Half-Solitons as Magnetic Charges
Towards polariton magnetricity

Theory:
Institut Pascal : H. Flayac, D. D. Solnyshkov and G. Malpuech

Experiment:
L.K.B. : R. Hivet, T. Boulier, D. Andreoli, E. Giacobino and A. Bramati
L.P.N. : D. Tanese, J. Bloch and A. Amo

Outline

- Solitons in scalar BECs
  - Solitons (1D)
  - Oblique solitons (2D)
- 2 component spinor BEC
  - Pseudospin dynamics
  - Half-solitons
  - Magnetic charges
- Towards magnetricity
  - Polariton condensate
  - Half-solitons imprinting
  - Experimental evidence
- Summary

Outline

- Solitons in scalar BECs
  - Solitons (1D)
  - Oblique solitons (2D)
- 2 component spinor BEC
  - Pseudospin dynamics
  - Half-solitons
  - Magnetic charges
- Towards magnetricity
  - Polariton condensate
  - Half-solitons imprinting
  - Experimental evidence
- Summary

Bose-Einstein Condensate (BEC)

Complex wavefunction

$$\psi\left(\vec{r},t\right)=\sqrt{n\left(\vec{r},t\right)}e^{i\theta\left(\vec{r},t\right)}$$

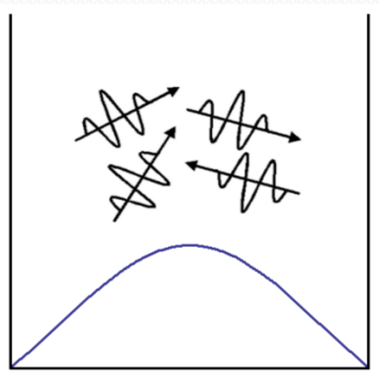
Ψ : macroscopic wavefunction
n : density of the Bose Gas
θ : phase of the wavefunction

Quantum Fluid

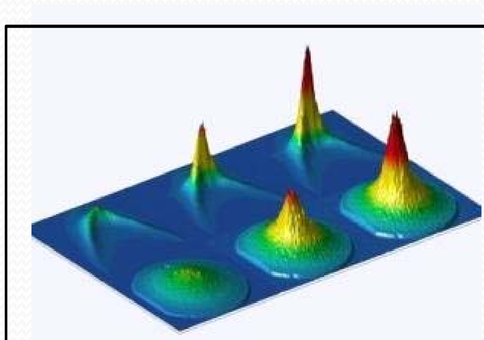
Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+\alpha\left|\psi\right|^2\psi$$

m : mass of the particles
α : interaction constant



Macroscopic coherent state



Gray Solitons in 1D

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + \alpha |\psi|^2 \psi$$

$$\alpha > 0$$

Gray Solitons in 1D

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \alpha|\psi|^2\psi$$

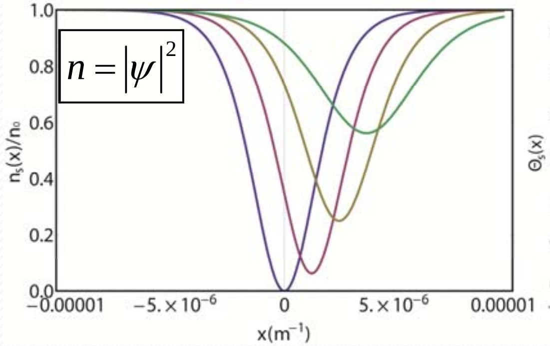
$$\alpha > 0$$

$$\psi(x) = \sqrt{n_0}\left[\sqrt{1-\frac{v^2}{c^2}}\tanh\left(\frac{x-vt}{\xi}\sqrt{1-\frac{v^2}{c^2}}\right) + i\frac{v}{c}\right]$$

