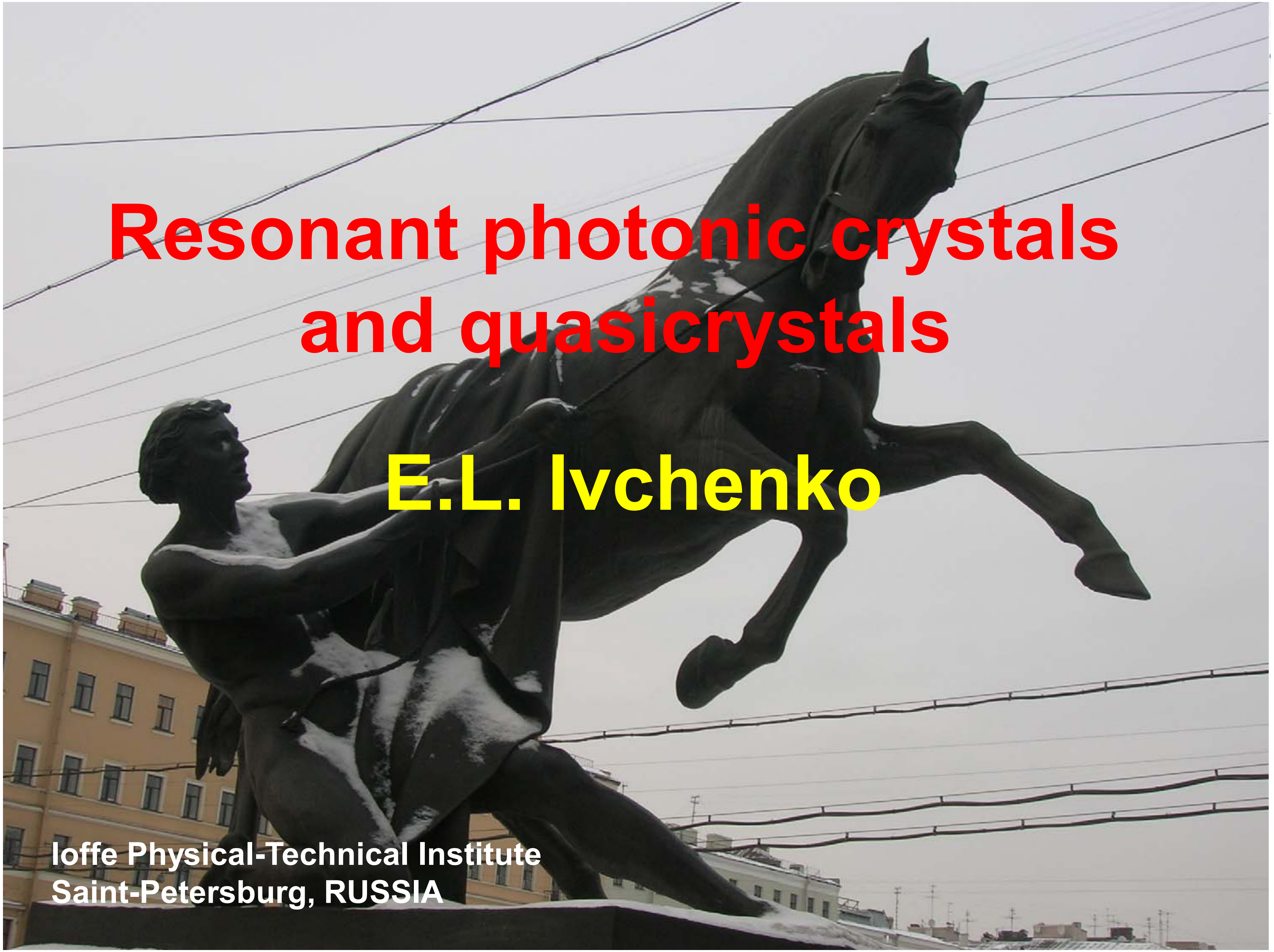

Resonant photonic crystals and quasicrystals

E.L. Ivchenko

Ioffe Physical-Technical Institute
Saint-Petersburg, RUSSIA

Resonant photonic crystals and quasicrystals

- Introduction. Resonant Bragg QWs
- QWs, Optical Lattices, Nuclear Resonances
- Superradiant and Photonic-Crystal Regimes
- Experimental Iℓℓustration
- Resonant Fibonacci QW Chains
- Time-Resolved and Nonlinear Properties

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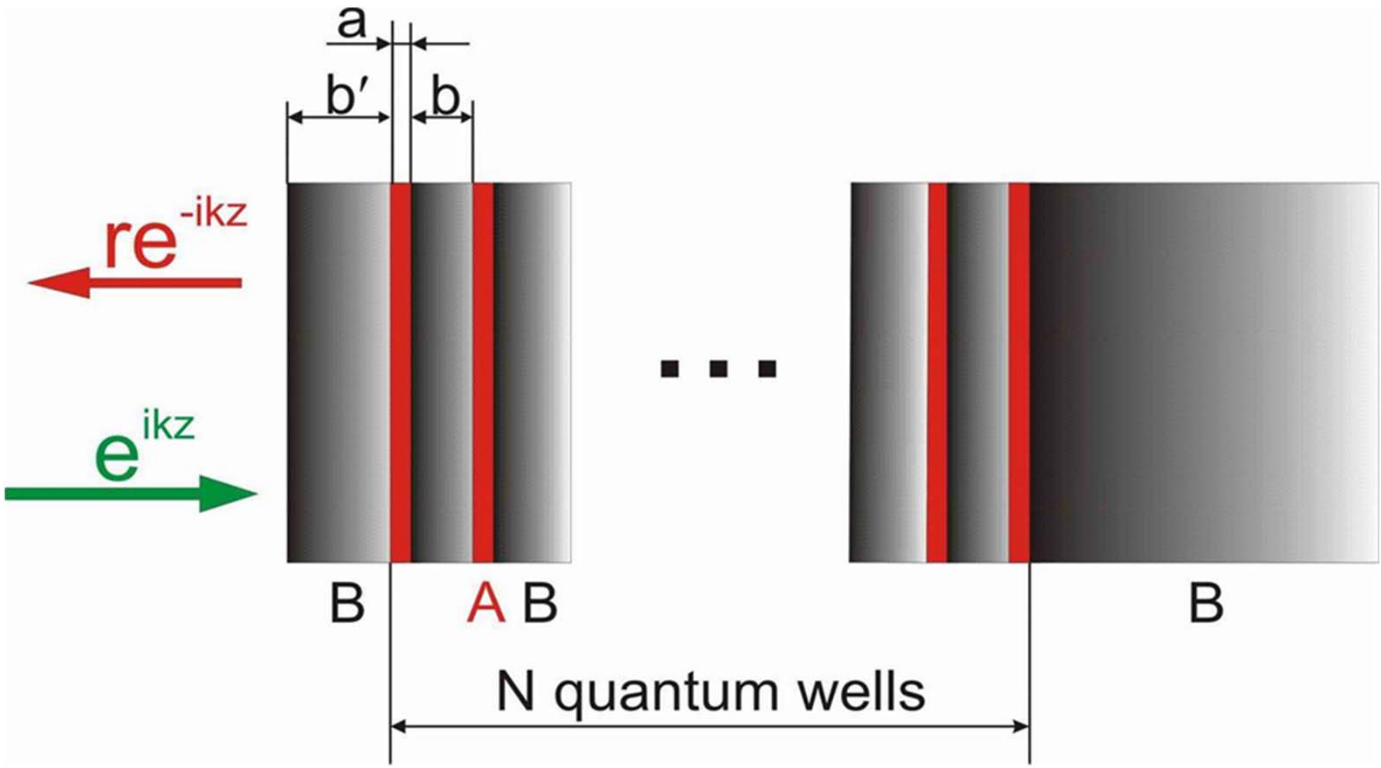
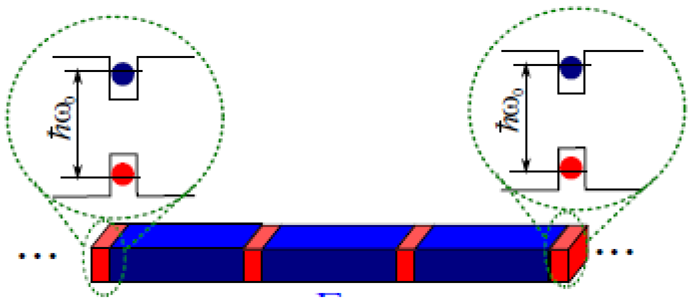
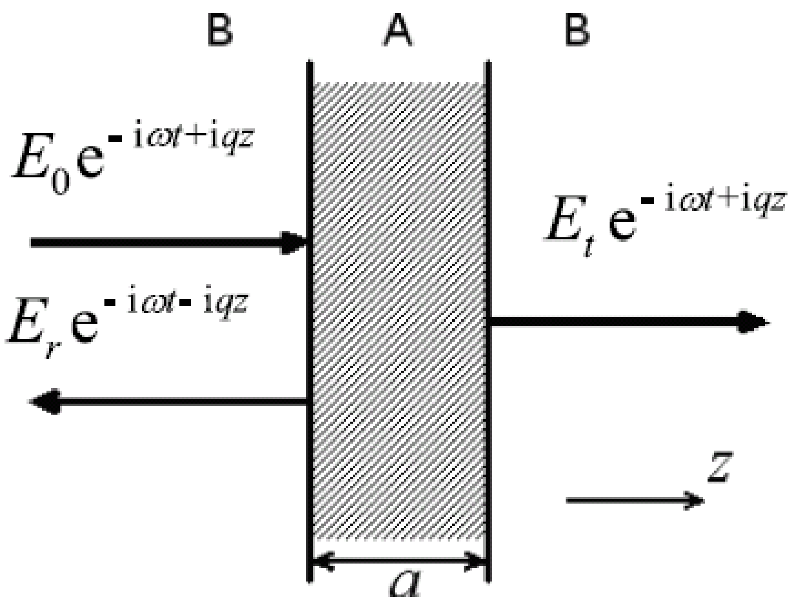
Light Reflection from MQWs





Period $d = a + b$, cap-layer thickness b'

Reflection from a single QW



$$r = \frac{E_r}{E_0} = \frac{i\Gamma_0}{\omega_0 - \omega - i(\Gamma_0 + \Gamma)}, \quad t = 1 + r$$

Andreani, Tassone, Bassani (1991)

Infinite Periodic QW Structure

$$\cos Kd = \cos qd - \frac{\Gamma_0}{\omega_0 - \omega - i\Gamma} \sin qd$$

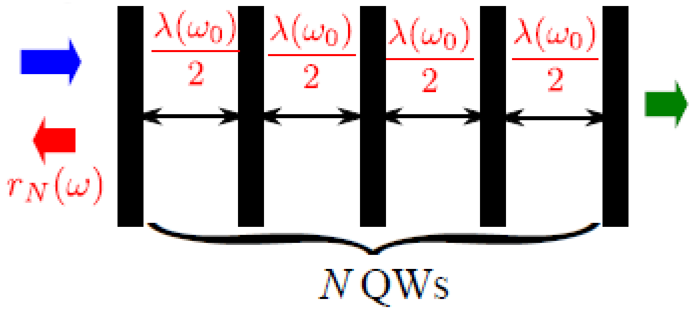
$$q(\omega) = \frac{\omega}{c} n_b \qquad -\pi/d < \mathrm{Re}\{K\} \leq \pi/d$$

Ivchenko, 1991

Resonance Bragg condition:

$$q(\omega_0) d = \pi \quad \text{or} \quad \frac{\omega_0}{c} n_b d = \pi$$

Light Reflection from *N* QWs



$$\frac{\omega_0}{c} n_b (a+b) = \pi$$

$$r_N = \frac{\mathrm{i} N \Gamma_0}{\omega_0 - \omega - \mathrm{i}(N\Gamma_0 + \Gamma)}$$

Ivchenko, Nesvizhskii, Jorda 1994

Light Reflection from *N* QWs

$$\frac{\omega_0}{c} n_b (a+b) = \pi$$

oblique incidence:

$$\frac{\omega_0}{c} n_b d \cos\theta = \pi$$

m-th order of reflection:

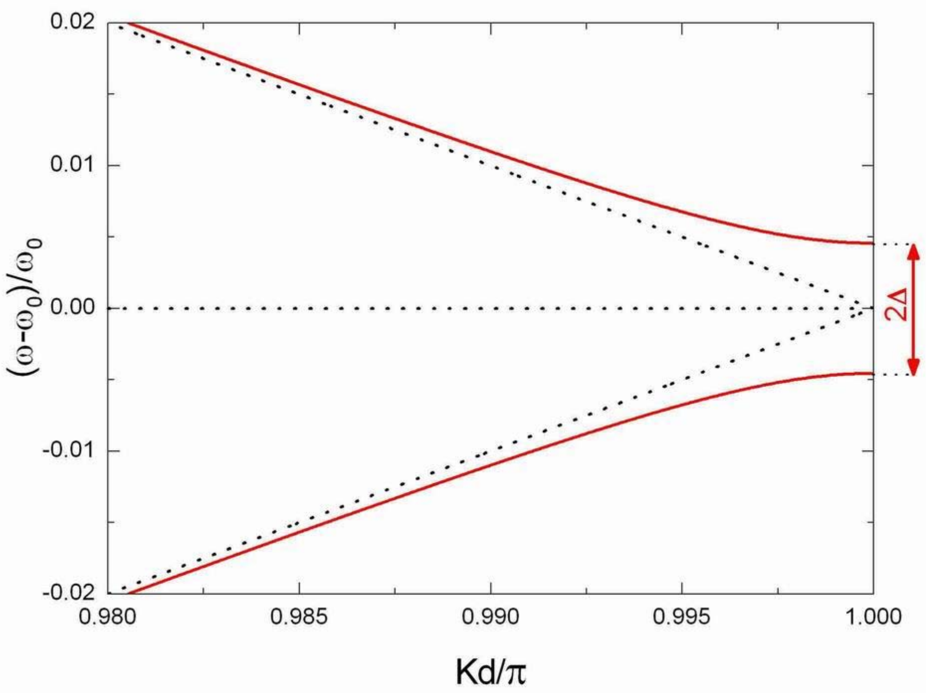
$$\frac{\omega_0}{c} n_b d \cos\theta_m = \pi m$$

$$r_N = \frac{\mathrm{i} N \Gamma_0}{\omega_0 - \omega - \mathrm{i}(N\Gamma_0 + \Gamma)}$$

Effect of **multi**reflection

Exciton-Polariton Dispersion in Resonant Bragg QW Structure

$$q(\omega_0)d = \pi \quad \text{or} \quad \frac{\omega_0}{c} n_b d = \pi$$



$$\Delta = \sqrt{\frac{2\Gamma_0 \omega_0}{\pi}}$$

$$\cos Kd = \cos qd - \frac{\Gamma_0}{\omega_0 - \omega - i\Gamma} \sin qd$$

Ivchenko, Willander (1999)

Concluding the INTRODUCTION

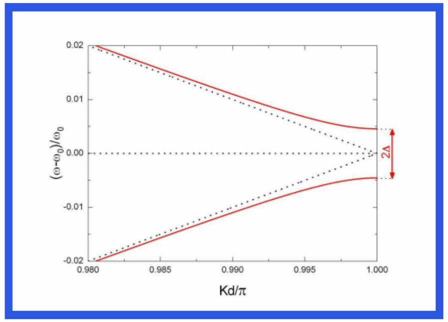
1.

$$\cos Kd = \cos qd - \frac{\Gamma_0}{\omega_0 - \omega - i\Gamma} \sin qd$$

2.

$$r_N = \frac{\mathrm{i} N \Gamma_0}{\omega_0 - \omega - \mathrm{i}(N\Gamma_0 + \Gamma)}$$

3.



$$\Delta = \sqrt{\frac{2\Gamma_0\omega_0}{\pi}}$$

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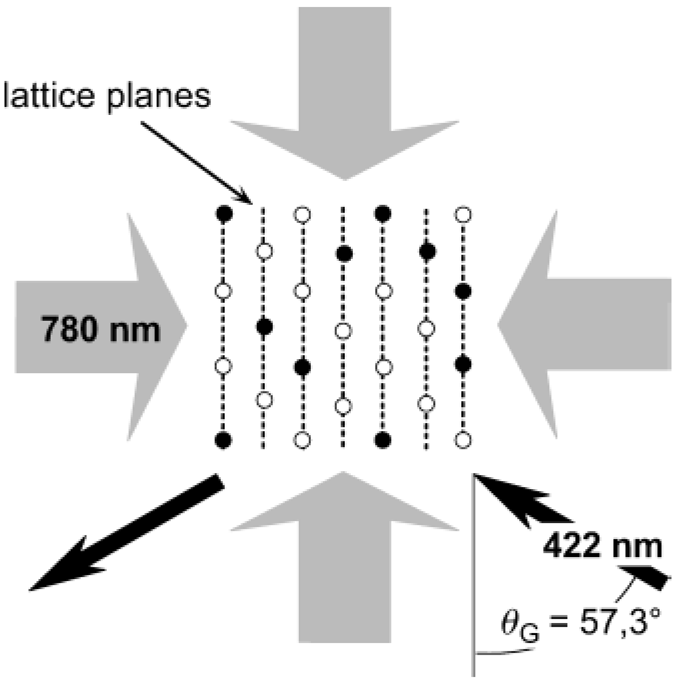
Optical Lattices of Cold Atoms

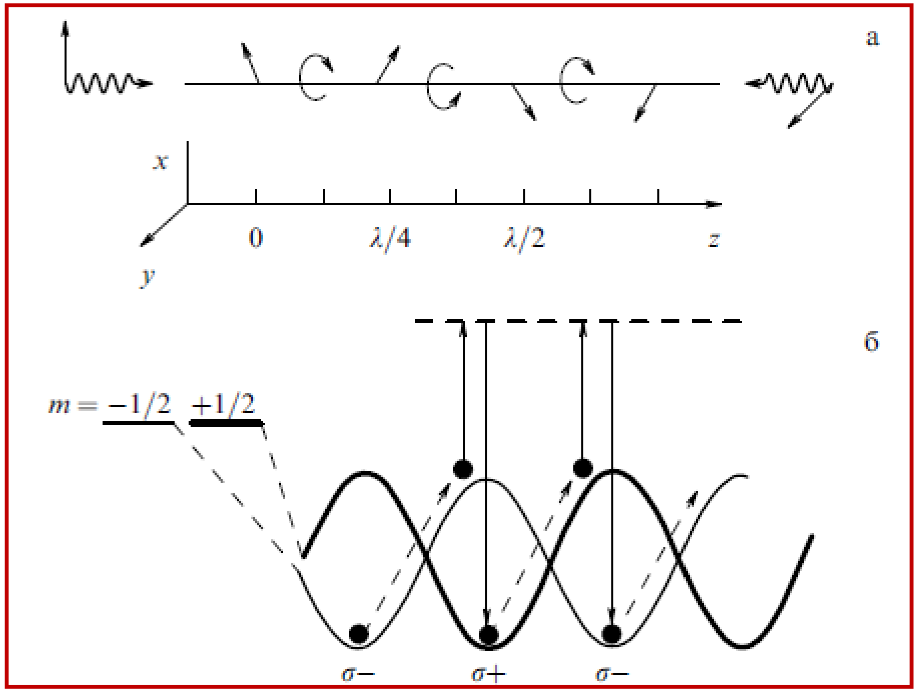


Weidemüller et al. 1998

2D schematic presentation of Bragg diffraction from optical lattices. The atomic lattice is formed in the intersection of six laser beams, with the pair of laser beams perpendicular to the drawing plane not shown here. The lattice constant is determined by the wavelength of the lattice field (λ_L= 780 nm). Occupied lattice sites are represented by filled circles. A laser beam of shorter wavelength (λ_B= 422 nm) is diffracted from the lattice planes when the Bragg condition is fulfilled.

Laser Cooling and Trapping of Neutral Atoms

Steven Chu, Claude Cohen-Tannoudji, William D. Phillips, Nobel prize 1997



The first beam configuration that was experimentally studied was the so-called 1D lin⊥lin configuration, in which two beams having crossed linear polarizations propagate in opposite directions.

