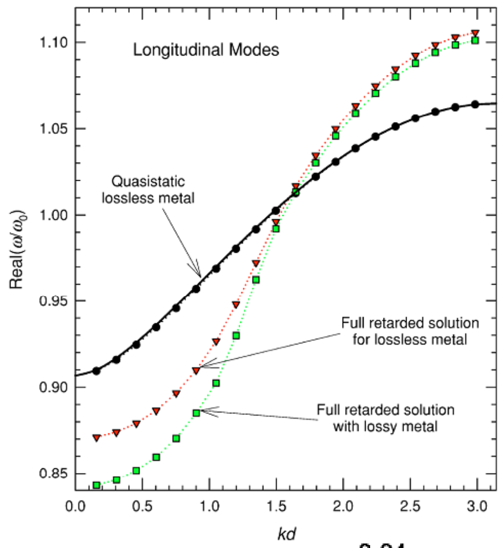
Dispersion relation for condensation

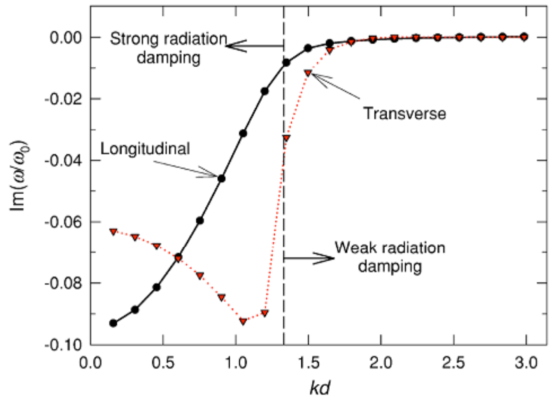


Chain of silver spheres
a=25nm
d=75nm

PRB, 70, 125429 ('04)







Exp.: PRB, 65, 193408 ('02)