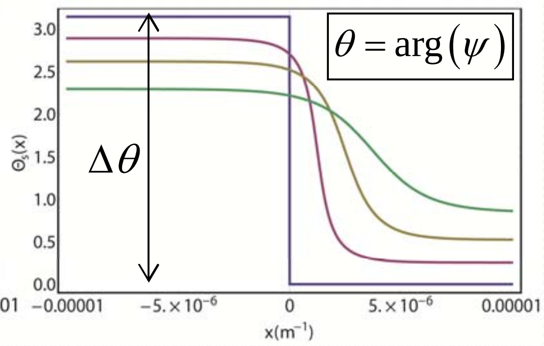
- v : soliton velocity/flow
- c : speed of sound
- ξ : healing length
- n_0 : density at infinity
- $\Delta\theta = 2\arccos(v/c) \in \{0, \pi\}$ (phase shift)

$$c = \sqrt{\frac{\alpha n}{m}}$$

$$\xi = \frac{\hbar}{\sqrt{m\alpha n}}$$



Density



Phase

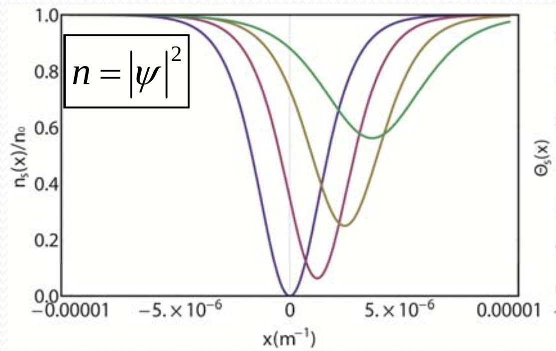
Gray Solitons in 1D

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\alpha|\psi|^2\psi$$ $\alpha>0$

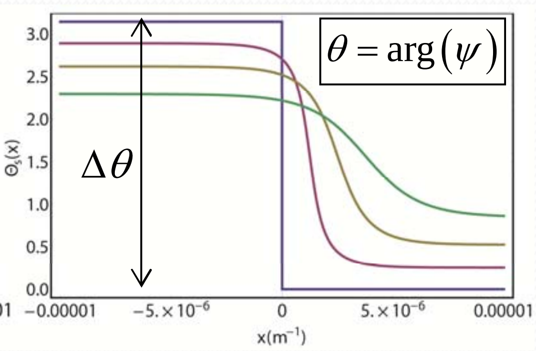
$$\psi(x)=\sqrt{n_0}\left[\sqrt{1-\frac{v^2}{c^2}}\tanh\left(\frac{x-vt}{\xi}\sqrt{1-\frac{v^2}{c^2}}\right)+i\frac{v}{c}\right]$$

- v : soliton velocity/flow
- c : speed of sound
- ξ : healing length
- n_0 : density at infinity
- $\Delta\theta = 2\arccos(v/c) \in \{0,\pi\}$ (phase shift)