Phillips, Nobel Lecture, December 1997

Optical Lattices of Cold Atoms

Intra-atomic optical transitions

$$5S_{1/2}(F=3) \to 5P_{3/2}(F'=4)$$

of the rubidium D_2 resonance line at $\lambda_L = 780.2$ nm
(F = total angular momentum)
In the electromagnetic field

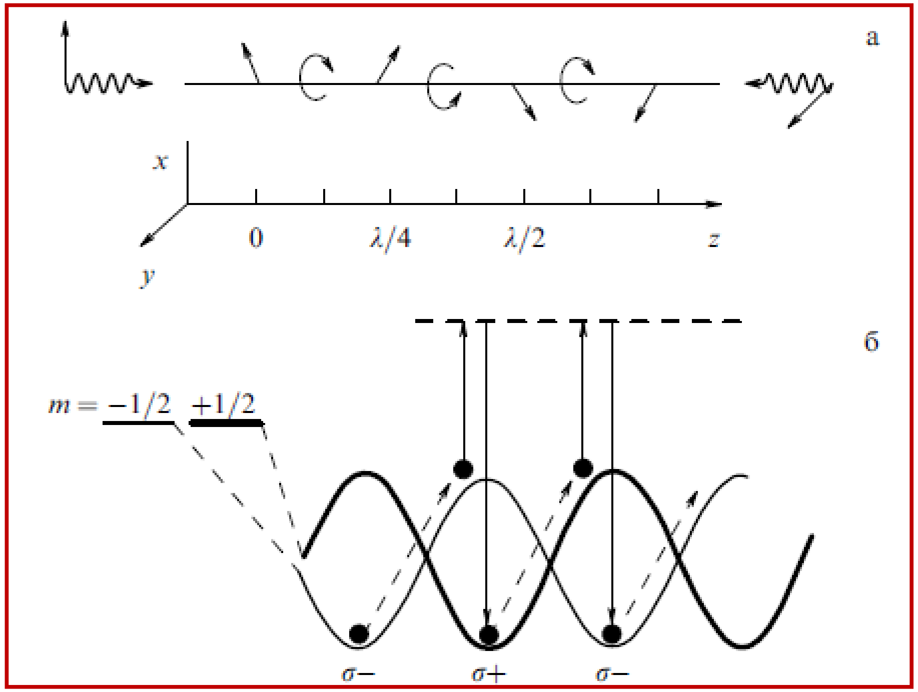
$$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{e}^{-\mathrm{i}\omega t}\boldsymbol{E}(\boldsymbol{r}) + \text{c.c.}\ ,$$

the energies of the ground-state sublevels $S_z = 1/2$ are renormalized. The field-induced correction to the atomic Hamiltonian has the form

$$\mathcal{H}(\boldsymbol{r}) = A\ \boldsymbol{E}^*(\boldsymbol{r}) \cdot \boldsymbol{E}(\boldsymbol{r}) + \mathrm{i}B\ \boldsymbol{\sigma} \cdot [\boldsymbol{E}^*(\boldsymbol{r}) \times \boldsymbol{E}(\boldsymbol{r})]$$

1D Optical Lattice of Cold Atoms

Steven Chu, Claude Cohen-Tannoudji, William D. Phillips, Nobel prize 1997

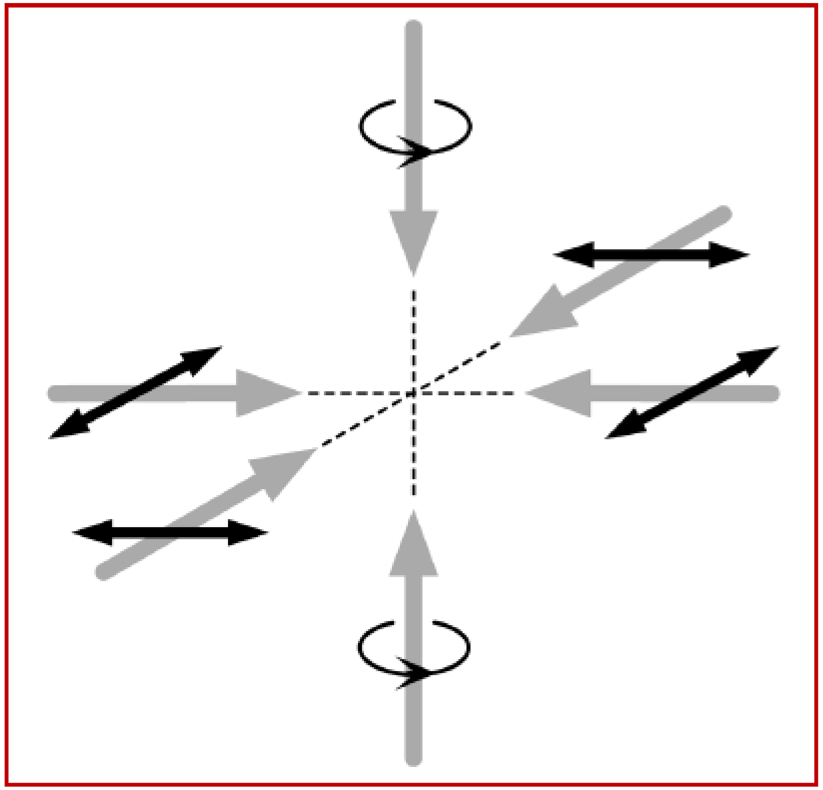


Phillips, Nobel Lecture, December 1997

The first beam configuration that was experimentally studied was the so-called 1D lin⊥lin configuration, in which two beams having crossed linear polarizations propagate in opposite directions.

$$\delta\varepsilon_{\pm 1/2} \propto mBE_0^2 \sin\frac{4\pi z}{\lambda}$$

3D Optical Lattices of Cold Atoms

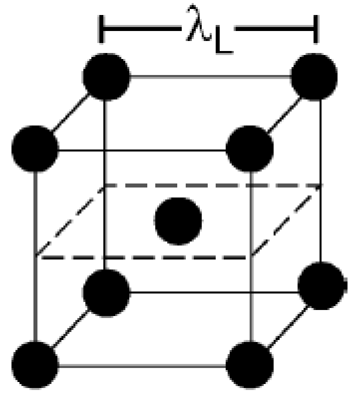


$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{e}^{\mathrm{i}\psi}\boldsymbol{e}_y \cos 2\pi x + \mathrm{e}^{-\mathrm{i}\psi}\boldsymbol{e}_x \cos 2\pi y + \mathrm{e}^{\mathrm{i}\varphi}\frac{\boldsymbol{e}_x + \mathrm{i}\boldsymbol{e}_y}{\sqrt{2}} \cos 2\pi z$$

(x,y,z) in units of λ_L)

The bcc structure is obtained for

$$\psi = \pi/4, \varphi = 0$$



The atomic lattice is formed in the intersection of six laser beams

Resonant Bragg systems

optical lattices of cold atoms

multiple quantum wells

nuclear resonances for γ-quanta

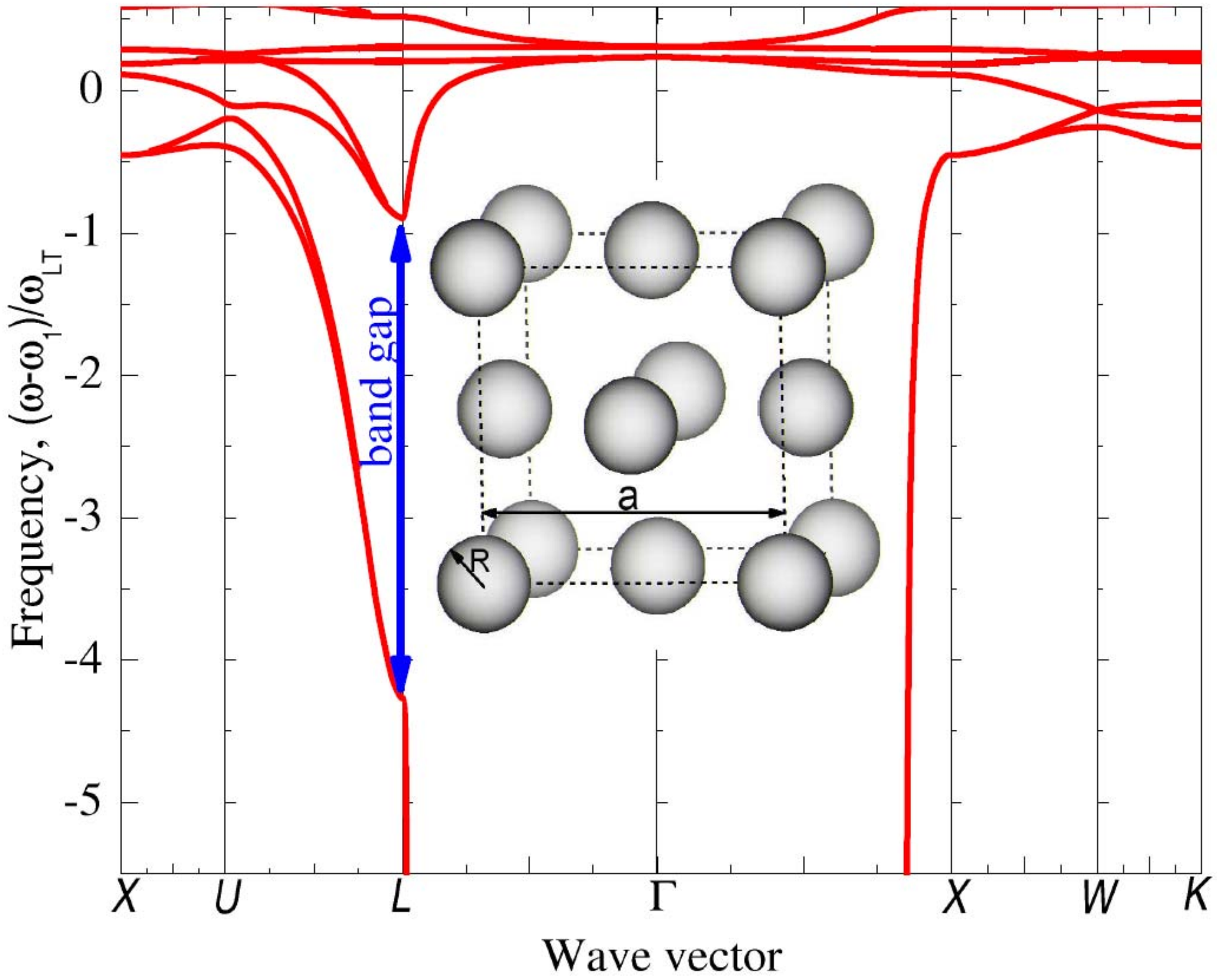
$$|\boldsymbol{q}(\omega_0)| = |\boldsymbol{q}(\omega_0) + \boldsymbol{b}|$$

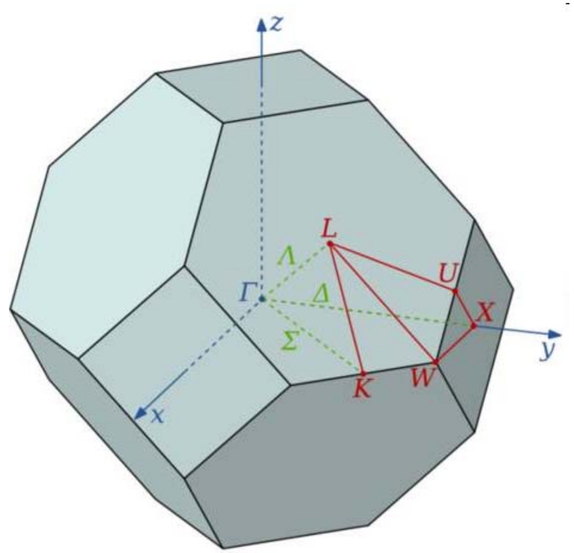
quantum-dot photonic crystals

linear resonator chain

coupled-resonator optical waveguide

Exciton polaritons in 3D photonic crystal

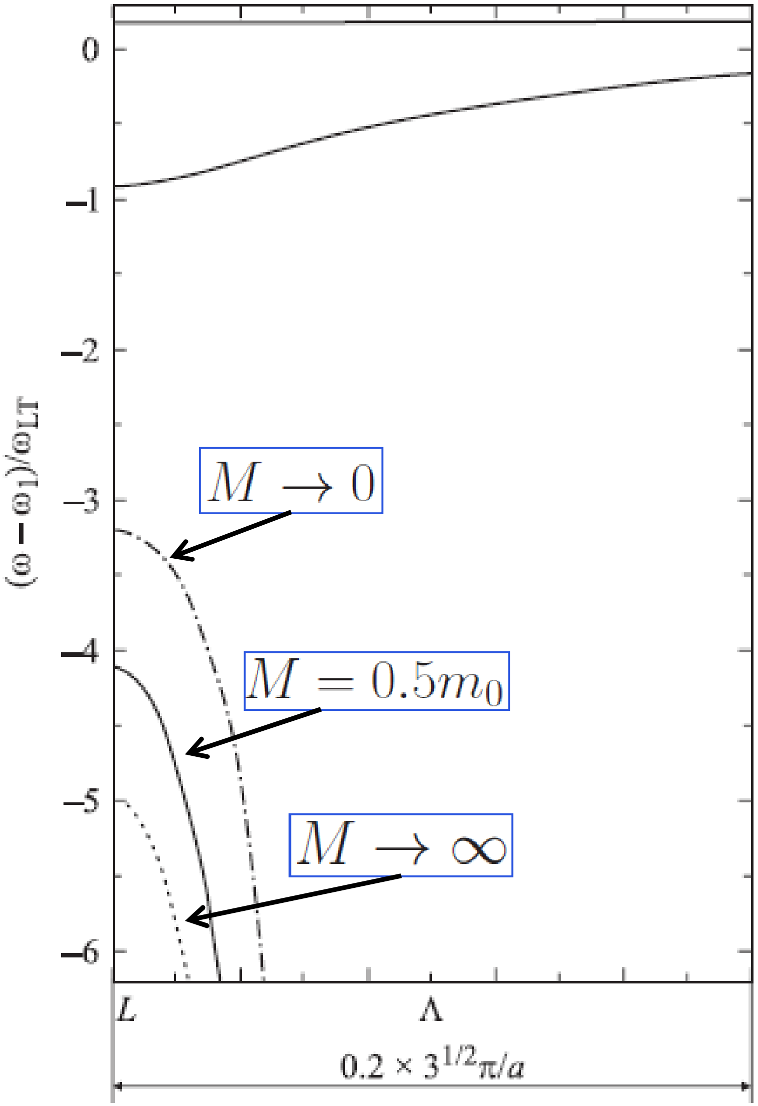




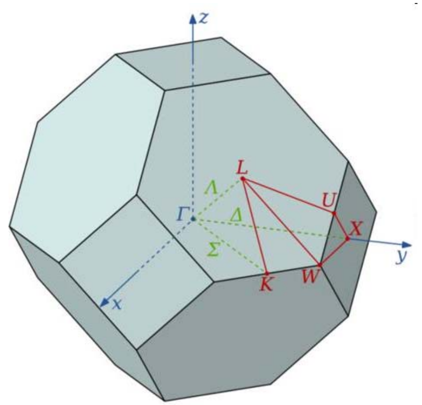
$$\frac{\omega_0}{c} n_b = \sqrt[3]{1.1} k_L = 1.032 \frac{\sqrt{3}\pi}{a}$$

Ivchenko, Fu, Willander 2000
Ivchenko, Poddubny 2006

Exciton polaritons in 3D photonic crystal

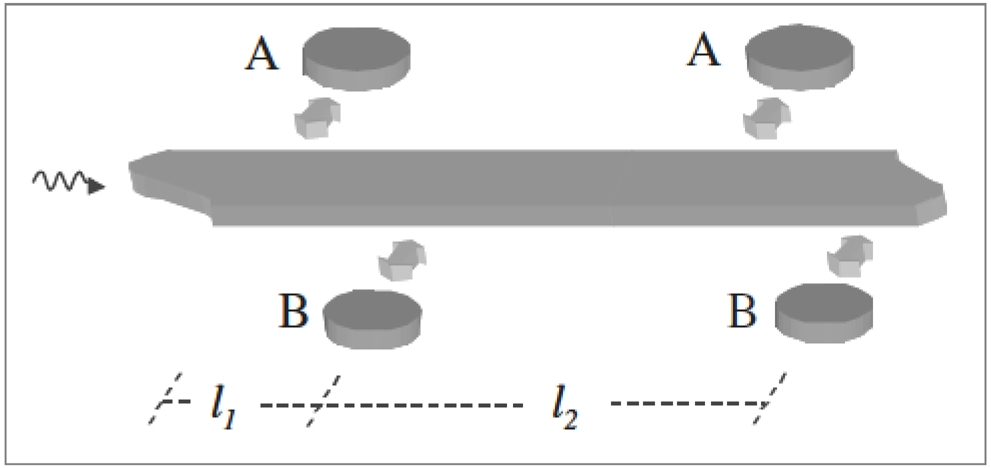


$$\frac{\omega_0}{c} n_b = \sqrt[3]{1.1} k_L = 1.032 \frac{\sqrt{3}\pi}{a}$$



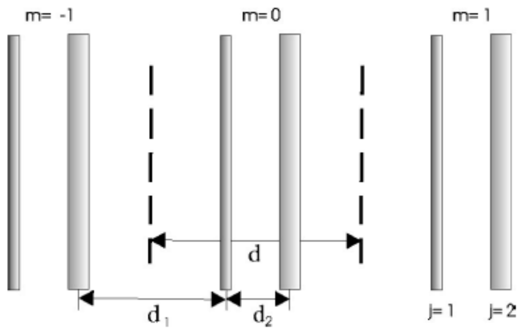
Ivchenko, Poddubny 2006

Coupled-resonator optical waveguide



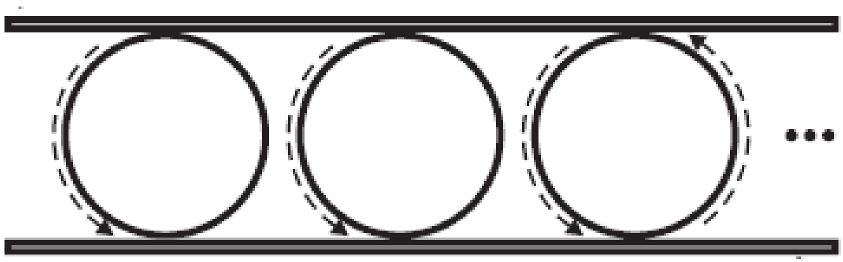
Yanik, Suh, Wang, Fan, PRL 2004

$$\cos(kl) = \cos(\beta l) + \frac{C_+}{(\omega - \omega_A)} + \frac{C_-}{(\omega - \omega_B)}$$



Ivchenko, Voronov, Erementchouk, Deych, Lisyansky, PRB 2004

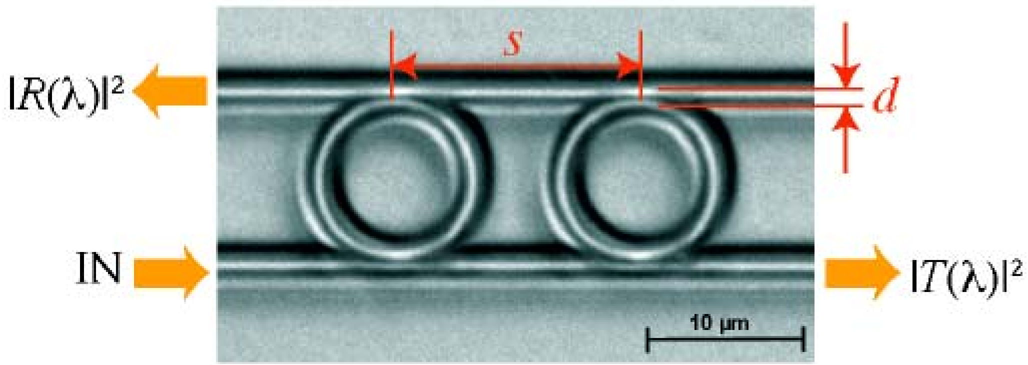
Chain of ring resonators



$$t \simeq \frac{(\omega - \omega_0)^2}{N^2\gamma^2 + (\omega - \omega_0)^2}$$

Matsko, Savchenkov, Liang, Ilchenko, Seidel, Maleki, Optics Express 2009

The resonators are connected to the external environment through single mode waveguides. When the periodic ity of the chain satisfies the Bragg condition and when the incoming light is resonant with the resonator modes.