$$c=\sqrt{\frac{\alpha n}{m}} \qquad \xi=\frac{\hbar}{\sqrt{m\alpha n}}$$



Density

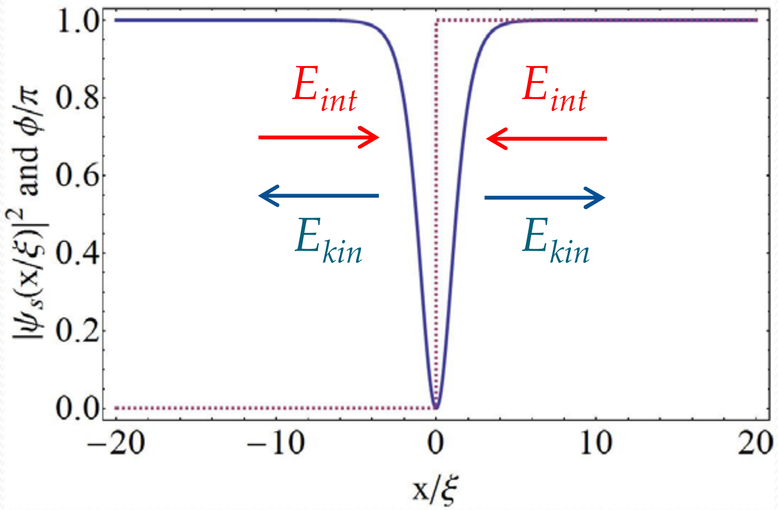


Phase

$0<v<c$ $0<\Delta\theta<\pi$

Dark Soliton: $v=0,\ \Delta\theta=\pi$

$$\psi_{DS}(x/\xi)=\sqrt{n_0}\tanh(x) =\sqrt{n_0}\,|\tanh(x)|\,e^{i\pi H(x)}$$



Solitons as Relativistic Particles

$$\psi(x) = \sqrt{n_0}\left[\sqrt{1-\frac{v^2}{c^2}}\tanh\left(\sqrt{1-\frac{v^2}{c^2}}\,\frac{x-vt}{\xi}\right) + i\frac{v}{c}\right]$$

Can be rewritten as:

$$\gamma = 1/\sqrt{1-v^2/c^2}$$

$$\psi(x) = \sqrt{n_0}\left[\gamma^{-1}\tanh\left(\frac{x-vt}{\gamma\xi}\right) + i\frac{v}{c}\right]$$

Solitons as Relativistic Particles

$$\psi(x)=\sqrt{n_0}\left[\sqrt{1-\frac{v^2}{c^2}}\tanh\left(\sqrt{1-\frac{v^2}{c^2}}\,\frac{x-vt}{\xi}\right)+i\frac{v}{c}\right]$$

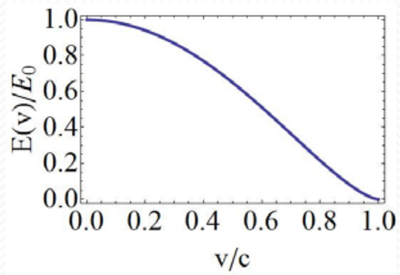
Can be rewritten as:

$$\gamma=1/\sqrt{1-v^2/c^2}$$

$$\psi(x)=\sqrt{n_0}\left[\gamma^{-1}\tanh\left(\frac{x-vt}{\gamma\xi}\right)+i\frac{v}{c}\right]$$

Dispersion relation:

$$E_s(v)=\frac{4}{3}\hbar c n_0\left(1-\frac{v^2}{c^2}\right)^{3/2}$$



Mass (v=0):

$$m_s^0=-\frac{4\hbar n_0}{c}<0$$

Mass (v<<c):

$$m_s=\frac{m_s^0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Solitons as Relativistic Particles

$$\psi(x) = \sqrt{n_0}\left[\sqrt{1-\frac{v^2}{c^2}}\tanh\left(\sqrt{1-\frac{v^2}{c^2}}\,\frac{x-vt}{\xi}\right) + i\frac{v}{c}\right]$$

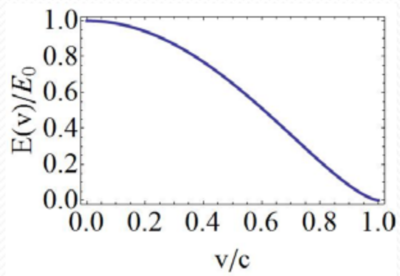
Can be rewritten as:

$$\gamma = 1/\sqrt{1-v^2/c^2}$$

$$\psi(x) = \sqrt{n_0}\left[\gamma^{-1}\tanh\left(\frac{x-vt}{\gamma\xi}\right) + i\frac{v}{c}\right]$$

Dispersion relation:

$$E_s(v) = \frac{4}{3}\hbar c n_0\left(1-\frac{v^2}{c^2}\right)^{3/2}$$



Mass (v=0):

$$m_s^0 = -\frac{4\hbar n_0}{c} < 0$$

Mass (v<<c):