Xu, Sandhu, Povinelli, Shakya, Fan, Lipson, PRL 2006

Nuclear resonances for γ-quanta

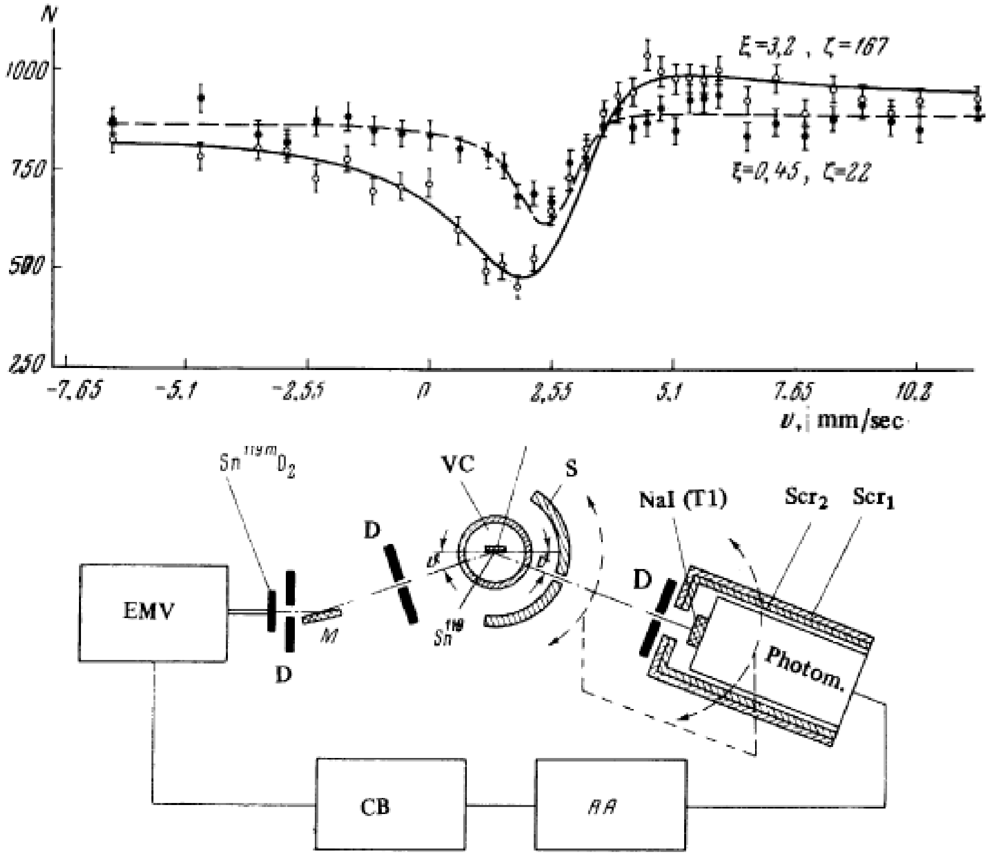
In the resonant gamma diffraction spectroscopy, the emitting and diffracting nuclei are the same, e.g., the Sn nuclei are both in a SnO_2 oxide layer (source) and a Sn monocrystal (diffracting material). The source (either the sample) is moving with various velocities v to produce a Doppler effect and scan the gamma ray energy through a given range. A typical range of velocities is around ± 10 mm/s (for ^{57}Fe 1 mm/s = 48.075 neV). In the resulting spectra, gamma-ray diffraction intensity is plotted as a function of the source velocity.

Voitovetskii, Korsunskii, Novikov, Pazhin, Diffraction of resonance γ rays by nuclei and electrons in tin single crystals, Sov. Phys. JETP 27 (1968)

Yu. Kagan, Theory of coherent phenomena and fundamentals in nuclear resonant scattering, Hyperfine Interactions 123/124, 83 (1999)

Nuclear resonances for γ-quanta

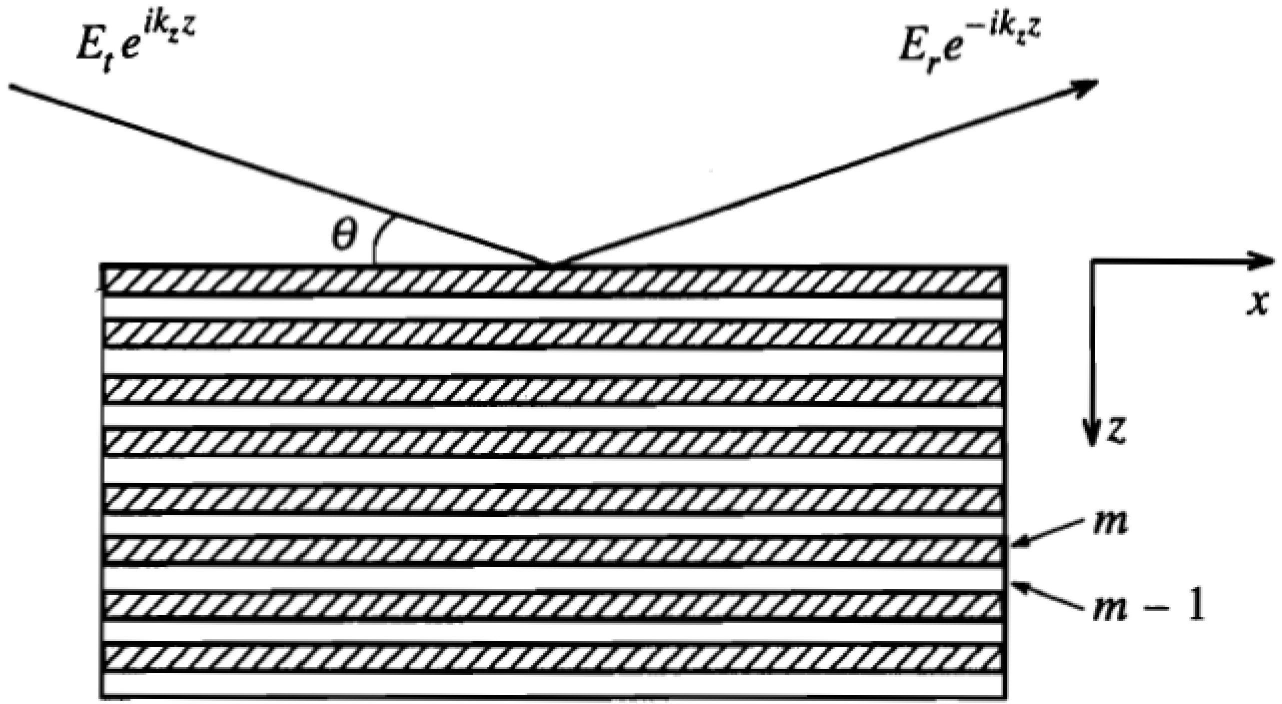
$\hbar\omega_0 = 24.8$ keV



Voitovetskii, Korsunskii, Novikov, Pazhin, Diffraction of resonance γ rays by nuclei and electrons in tin single crystals, Sov. Phys. JETP 27 (1968)

Yu. Kagan, Theory of coherent phenomena and fundamentals in nuclear resonant scattering, Hyperfine Interactions 123/124, 83 (1999)

Nuclear resonances for γ-quanta



Nuclear transition of the ^{57}Fe isotope
$\hbar\omega_0 = 14.413$ keV,
grazing angle
$\theta = 11.41$ mrad
$2(\omega_0 / c)\sin\theta = 2\pi/d$
d_{57}=10 Å, d_{56}=30 Å

Chumakov, Niesen, Nagy, Alp, "Nuclear resonant scattering of synchrotron radiation by multilayer structures", Hyperfine Interactions 123/124, 427 (1999)

Model nuclear periodic multilayers $[^{57}\mathrm{Fe}(d_{57})/^{56}\mathrm{Fe}(d_{56})]\cdot N$ *on a glass substrate. Here N is* the number of periods.

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Transfer matrix through a single QW

The transfer matrix connects the amplitudes of the electric field at the points $z = \pm d/2$

$$\begin{bmatrix} E'_+ \\ E'_- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} E_+ \\ E_- \end{bmatrix} .$$

The amplitudes are defined as follows: the incoming and outgoing waves on the left-hand side are, respectively,

$$E_+ \exp\left[\mathrm{i}q(z + d/2)\right] \text{ and } E_- \exp\left[-\mathrm{i}q(z + d/2)\right]$$

and those on the right-hand side are

$$E'_- \exp\left[-\mathrm{i}q(z - d/2)\right] \text{ and } E'_+ \exp\left[\mathrm{i}q(z - d/2)\right] .$$

The components T_{ij} are related to the reflection and transmission coefficients $\tilde{r} = \mathrm{e}^{\mathrm{i}qd} r_{\mathrm{QW}}$, $\tilde{t} = \mathrm{e}^{\mathrm{i}qd} t_{\mathrm{QW}}$ by

$$\hat{T} = \frac{1}{\tilde{t}} \begin{bmatrix} \tilde{t}^2 - \tilde{r}^2 & \tilde{r} \\ -\tilde{r} & 1 \end{bmatrix} .$$

Transfer matrix through a single QW

The components T_{ij} are related to the reflection and transmission coefficients $\tilde{r} = \mathrm{e}^{\mathrm{i}qd} r_{\mathrm{QW}}$, $\tilde{t} = \mathrm{e}^{\mathrm{i}qd} t_{\mathrm{QW}}$ by

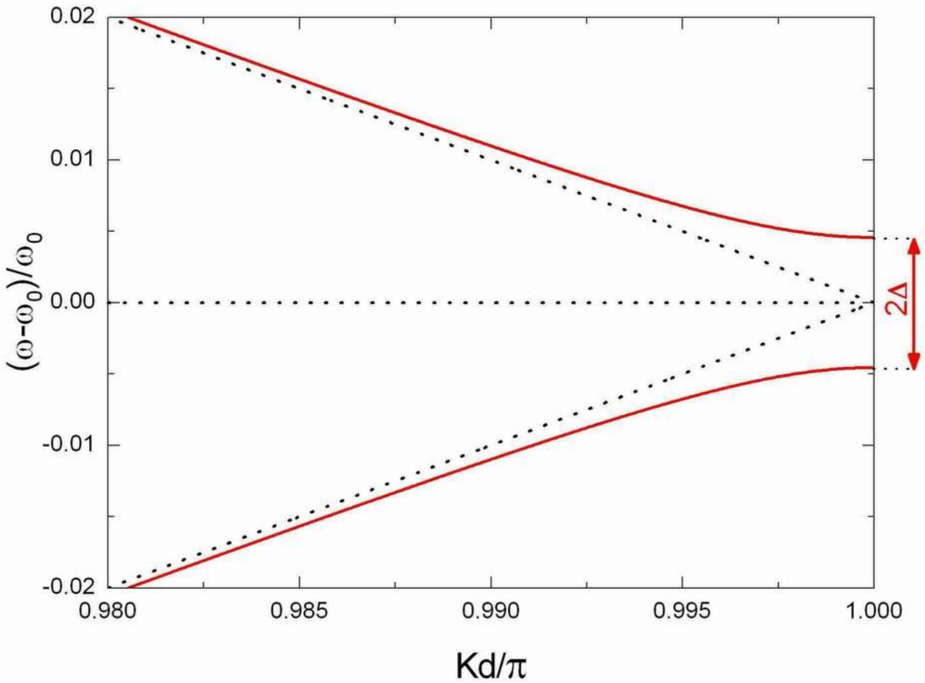
$$\hat{T} = \frac{1}{\tilde{t}} \begin{bmatrix} \tilde{t}^2 - \tilde{r}^2 & \tilde{r} \\ -\tilde{r} & 1 \end{bmatrix} .$$

$$\cos Kd = \frac{1}{2}\mathrm{Tr}(\hat{T}) = \frac{\tilde{t}^2 - \tilde{r}^2 + 1}{2\tilde{t}} = \cos qd + \mathrm{i} \sin qd \frac{r_{\mathrm{QW}}}{1 + r_{\mathrm{QW}}}$$

$$\cos Kd = \cos qd - \frac{\Gamma_0}{\tilde{\omega}_0 - \omega - \mathrm{i}\Gamma} \sin qd$$

Exciton-Polariton Dispersion in Resonant Bragg QW Structure

$$\omega - \omega_0 = \pm\sqrt{\frac{2}{\pi}\Gamma_0\omega_0 + \omega_0^2\left(\frac{Kd}{\pi} - 1\right)^2}$$



$$\Delta = \sqrt{\frac{2\Gamma_0\omega_0}{\pi}}$$

$Ga_{0.96}In_{0.04}As/GaAs$: $\hbar\Gamma_0 = 0.027$ meV, $\Delta = 5$ meV

$CdTe/Cd_xZn_{1-x}Te$: $\hbar\Gamma_0 = 0.12$ meV, $\Delta = 11$ meV

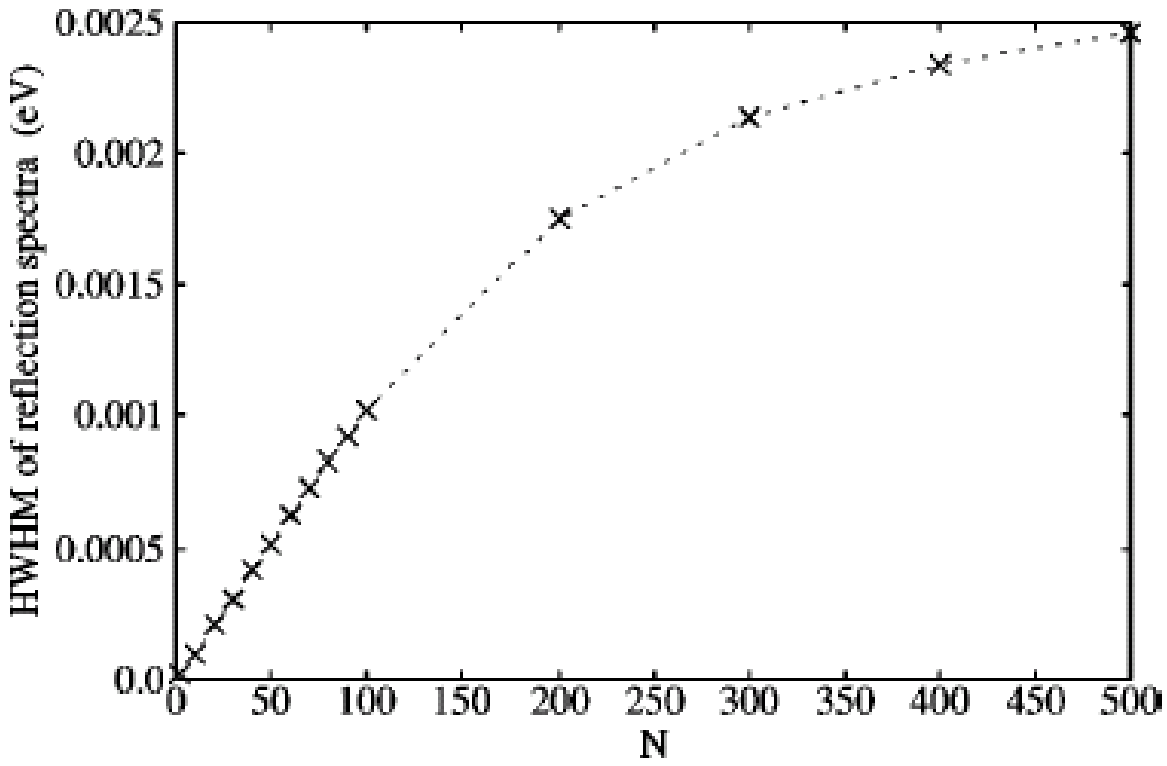
Two regimes of light reflection

With increasing the number of wells
the reflection-peak halfwidth changes

from $N\Gamma_0 + \Gamma$ (superradiant regime)
to $\Delta = \sqrt{\frac{2}{\pi}\Gamma_0\omega_0}$ (photonic-crystal regime)

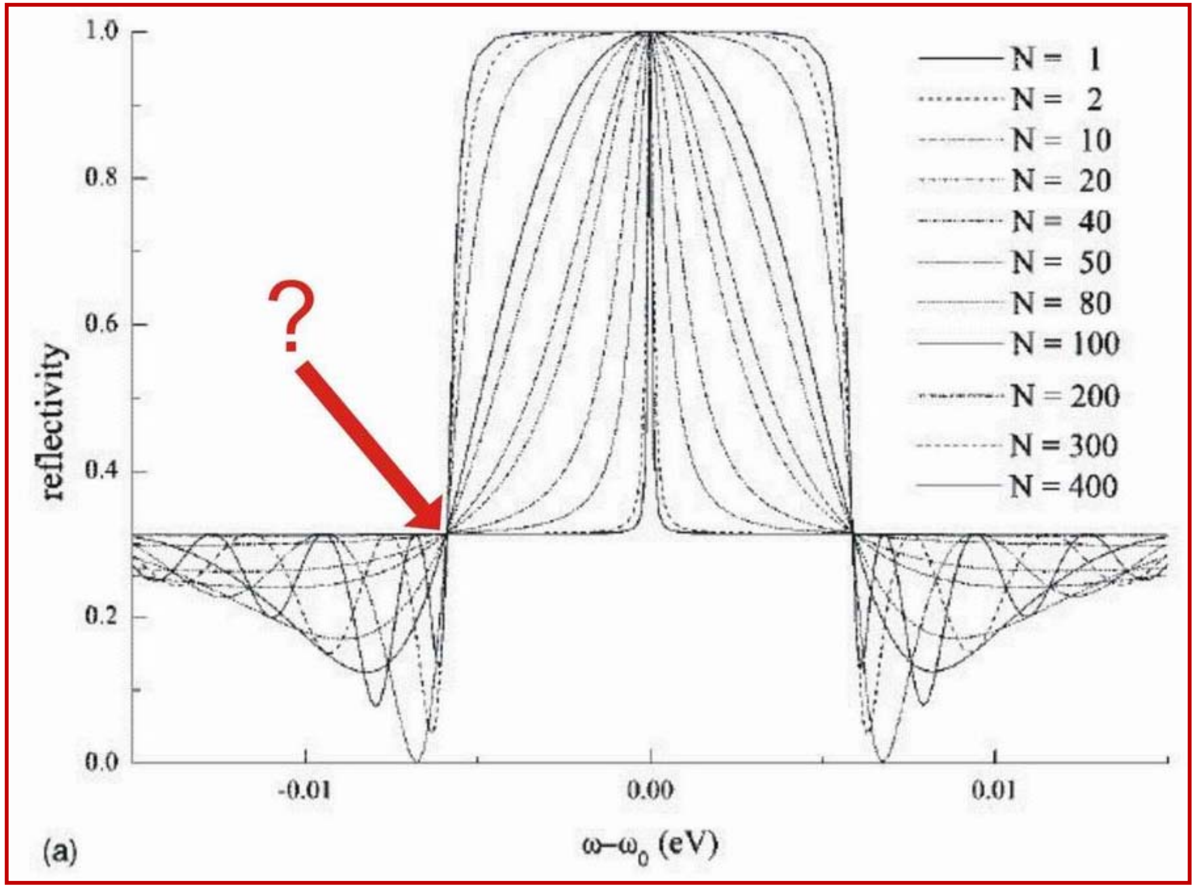
$$N\Gamma_0 \ll \Delta = \sqrt{2\Gamma_0\omega_0/\pi} \quad \text{or} \quad N\sqrt{\Gamma_0/\omega_0} \ll 1$$

Reflection spectral width as a function of the QW number



TOMOE IKAWA AND KIKUO CHO PHYSICAL REVIEW B **66**, 085338 (2002)

Reflection spectral width as a function of the QW number



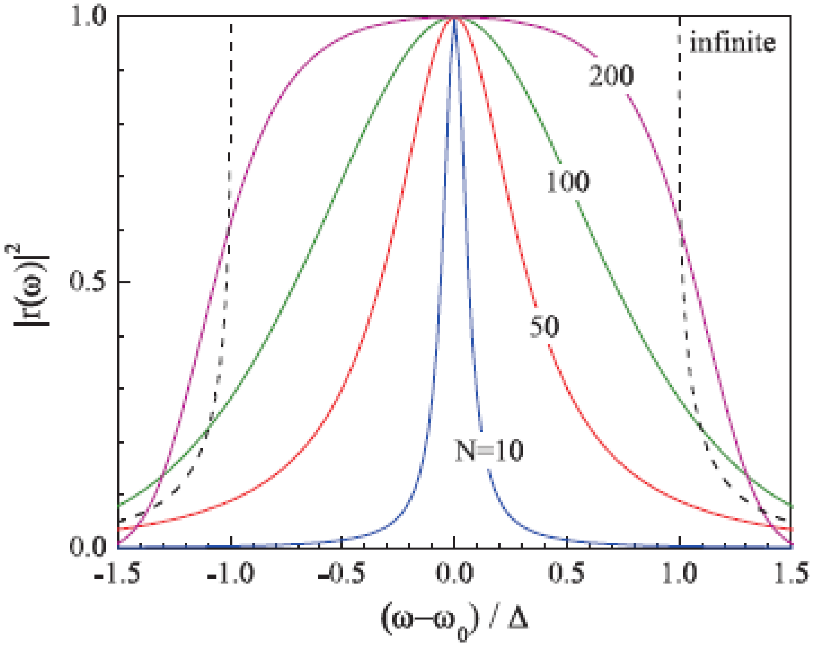
$a = 8$ nm, $\varepsilon_b = 12.6$,
$\hbar\omega_0 = 1.5152$ eV,
$\hbar\Gamma_0 = 33$ μeV

Pilozzi, D' Andrea, Cho, Phys. Rev. B **69**, 205311 (2004)

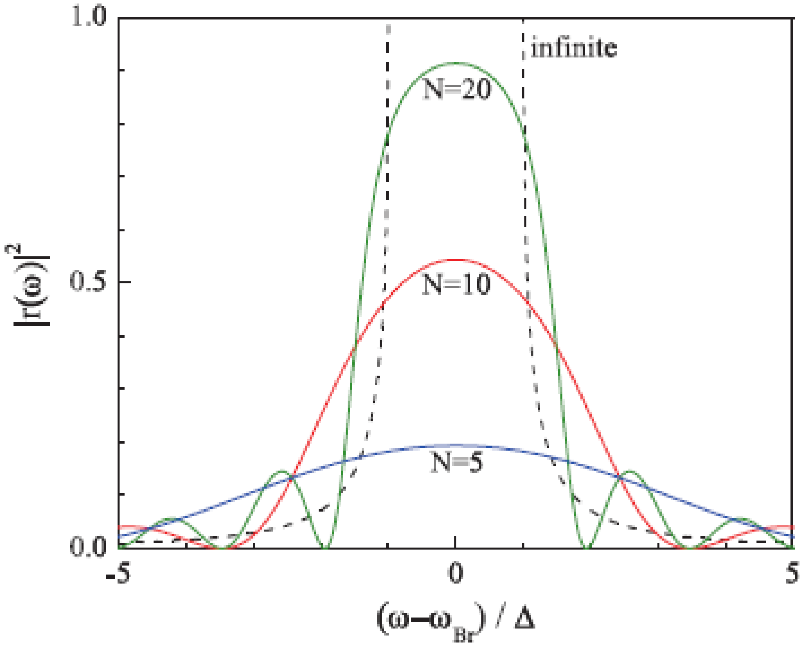
Voronov, Ivchenko, Poddubny, Chaldyshev, FTT **49**, 1710 (2006)

Comparison of resonant and nonresonant Bragg structures

Resonant Bragg MQWs



Nonresonant lattice $ABAB\ldots$



$\Gamma = 0$

$$\Delta = \sqrt{\frac{2}{\pi}\omega_0\Gamma_0}$$

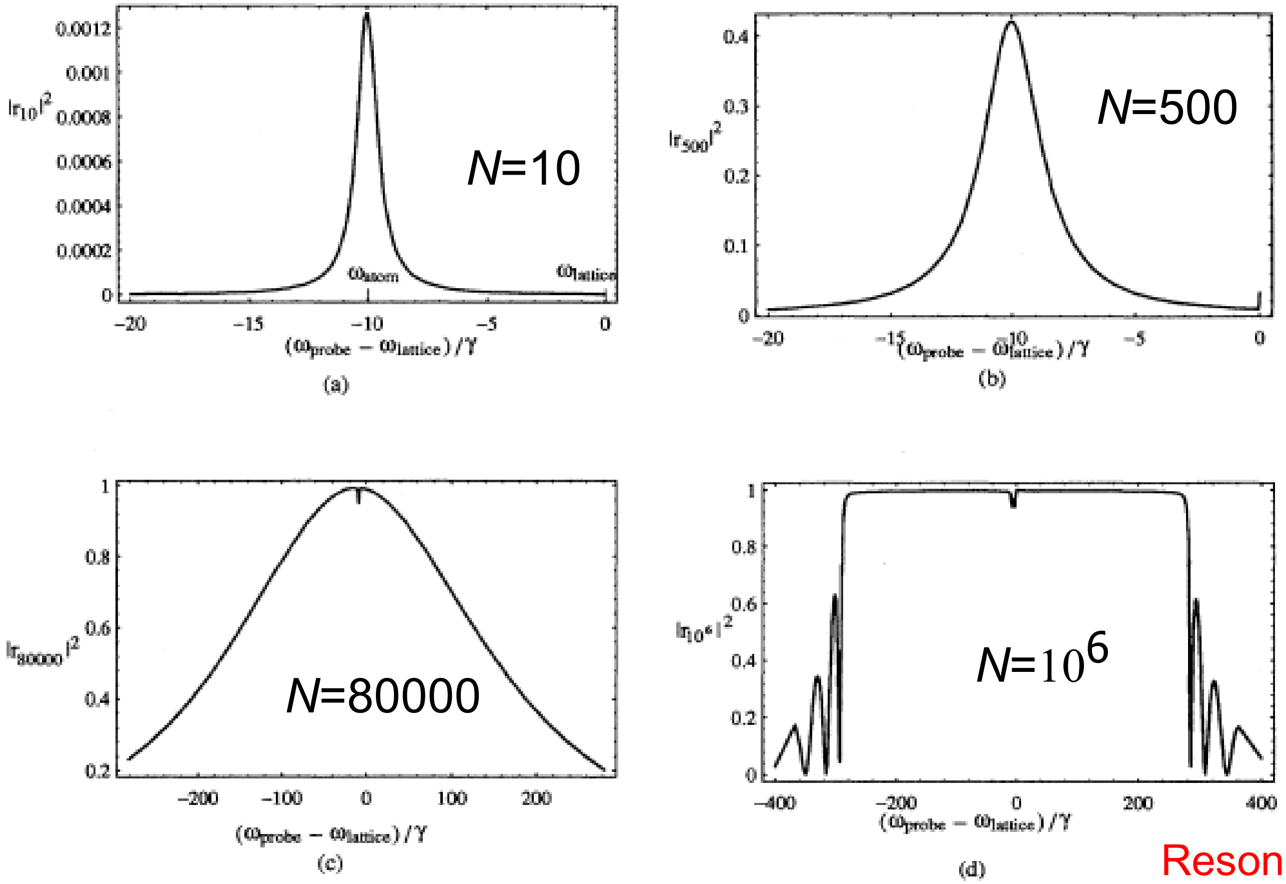
$n_A = 2.916,\ n_B = 2.653$

$$\Delta = \frac{2}{\pi}\frac{n_A - n_B}{n_A + n_B}\omega_{\rm Br}$$

Poshakinskiy 2012

Reflection from atomic lattice

Reflection spectra from a lattice of Cs atoms for various numbers of atomic planes



$$\Gamma_0 = 3.7 \times 10^{-3}\, \Gamma$$

$$\omega_{Br} = \omega_0 + 20\, \Gamma$$

$$\omega_{Br} = \frac{c}{n_b} \frac{\pi}{d}$$

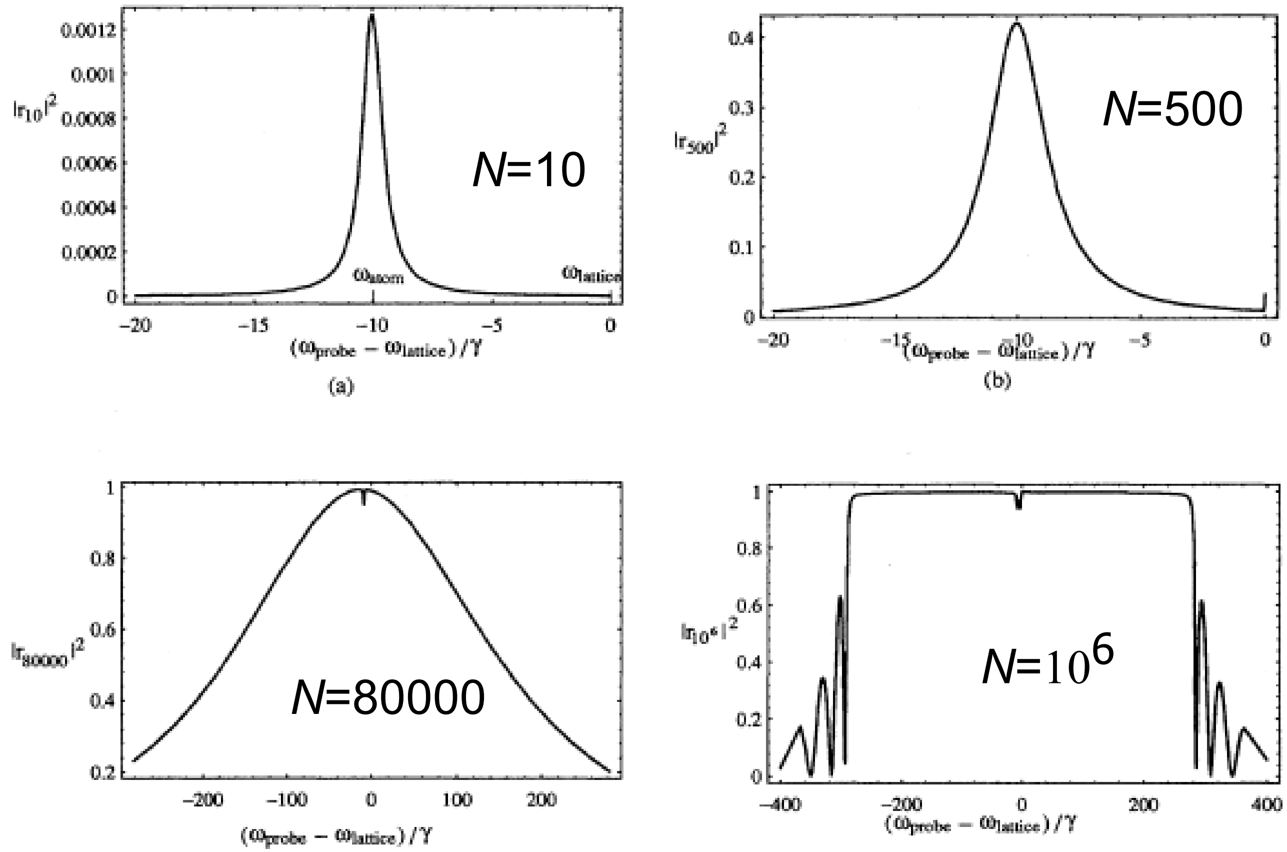
Resonant Bragg condition:

$\omega_{Br} = \omega_0$, damping $= N\Gamma_0 + \Gamma$

Deutsch, Spreeuw, Rolston, Phillips, PRB 1995

Two regimes of light reflection

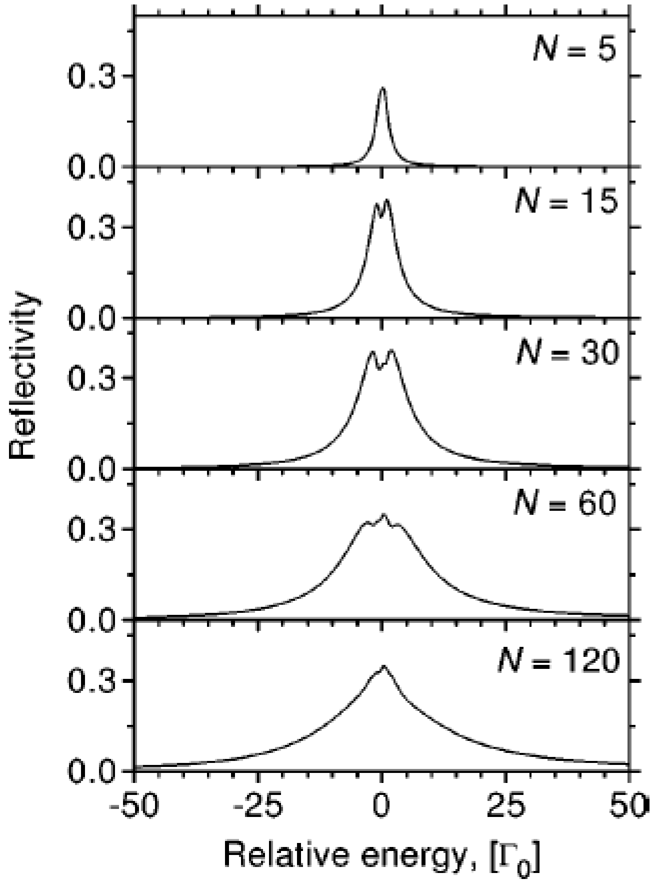
Reflection spectra from a lattice of Cs atoms for various numbers of atomic planes



$$\xi_0 \gamma = 2\Gamma_0$$

$$\Delta = \sqrt{\frac{2\omega_0 \Gamma_0}{\pi}} \longleftrightarrow \Delta\omega_{\mathrm{max}} = \sqrt{\frac{\xi_0 \gamma \omega_{\mathrm{atom}}}{\pi}}$$

Nuclear Bragg reflection



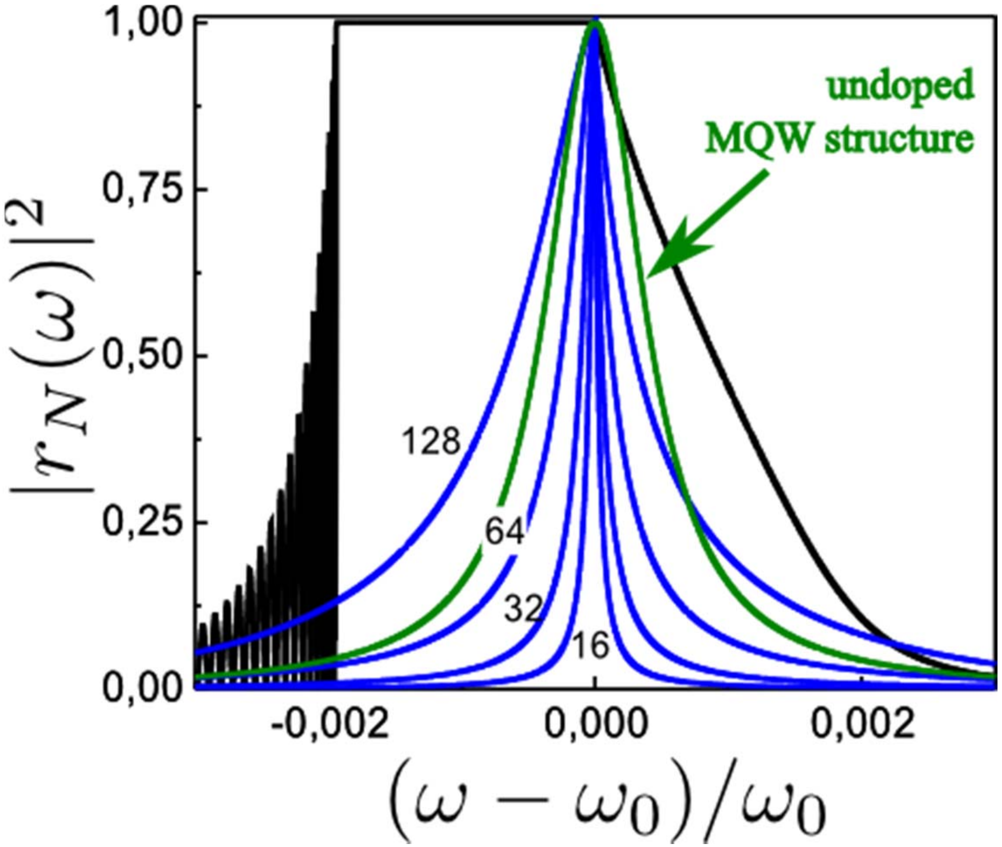
Computer simulation of the energy spectra of the first-order nuclear Bragg reflection for a

$[^{57}Fe(d_{57})/^{56}Fe(d_{56})]\cdot N$

multilayer with different number of periods *N.*

Chumakov et al. 1999

Exciton in the 2D Fermi sea



$$\boldsymbol{P}(z) = \int \pi(\omega, z, z')\boldsymbol{E}(z')\mathrm{d}z'$$

$$\pi(\omega, z, z') = \frac{1}{4}\hbar\kappa_b\omega_{LT}a_B^3\Phi(z)\Phi(z')G(\omega)$$

$$G(\omega) = \frac{1}{\mathcal{S}}\sum_{\boldsymbol{k}}\left|\frac{M_k}{M_k^0}\right|^2\frac{1 - n_F(k)}{E_g + \dfrac{\hbar k^2}{2m} - \hbar\omega - \mathrm{i}0}$$

Averkiev, Glazov, Voronov (2012)

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Resonant optical spectra. Experiment

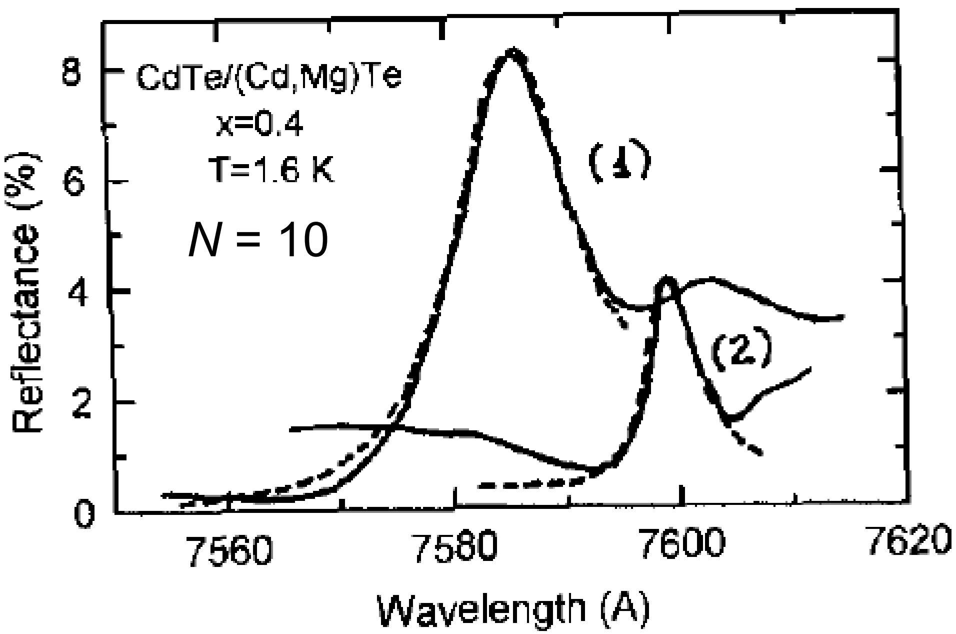
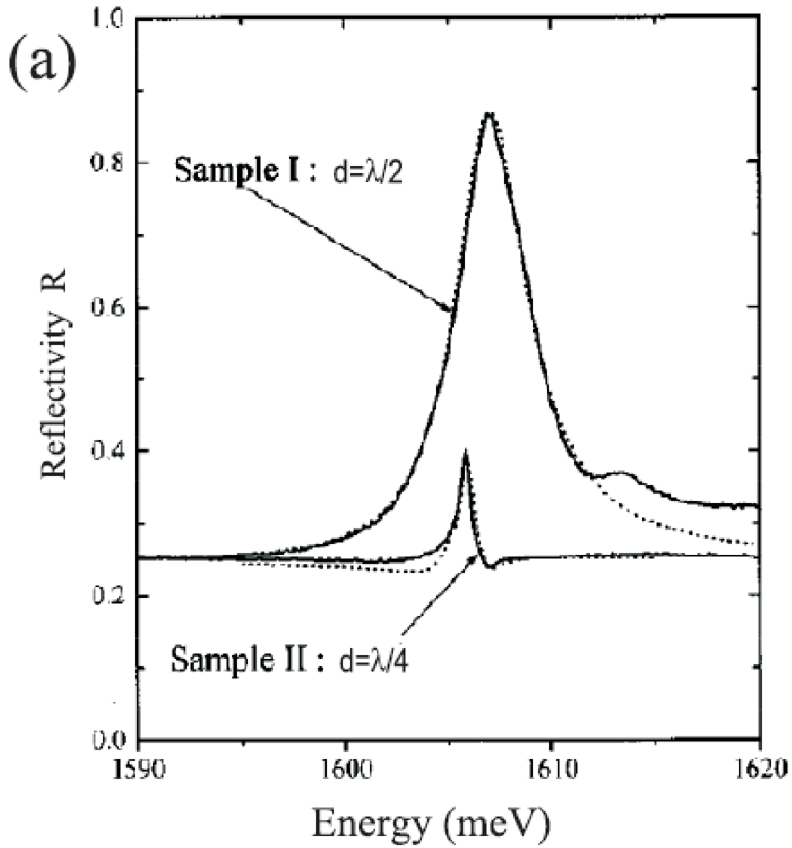


Fig.1. Reflectance from the Bragg (curve 1) and anti-Bragg (curve 2) MQW structures $CdTe/Cd_{1-x}Mg_xTe$ (x=0.4) at oblique incidence θ=68°. Dashed lines are the theoretical fit with parameters: $\hbar\omega_0$=1.633 eV, $\hbar\Gamma$=1.3 meV, $\hbar\Gamma_0$=0.15 meV for Bragg structure and $\hbar\omega_0$=1.631 eV, $\hbar\Gamma$=0.6 meV, $\hbar\Gamma_0$=0.2 meV for anti-Bragg structure.

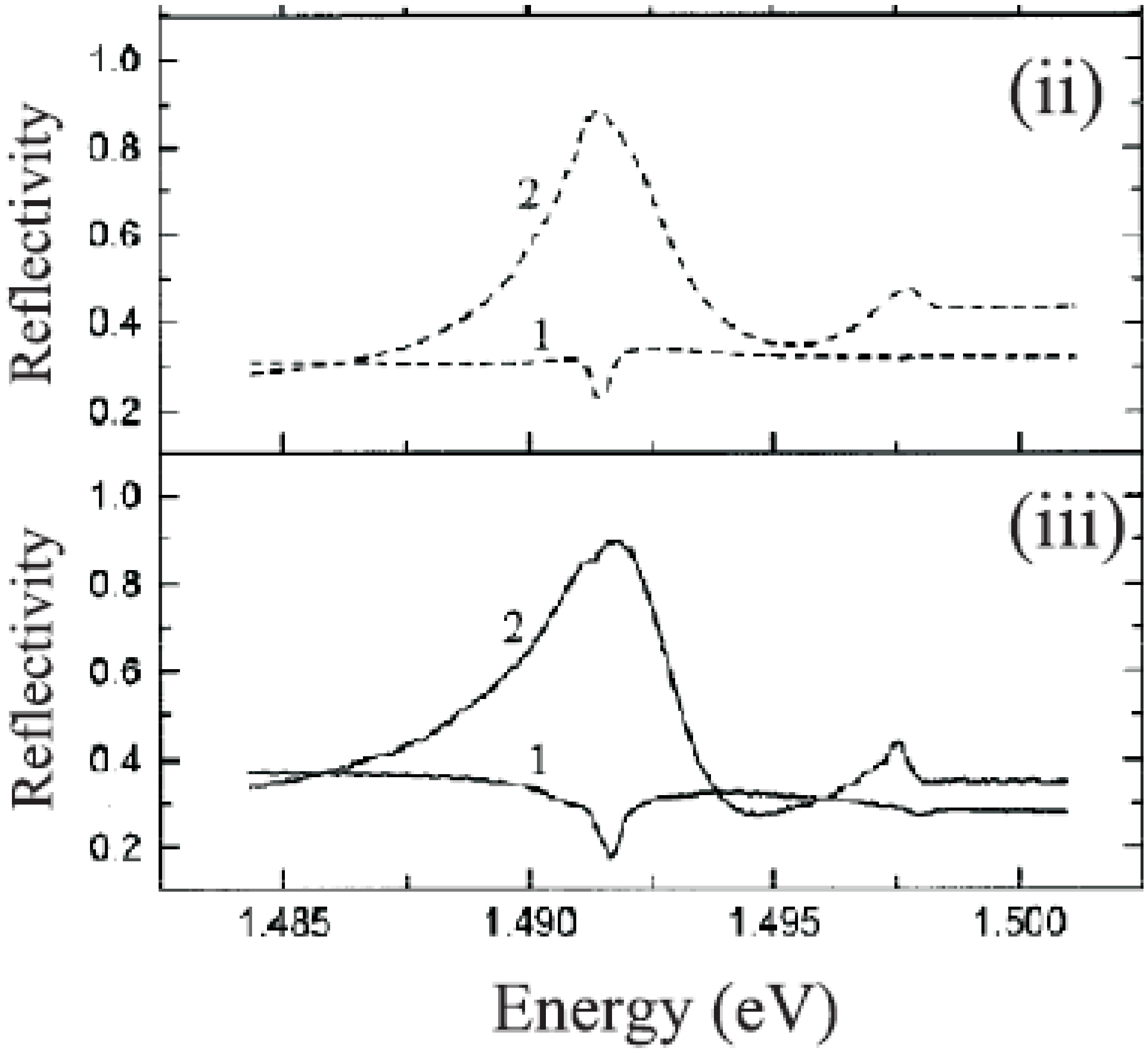
Kochereshko, Pozina, Ivchenko, Yakovlev, Waag, Ossau, Landwehr, Hellmann, Göbel, Superlattices&Microstructures 1994



$CdTe/Cd_xZn_{1-x}Te$, N = 10

Merle d'Aubigné, Wasiela, Marriete, Dietl 1994

Resonant optical spectra. Experiment



1 – $d = 0.85\ \lambda/2$

2 – Bragg structure

$In_{0.04}Ga_{0.96}As/GaAs$

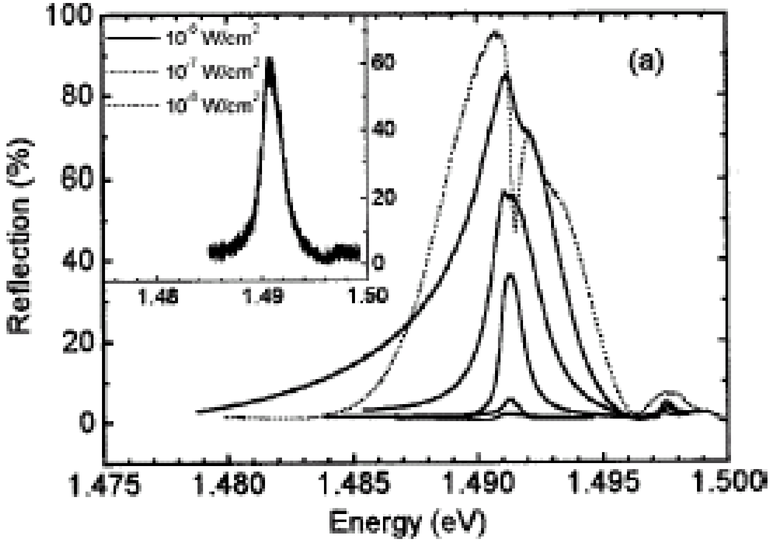
$N = 30$

Theory

Experiment

Ell et al. (1998)

Reflection spectral width as a function of the QW number



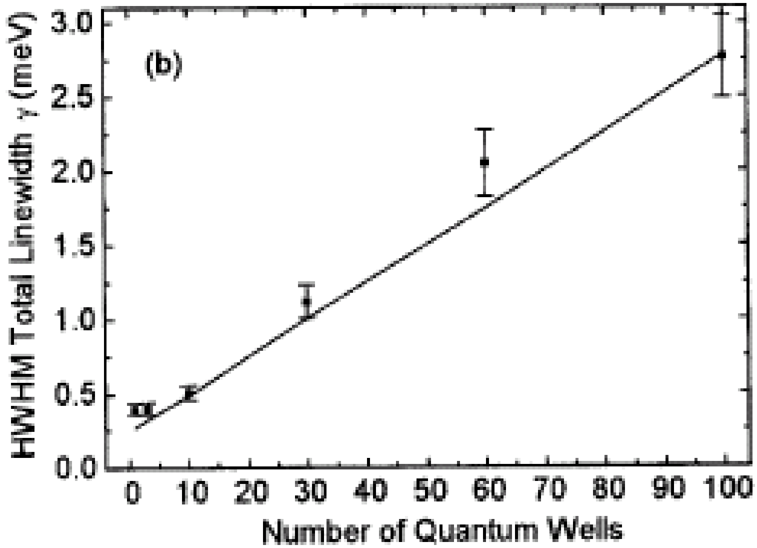


FIG. 4. (a) Increase of experimental reflection with N for 1, 3, 10, 30, 60, and 100 QW's with Bragg periodicity. The experimental measurements were done with an AR coating on the front and back.

$$\Gamma_0 = 27 \pm 2\ \mu\text{eV}$$

$$\Gamma = 0.32 \pm 0.03\ \text{meV}$$

halfwidth = $N\Gamma + \Gamma_0$

Prineas, Ell, Lee, Khitrova, Gibbs, and Koch (2000)

Room temperature spectra

APPLIED PHYSICS LETTERS **99**, 251103 (2011)

Optical lattices of InGaN quantum well excitons

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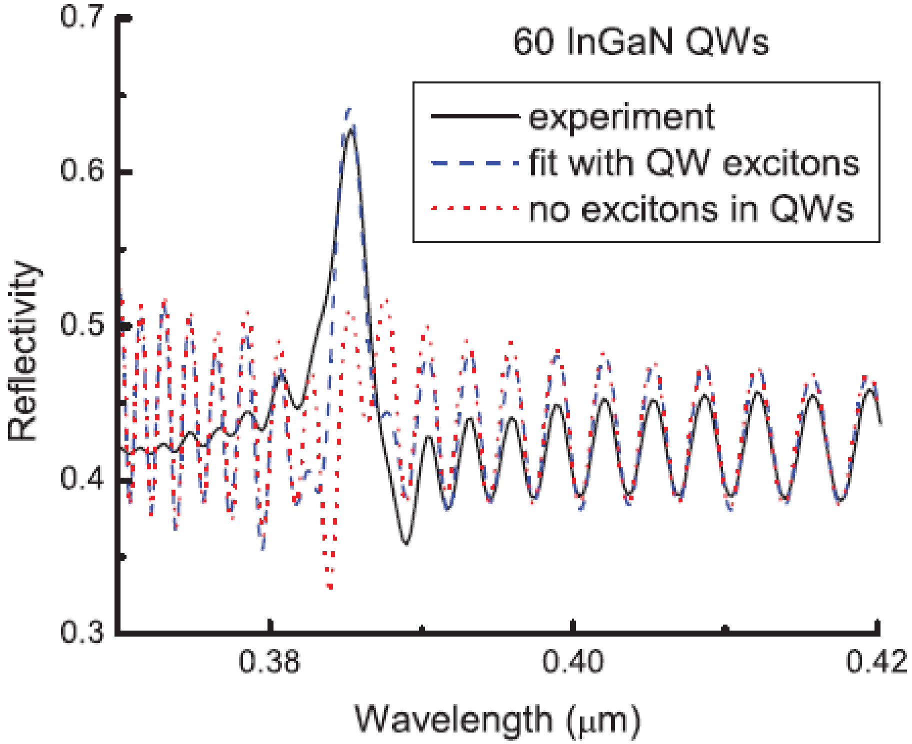
A. F. Tsatsulnikov,[1] M. A. Yagovkina,[1] Taek Kim,[2] and Youngsoo Park[2]



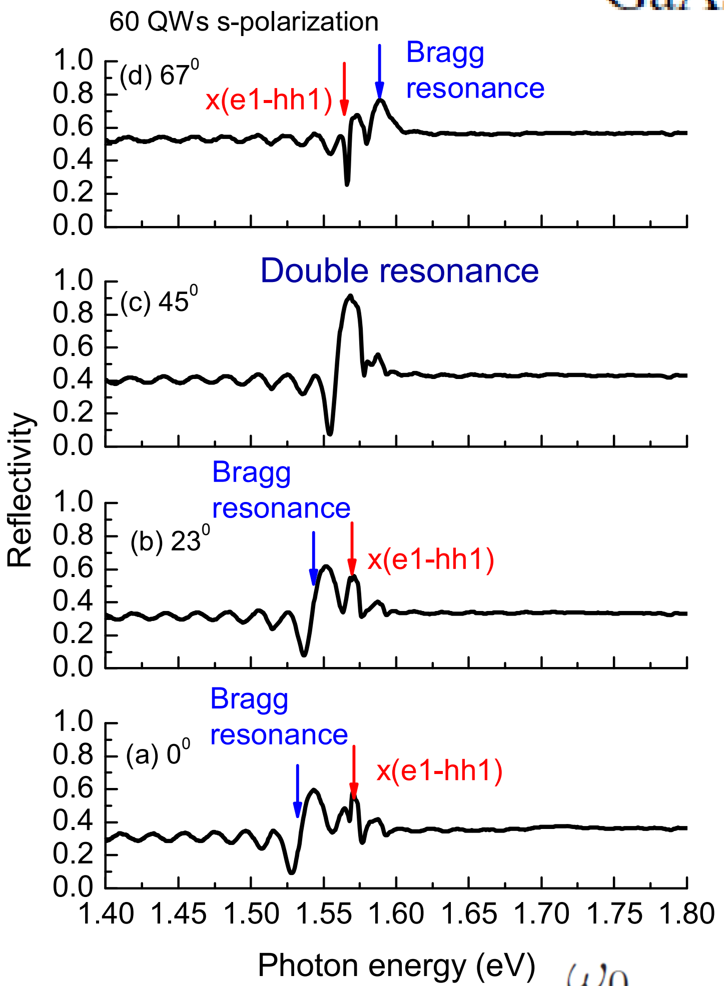
Experimental (solid black) and calculated (colored curves) spectra of the optical reflection from the sample with 60 InGaN QWs.

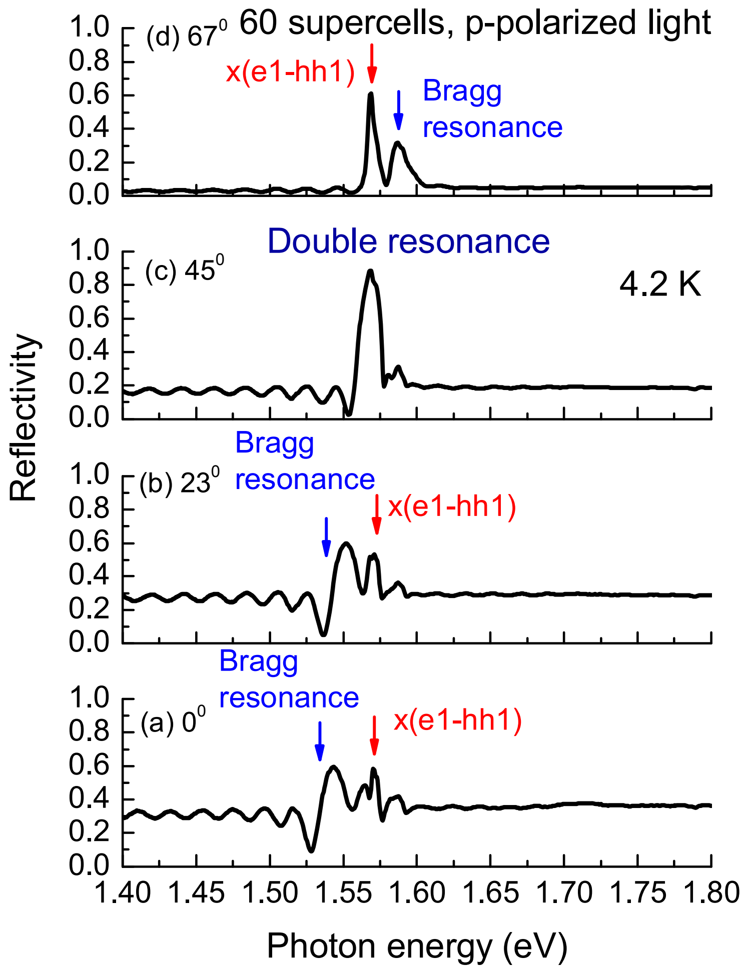
$\hbar\omega_0 = 3.22$ eV,
$\hbar\Gamma_0 = 0.17$ meV
$\hbar\Gamma = 27$ meV
room temperature

s-polarized light incident at 60º

Angular dependence of reflectivity

GaAs/AlGaAs





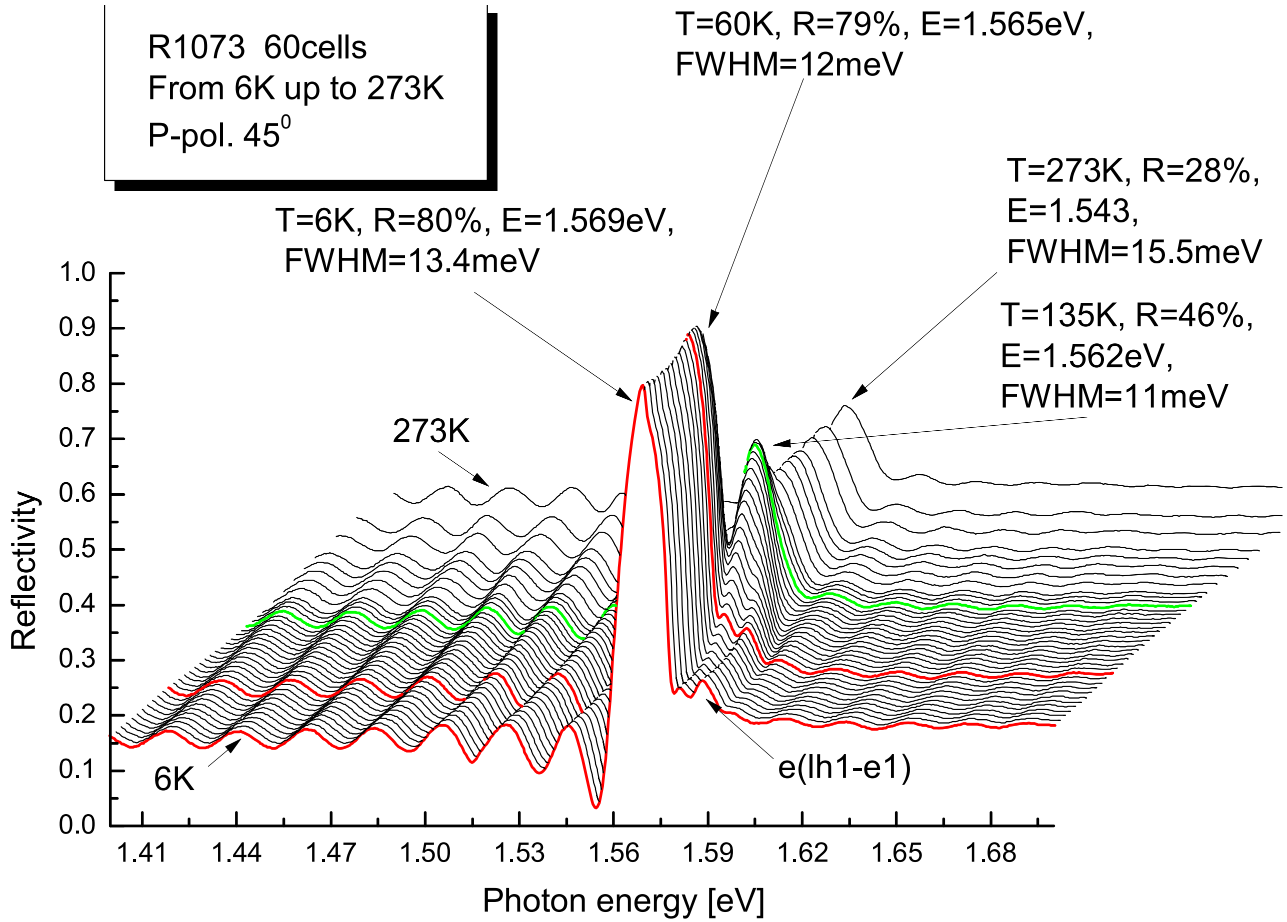
$\hbar\Gamma_0$ = 40 μeV

$$\frac{\omega_0}{c} n_b d \cos\theta = \pi$$

Chaldyshev et al. 2012

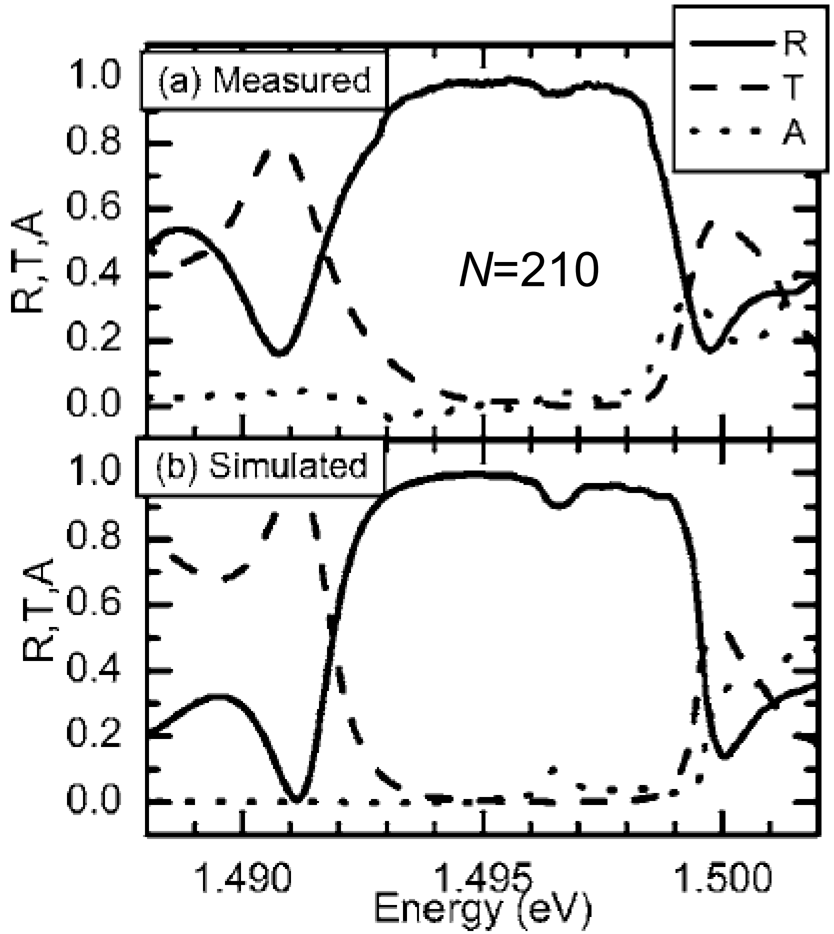
Temperature dependence

GaAs/AlGaAs



Chaldyshev et al. 2012

Light reflection from 210 QWs



$$A = 1 - R - T$$

$In_{0.025}Ga_{0.975}As/GaAs$

$\hbar\Gamma$ = 0.2 meV
$\hbar\Gamma_0$= 31 μeV
12 nm/103 nm

Bragg-spaced QW structure

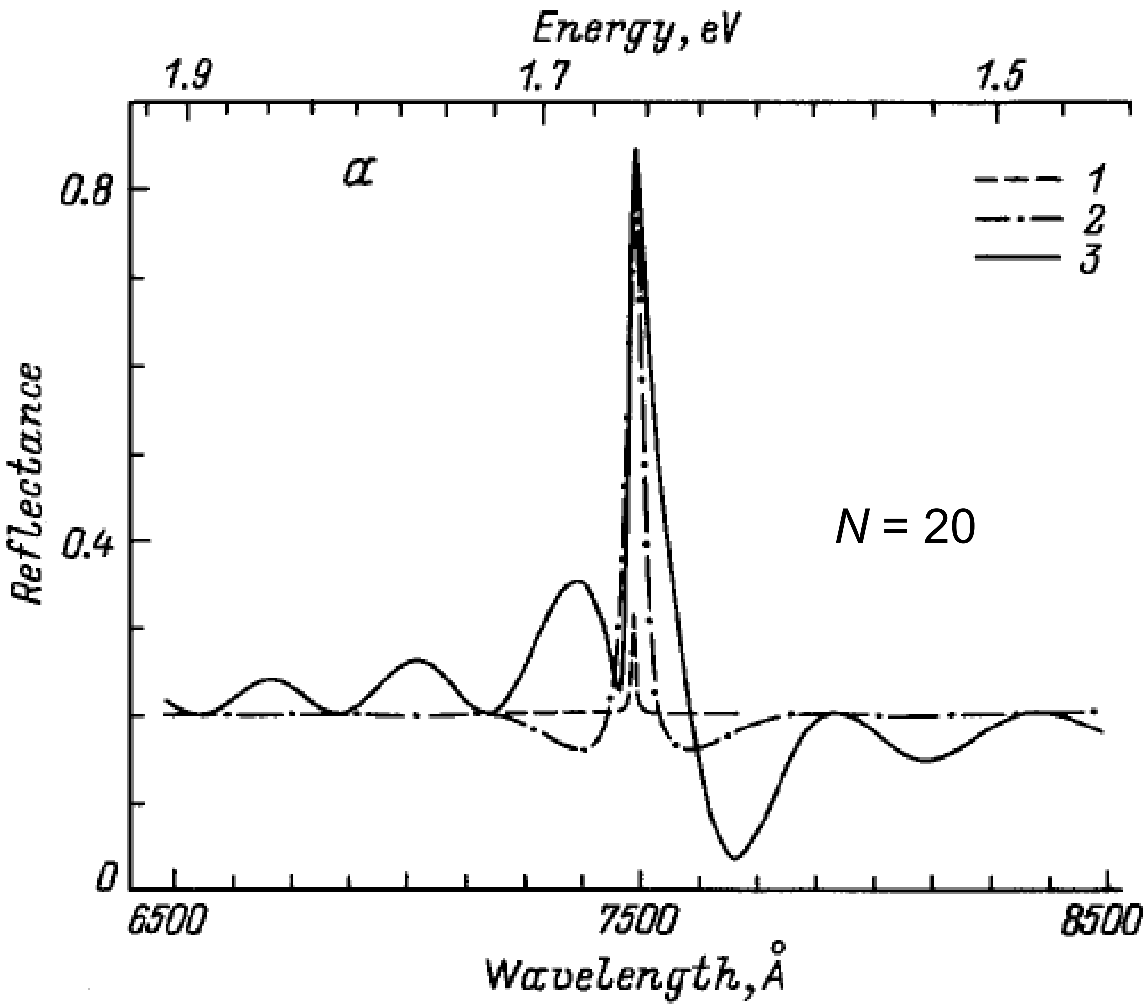
Prineas, Yildirim, Johnston, Reddy, PRB, 2006

Reflection coefficient from *N* QWs

$$r_N = \frac{\tilde{r}}{1 - \tilde{t}\,\dfrac{\sin(N-1)Kd}{\sin NKd}}$$

$$\cos Kd = \cos qd - \frac{\Gamma_0}{\omega_0 - \omega - i\Gamma}\,\sin qd$$

Effect of dielectric contrast

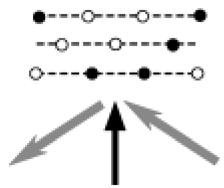


Ivchenko, Kochereshko, Platonov, Yakovlev, Waag, Ossau, Landwehr, 1997

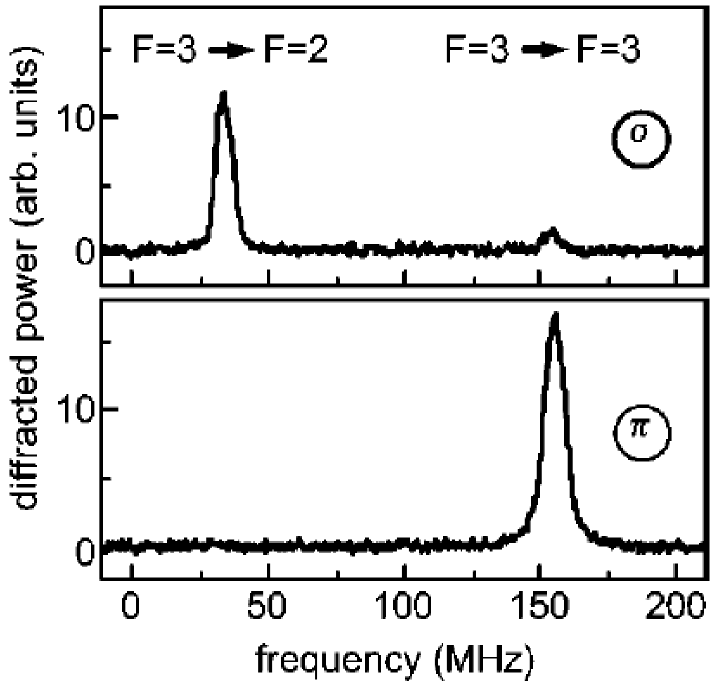
Diffraction in the Born approximation

Neglecting multidiffraction, one has for the diffraction efficiency

$$I_{\text{dif}} \propto \frac{E_0^2 N^2 |\boldsymbol{e}_f^* \hat{\boldsymbol{S}} \boldsymbol{e}_i|^2}{(\omega_0 - \omega)^2 + \Gamma^2} \, \delta_{\boldsymbol{q}_f, \boldsymbol{q}_i + \boldsymbol{b}}$$



Weidemüller et al. 1998



Spectrum of the Bragg diffracted power vs frequency of the incident blue light. The incident beam is linearly polarized (a) in the horizontal plane (s polarization) and (b) along the vertical direction (p polarization). The resonance frequencies of the $5S_{1/2}(F{=}3) \rightarrow 6P_{1/2}(F{=}2)$ and $(F{=}3)$ transitions are indicated above (λ = 422 nm).

^{85}Rb

Preparation of the lattice:

$5S_{1/2}(F{=}3) \rightarrow 5P_{3/2}(F{=}4)$

$\lambda_L = 780.2$ nm   ($\lambda_L \cos\theta \approx \lambda$)

Optical lattices. Multiple reflection

PRL 106, 223903 (2011) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2011

Photonic Band Gaps in One-Dimensionally Ordered Cold Atomic Vapors

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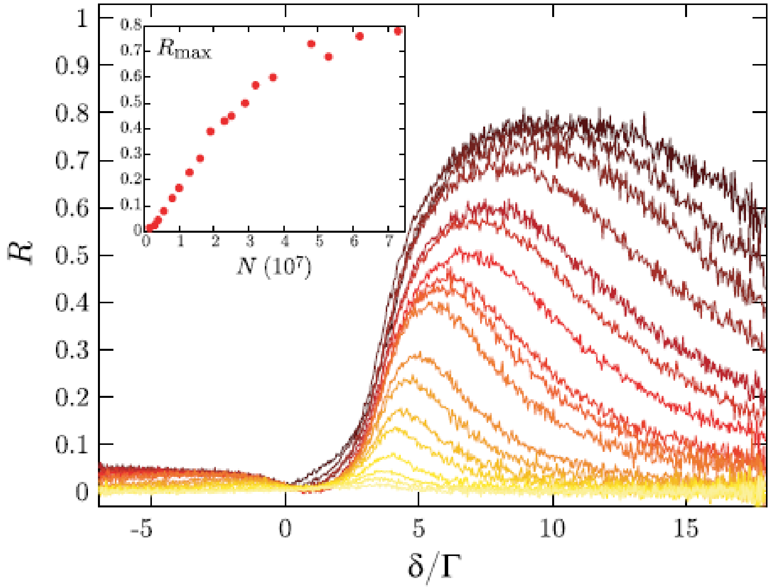
(Received 18 January 2011; revised manuscript received 4 April 2011; published 3 June 2011)

We experimentally investigate the Bragg reflection of light at one-dimensionally ordered atomic structures by using cold atoms trapped in a laser standing wave. By a fine-tuning of the periodicity, we reach the regime of multiple reflection due to the refractive index contrast between layers, yielding an unprecedented high reflectance efficiency of 80%. This result is explained by the occurrence of a photonic band gap in such systems, in accordance with previous predictions.



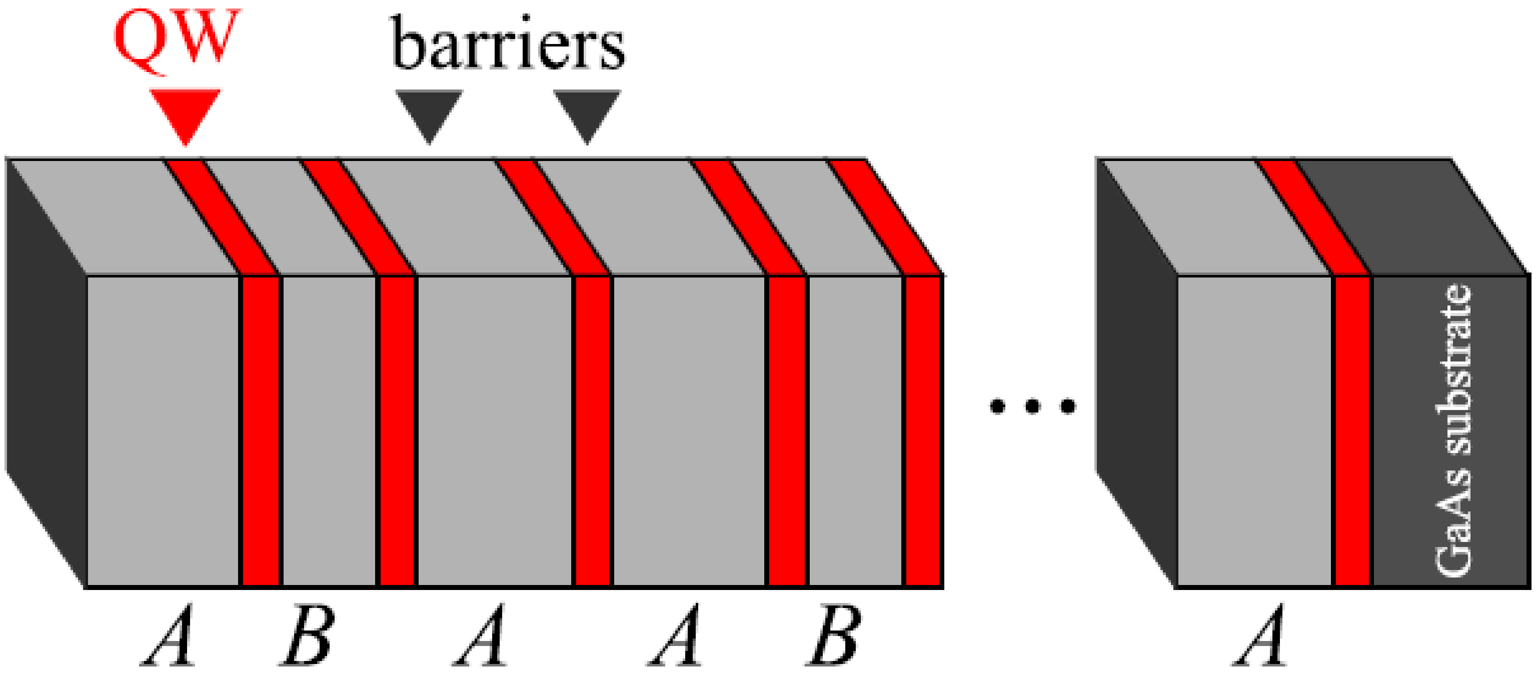
Spectra for different ^{85}Rb atom numbers ***N*** in the lattice (constant length, varying density)

Resonant photonic crystals and quasicrystals

- Introduction. Resonant Bragg QWs
- QWs, Optical Lattices, Nuclear Resonances
- Superradiant and Photonic-Crystal Regimes
- Experimental Illustration
- Resonant Fibonacci QW Chains
- Time-Resolved and Nonlinear Properties

Quasicrystals are one of the solid structural forms (in addition to crystals and amorphous solids) to be nonperiodic and possess the long-range order compatible with the Bragg diffraction.

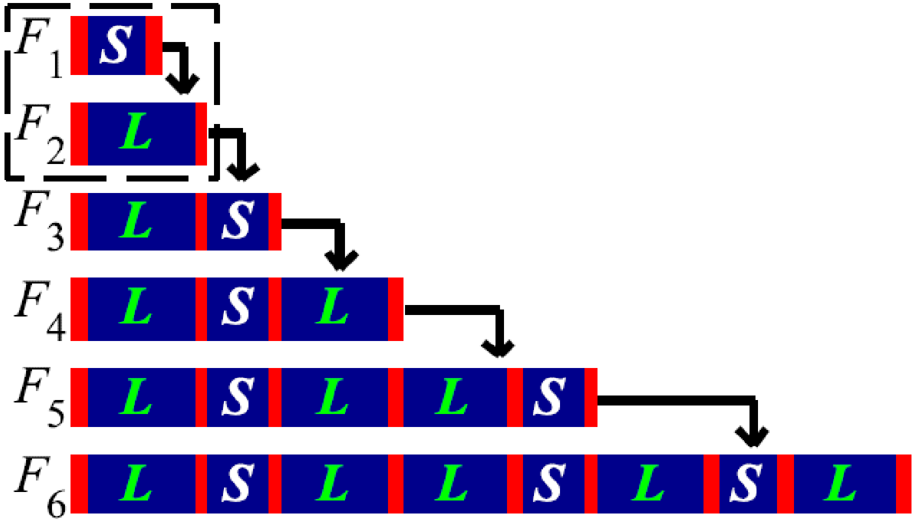
Fibonacci QW structures



Fibonacci chain

Fibonacci QW sequence

$$F_{j+1} = \{F_j, F_{j-1}\}$$



canonic Fibonacci:

$l/s = \tau$

golden mean

$$\tau = \frac{\sqrt{5}+1}{2} \approx 1.62$$

---

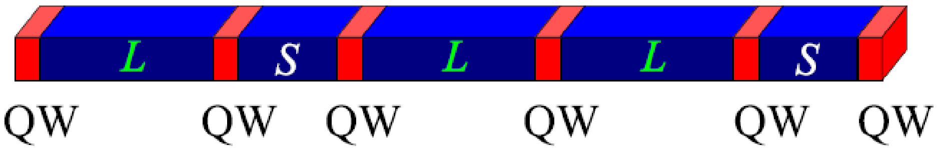
periodic:

$l/s = 1$

---

noncanonic Fibonacci: arbitrary l/s

Fibonacci multiple-QW structures



Structure factor

The structure factor of one-dimensional chain of sites z_j:

$$f(q) = \lim_{N\to\infty} f(q,N)\ ,\ \ f(q,N) = \frac{1}{N}\sum_{j=1}^{N} \mathrm{e}^{2\mathrm{i}qz_j}$$

Structure factor of the Fibonacci chain

ABAABABA... $a/b = \tau$

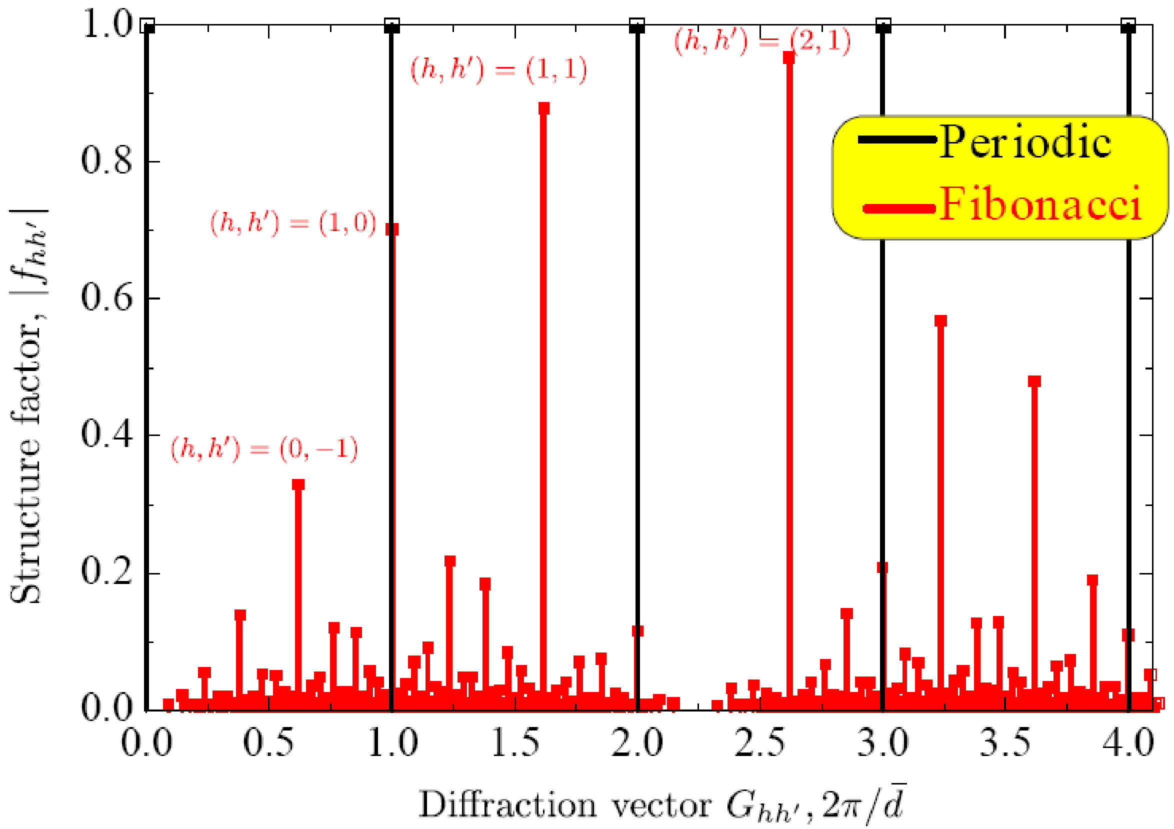
$$f(q) = \sum_{h,h'=-\infty}^{\infty} \delta_{2q,G_{hh'}} f_{hh'} \,, \qquad G_{hh'} = \frac{2\pi}{\bar{d}} \left(h + \frac{h'}{\tau} \right)$$

$$\bar{d} = b(3-\tau) \,, \; f_{hh'} = \frac{\sin S_{hh'}}{S_{hh'}} \mathrm{e}^{\mathrm{i}\theta_{hh'}}$$

$$S_{hh'} = \pi \frac{\tau(\tau h' - h)}{\tau + 2} \,, \; \theta_{hh'} = \frac{\tau - 2}{\tau} S_{hh'}$$

Structure factor of quasicrystal

$$f(q) \stackrel{\text{def}}{=} \lim_{N\to\infty} \frac{1}{N}\sum_{j=1}^{N} \exp(2iqz_j) = \begin{cases} 0 & \textbf{(disordered)} \\ \sum_{h,h'} \delta_{2q,G_{hh'}} f_{hh'} & \textbf{(quasicrystalline)} \\ \sum_h \delta_{2q,2\pi h/d} & \textbf{(periodic)} \end{cases}$$



Diffraction vectors of Fibonacci lattice

$$G_{hh'} = \frac{2\pi}{\bar{d}}\left(h + \frac{h'}{\tau}\right)$$

Resonance Bragg condition

1. Periodic quantum-well structure

$$q(\omega_0) = \pi \qquad \text{or} \qquad \frac{\omega_0}{c} n_b = \frac{\pi}{d}$$

In general,

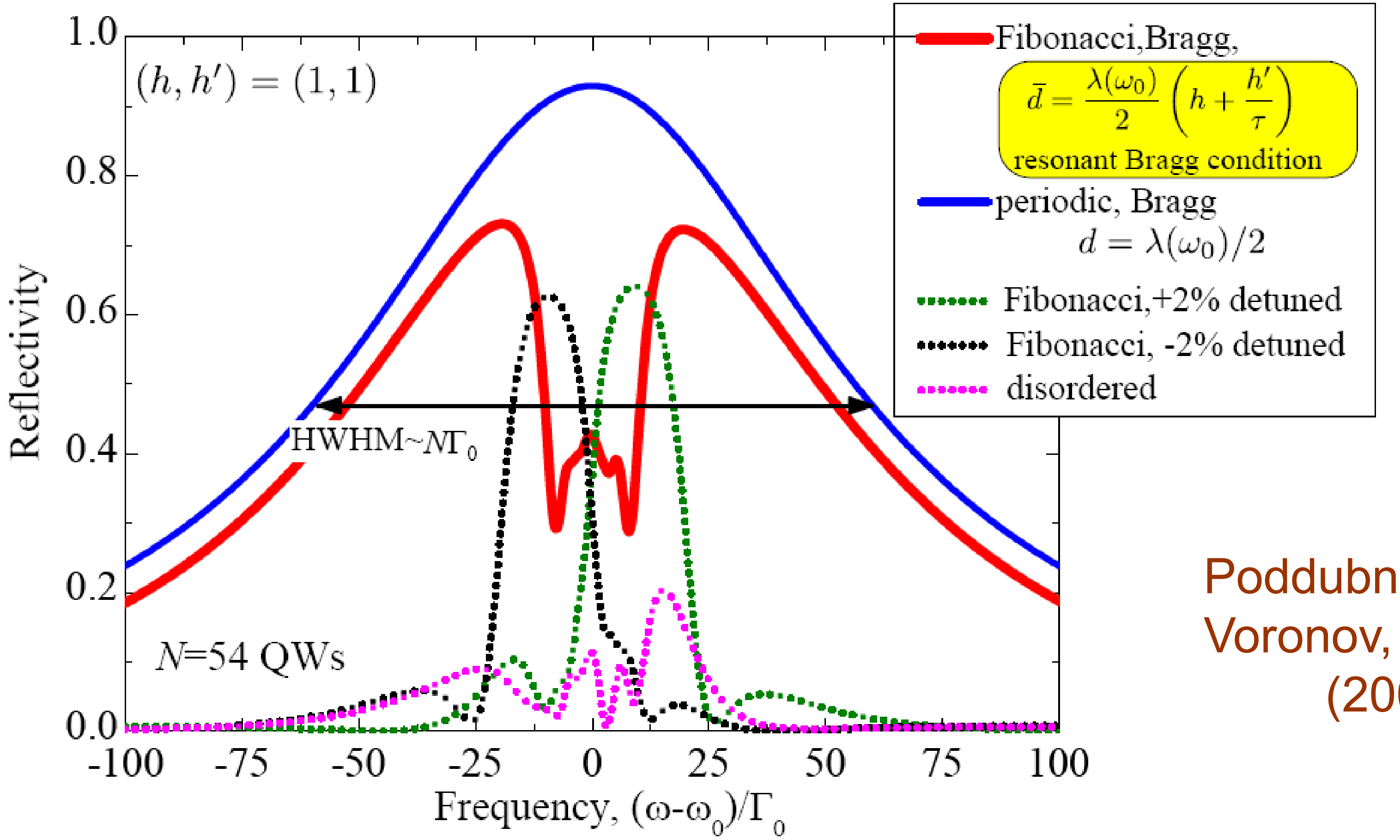
$$\frac{\omega_0}{c} n_b = \frac{\pi}{d} h \quad (h = 1, 2, 3 \ldots)$$

2. Fibonacci quantum-well structure

$$\frac{\omega_0}{c} n_b = \frac{G_{hh'}}{2} = \frac{\pi}{\bar{d}} \left(h + \frac{h'}{\tau} \right) \qquad (h = F_m, h' = F_{m-1})$$

Poddubny, Pilozzi, Voronov, Ivchenko, Phys. Rev. B 2008

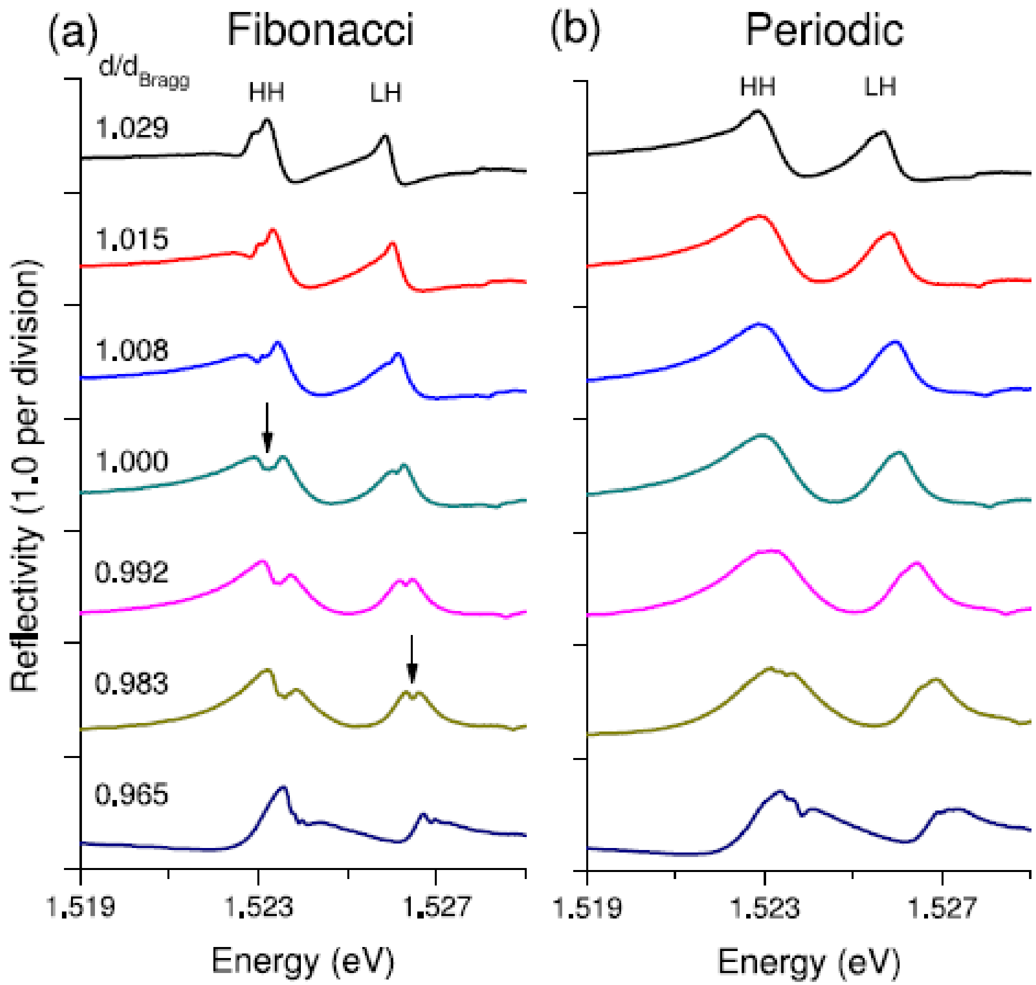
Reflection from Bragg Fibonacci QW structure



Poddubny, Pilozzi, Voronov, Ivchenko (2008)

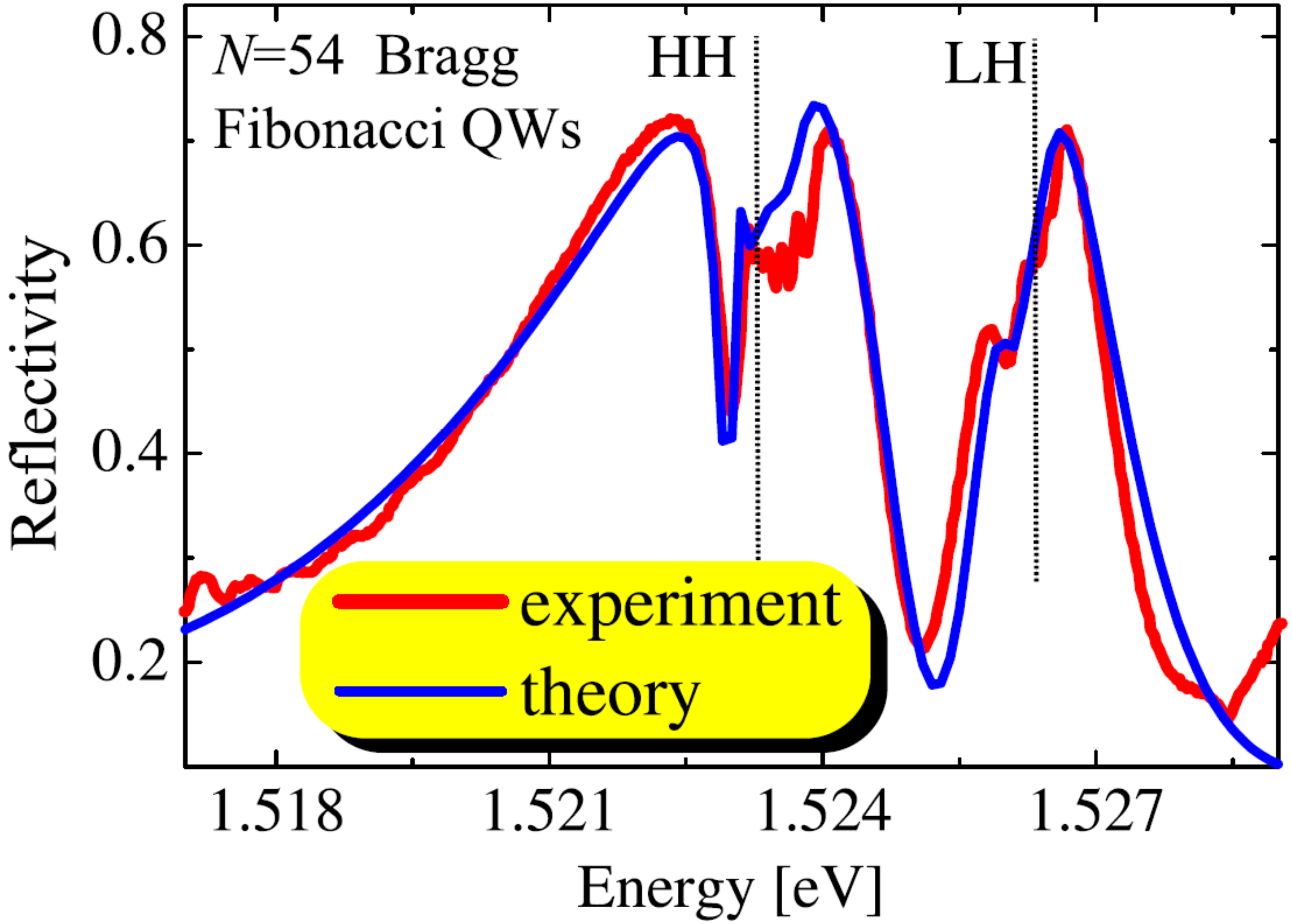
- Reflectivity grows when structure is tuned to Bragg condition
- Spectral HWHM is proportional to the number of QWs N
- Characteristic spectral dip is present around exciton resonance ω_0

Experiment, GaAs/AlGaAs QWs



Hendrickson, Richards, Sweet, Khitrova, Poddubny, Ivchenko, Wegener, Gibbs, Opt. Express 2008

Experiment and theory



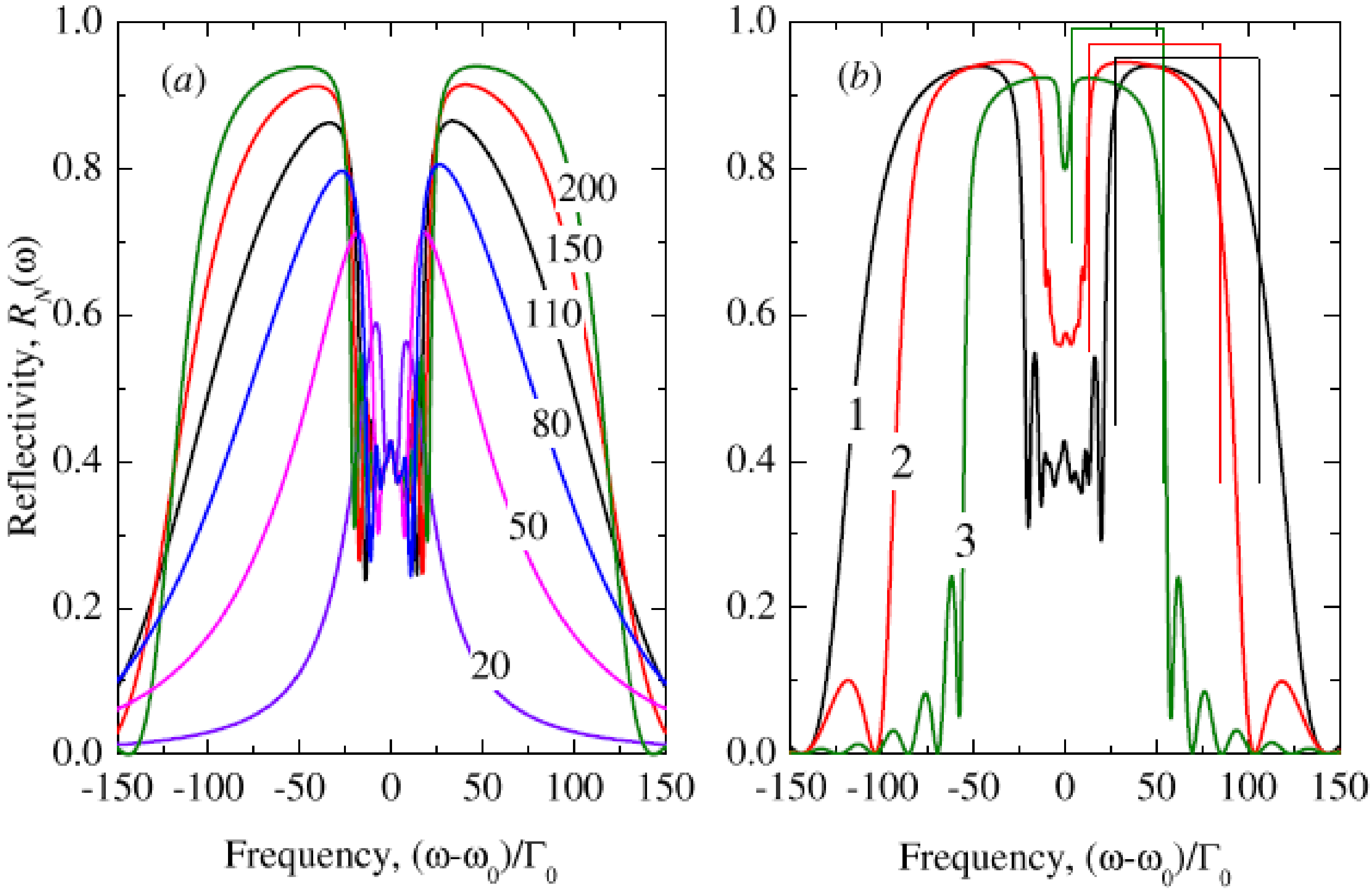
$\Gamma_0^{(HH)} \approx 25\ \mu$eV,$\Gamma_0^{(LH)} \approx 10\ \mu$eV, $\Gamma^{(HH)} \approx 180\ \mu$eV, $\Gamma^{(LH)} \approx 115\ \mu$eV

Werchner, Schafer, Kira, Koch, Sweet, Olitzky, Hendrickson, Richards, Khitrova, Gibbs, Poddubny, Ivchenko, Voronov, Wegener, Opt. Express (2009)

Calculated spectra

N=200

Poddubny, Pilozzi, Voronov, Ivchenko (2008)



(h,h')
(1,1)
(2,1)
(5,3)

Two-wave approximation

In a periodic system

$$E(z) = \mathrm{e}^{\mathrm{i}Kz} \sum_n \mathrm{e}^{\mathrm{i}b_n z} E_{b_n} \qquad \left(b_n = \frac{2\pi n}{d} \right)$$

$$\begin{aligned} \left(-\frac{d^2}{dz^2} - q^2 \right) E(z) &= \frac{2q\Gamma_0}{\omega_0 - \omega - \mathrm{i}\Gamma} \sum_{j=1}^{N} \delta(z - z_j) E(z) \\ &= \frac{2q\Gamma_0}{d(\omega_0 - \omega - \mathrm{i}\Gamma)} \sum_n \mathrm{e}^{\mathrm{i}b_n z} f_n^* E(z) \end{aligned}$$

$$E(z) = E_K \mathrm{e}^{\mathrm{i}Kz} + E_{K-b}\, \mathrm{e}^{\mathrm{i}(K-b)z}$$

Two-wave approximation

$$E(z) = E_K e^{iKz} + E_{K-b}\, e^{i(K-b)z}$$

$$(K^2 - q^2)E_K = \frac{2q\Gamma_0}{\omega_0 - \omega - i\Gamma}(E_K + f_1^* E_{K-b_1})$$

$$\left[|K - b_1|^2 - q^2)\right] E_{K-b_1} = \frac{2q\Gamma_0}{\omega_0 - \omega - i\Gamma}(f_1 E_K + E_{K-b_1})$$

$$\omega_{\rm out}^{\pm} = \omega_0 \pm \Delta\sqrt{\frac{1+|f_1|^2}{2}}$$

$$\omega_{\rm in}^{\pm} = \omega_0 \pm \Delta\sqrt{\frac{1-|f_1|^2}{2}}$$

Generalized Bloch-like functions in quasicrystals

$$\Delta \boldsymbol{E}(\boldsymbol{r}) - \text{grad div} \boldsymbol{E}(\boldsymbol{r}) = -\left(\frac{\omega}{c}\right)^2 \boldsymbol{D}(\boldsymbol{r})$$

$$\left(-\frac{d^2}{dz^2} - q^2\right) E(z) = \frac{2q\Gamma_0}{\omega_0 - \omega - \mathrm{i}\Gamma} \sum_{j=1}^{N} \delta(z - z_j) E(z)$$

$$= \frac{2q\Gamma_0}{\bar{d}(\omega_0 - \omega - \mathrm{i}\Gamma)} \sum_{hh'} \mathrm{e}^{\mathrm{i}G_{hh'}z} f^*_{hh'} E(z)$$

$$E_K(z) = \mathrm{e}^{\mathrm{i}Kz} \sum_{hh'} \mathrm{e}^{\mathrm{i}G_{hh'}z} E_{G_{hh'}}$$

In a periodic system

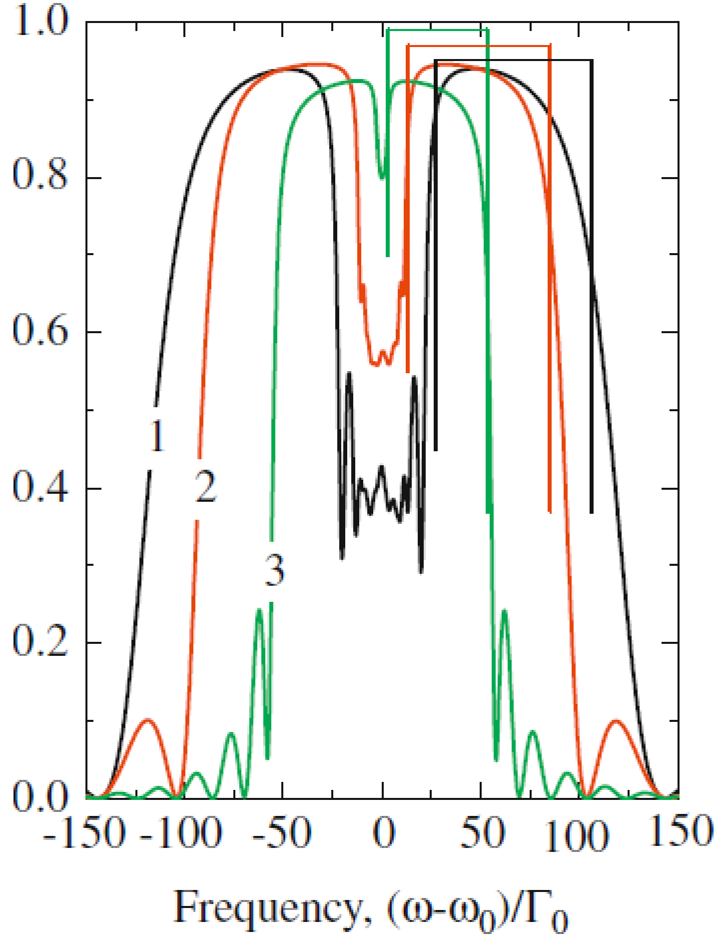
$$E(z) = \mathrm{e}^{\mathrm{i}Kz} \sum_{h} \mathrm{e}^{\mathrm{i}G_h z} E_{G_h} \qquad \left(G_h = \frac{2\pi}{d} h\right)$$

Two-wave approximation

$$E(z) = E_K e^{iKz} + E_{K-G}\, e^{i(K-G)z}$$

$$\omega_{out}^{\pm} = \omega_0 \pm \Delta \sqrt{\frac{1 + |f_{hh'}|}{2\,(h + h'/\tau)}}$$

$$\omega_{in}^{\pm} = \omega_0 \pm \Delta \sqrt{\frac{1 - |f_{hh'}|}{2\,(h + h'/\tau)}}$$



- Introduction. Resonant Bragg QWs
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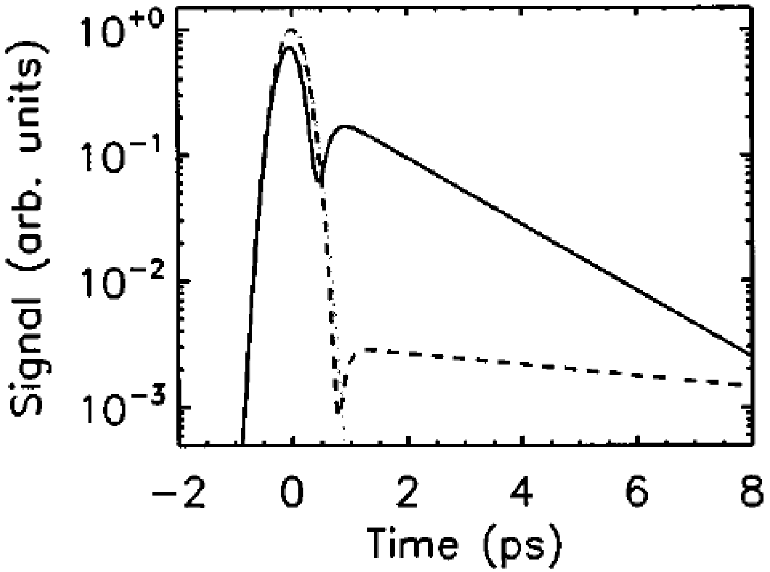
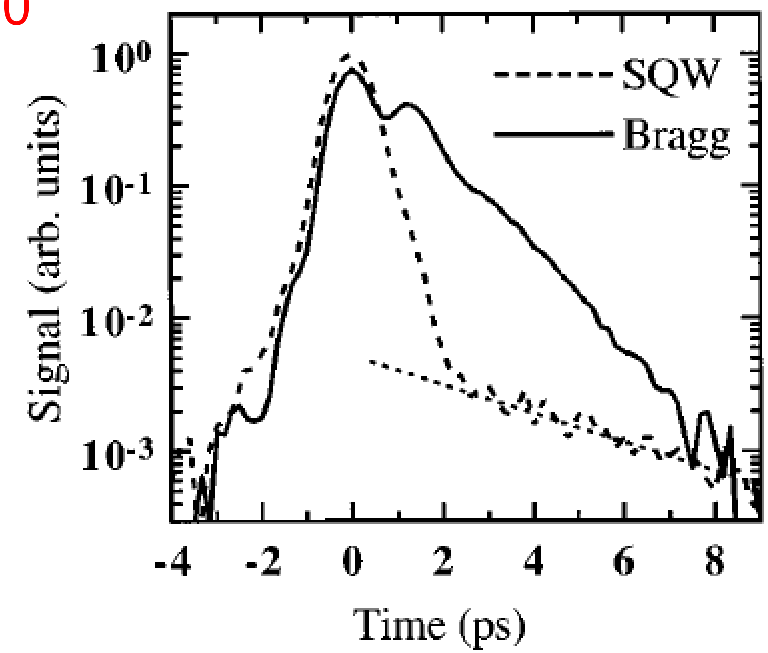
Time-resolved reflectivity from resonant Bragg QW structure

Theory



N = 10

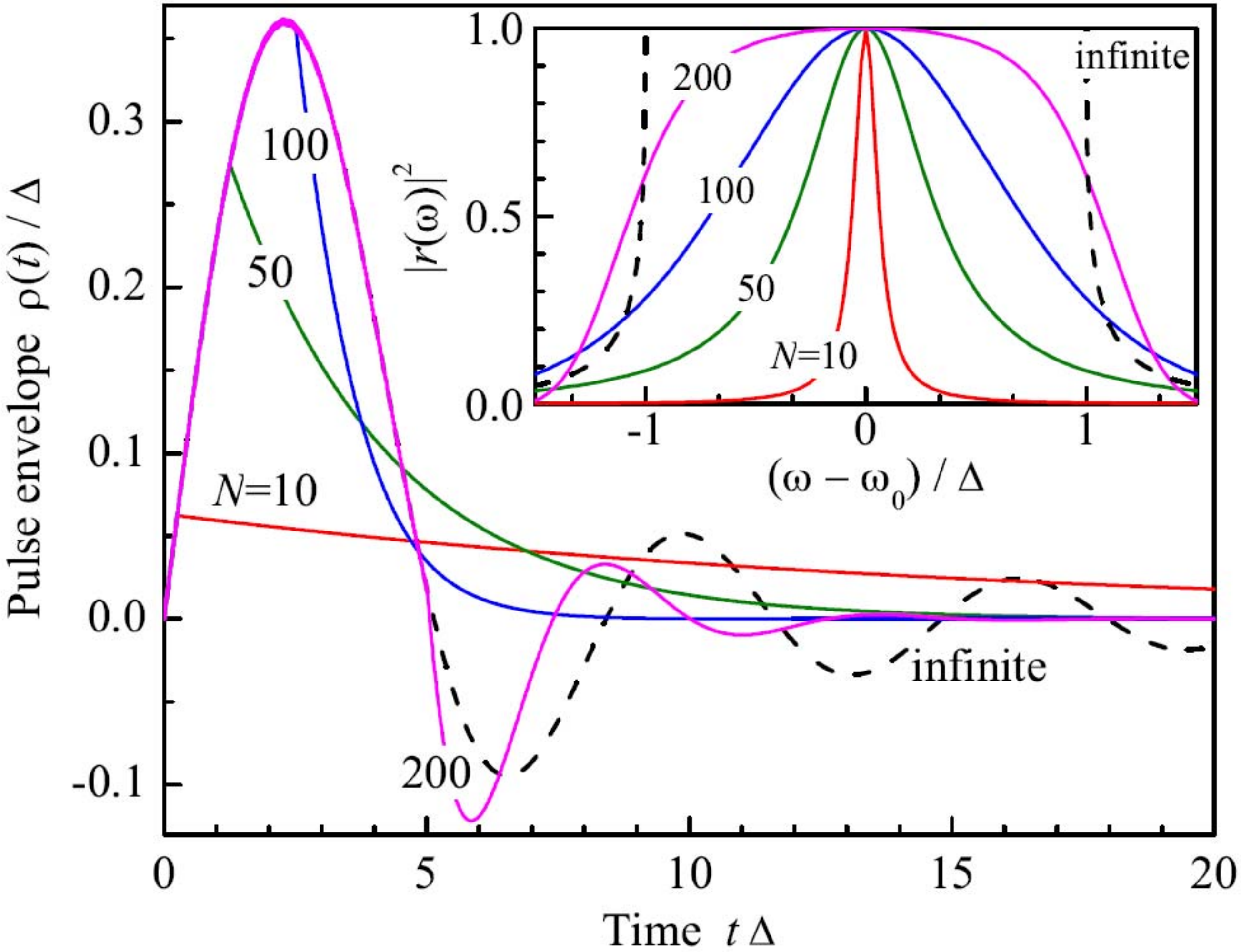
Experiment



Comparison of the time-resolved reflected pulses from a the Bragg structure (solid line), and from a single quantum well (dashed line), at low excitation on the semilogarithmic scale.

Haas, Stroucken, Hübner, Kuhl, Grote, Knorr, Jahnke, Koch, Hey, Ploog, Phys. Rev. 1998

Time-resolved reflectivity from resonant Bragg QW structure



Superradiant regime (small QW number N)

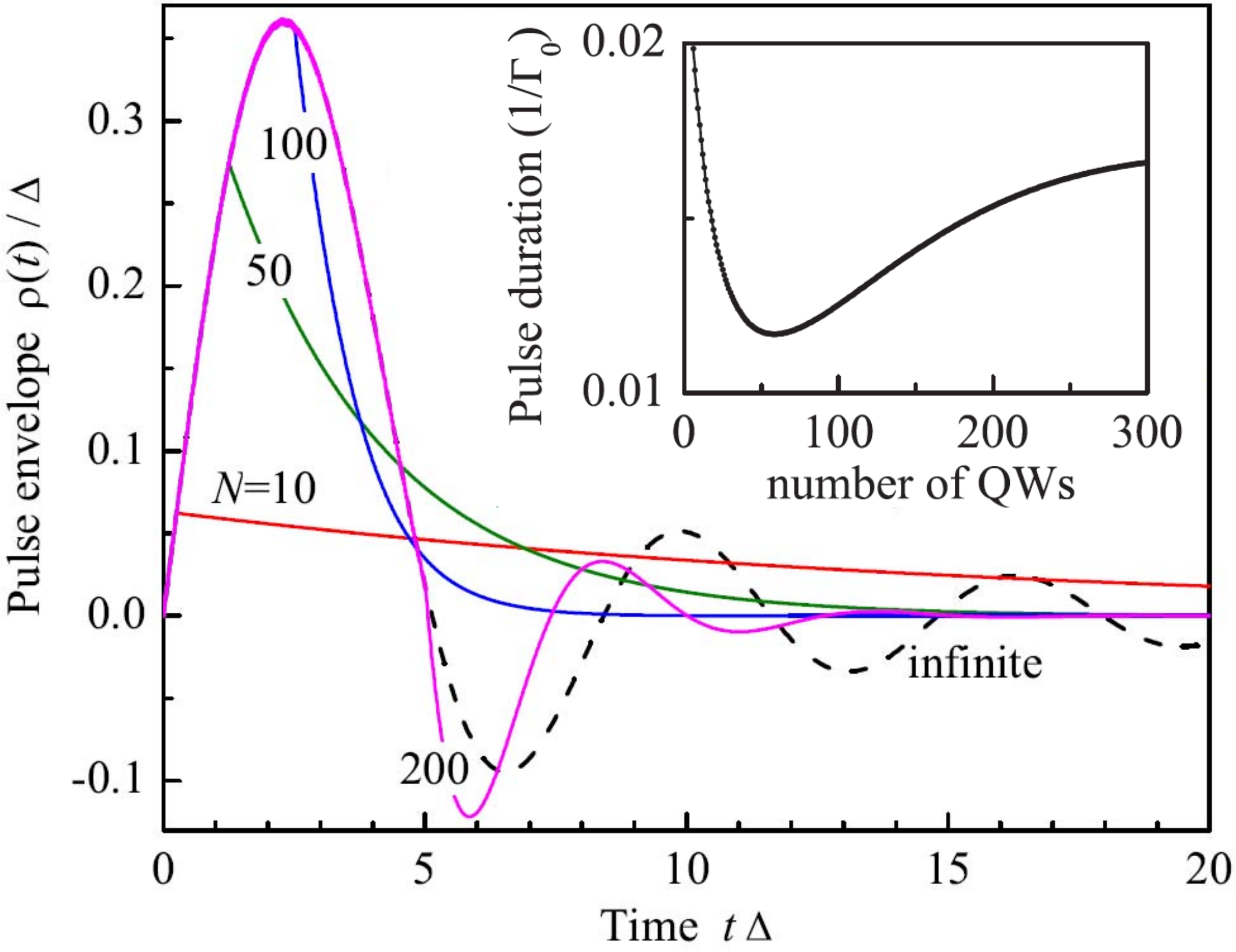
$$\rho_N(t) = N\Gamma_0 e^{-N\Gamma_0 t}$$

Semiinfinite structure

$$\rho_\infty(t) = \frac{2}{t} J_2(t\Delta)$$

Poshakinskiy, Poddubny, Tarasenko (2012)

Time-resolved reflectivity from resonant Bragg QW structure



Superradiant regime (small QW number *N*)

$$\rho_N(t) = N\Gamma_0 e^{-N\Gamma_0 t}$$

Semiinfinite structure

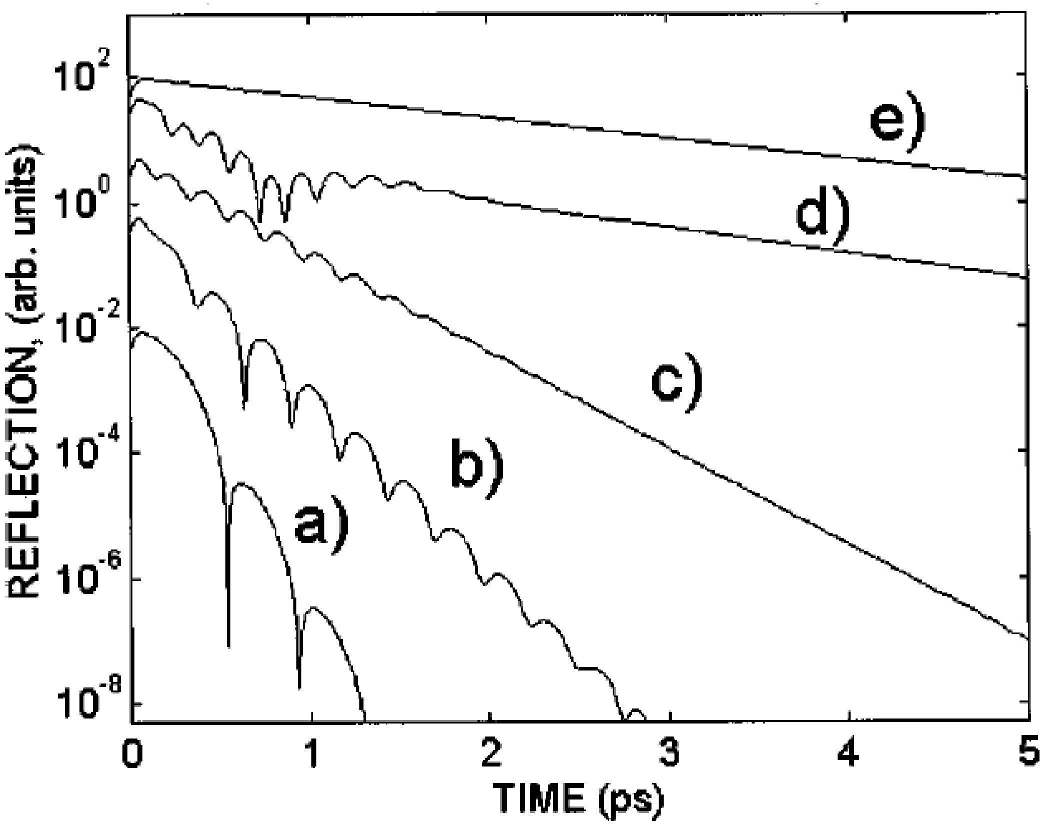
$$\rho_\infty(t) = \frac{2}{t} J_2(t\Delta)$$

Reflected pulse duration

$$\frac{\int |\rho(t)|^2 \, t \, dt}{\int |\rho(t)|^2 \, dt}$$

Poshakinskiy, Poddubny, Tarasenko (2012)

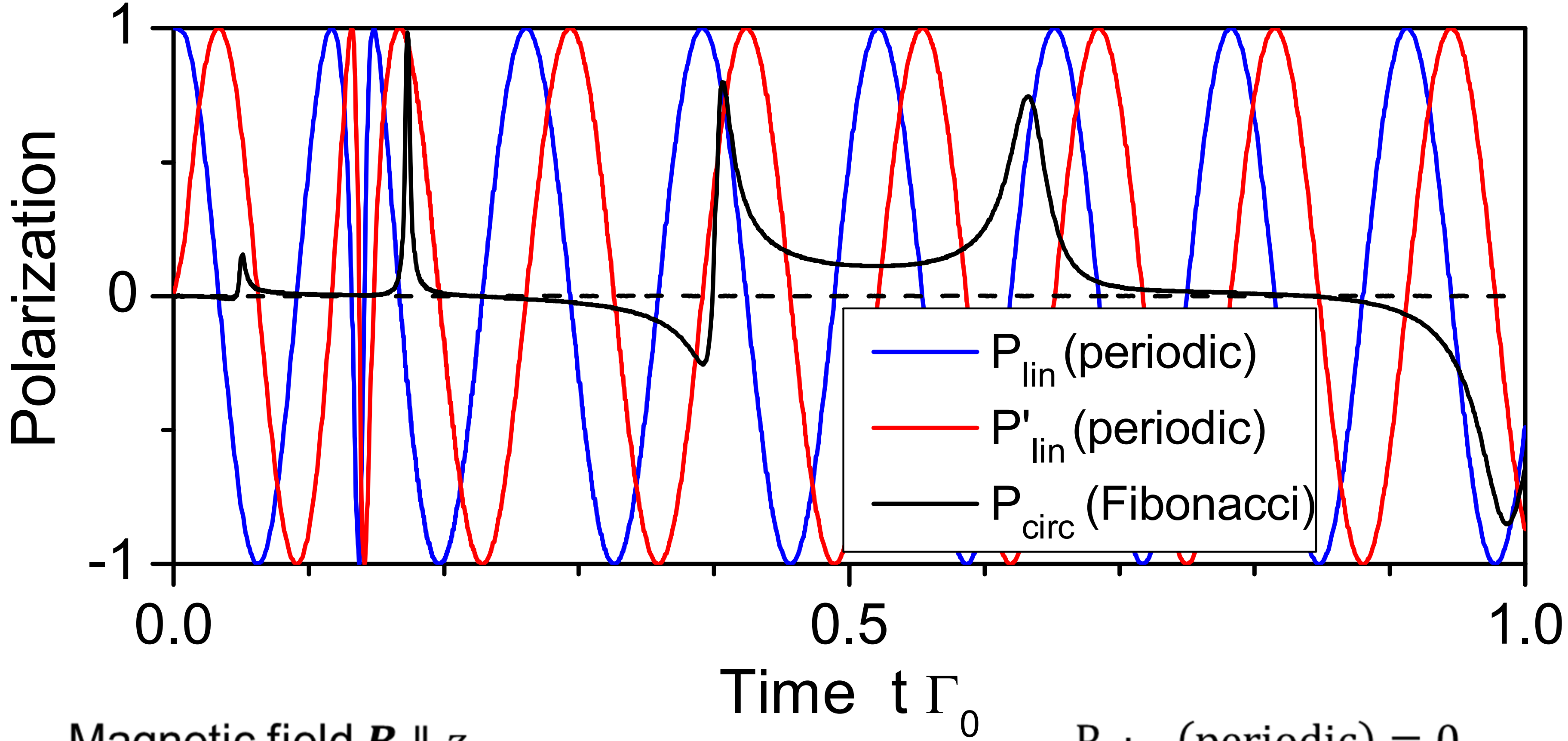
Effect of inhomogeneous broadening



Malpuech and Kavokin,
Appl. Phys. Lett. 2000

Time-resolved reflection of GaN/AlGaN quantum well structures: (a) SQW with D = 5 meV; (b) MQW, N =10, D = 5 meV; (c) MQW, N=50, D = 5 meV; (d) MQW, N=100, D = 5 meV; (e) SQW with D = 0.

Time-resolved reflectivity from resonant Bragg QW structure



Magnetic field $\boldsymbol{B} \parallel z$

Zeeman splitting $\omega_+ - \omega_- = 50\Gamma_0$

$P_{\text{circ}}(\text{periodic}) = 0$

$P_{\text{circ}}(\text{Fibonacci}) \neq 0$

Poshakinskiy, Poddubny, Tarasenko (2012)