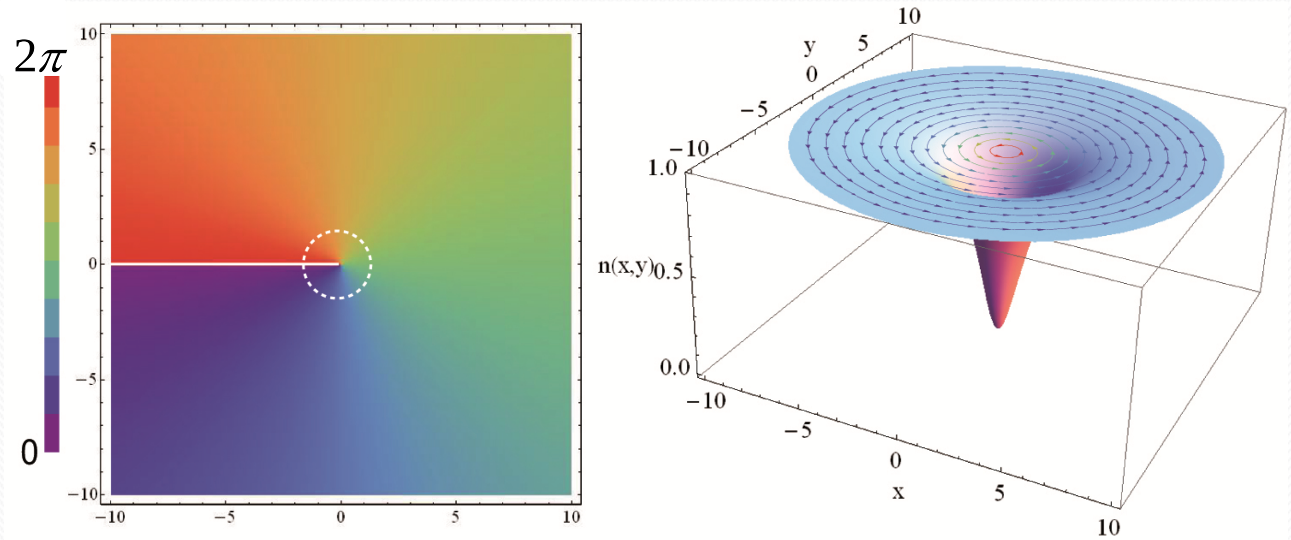
$$m_s = \frac{m_s^0}{\sqrt{1-\frac{v^2}{c^2}}}$$

The **size** and the **mass** are velocity dependent

$$l_s = \gamma\xi$$

$$m_s = \gamma m_s^0$$

Vortices in 2D



$$\int \vec{\nabla}\theta \cdot d\vec{s} = 2\pi l \Rightarrow \oint \vec{v} \cdot d\vec{s} = l\,\frac{h}{m}$$

- l : integer winding number
- v : is BEC velocity field

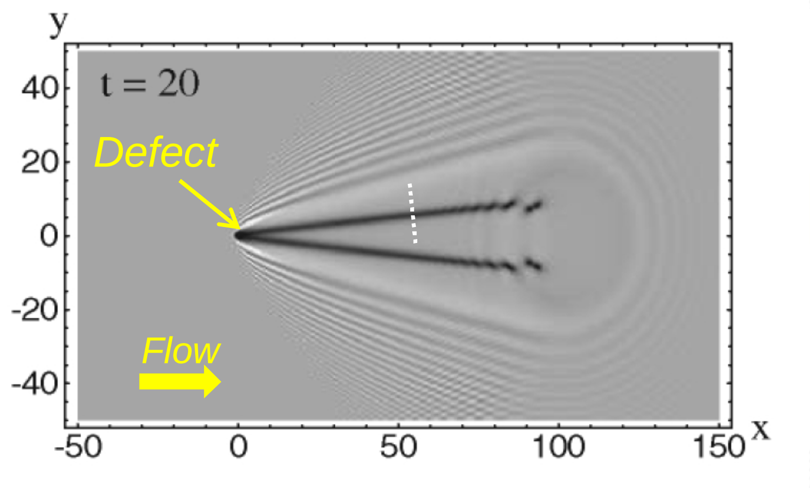
$$\psi_V(r,\phi) = f_l(r)e^{il\phi}$$

$$f_{\pm 1}(r) = \frac{r}{\sqrt{r^2+2}}$$

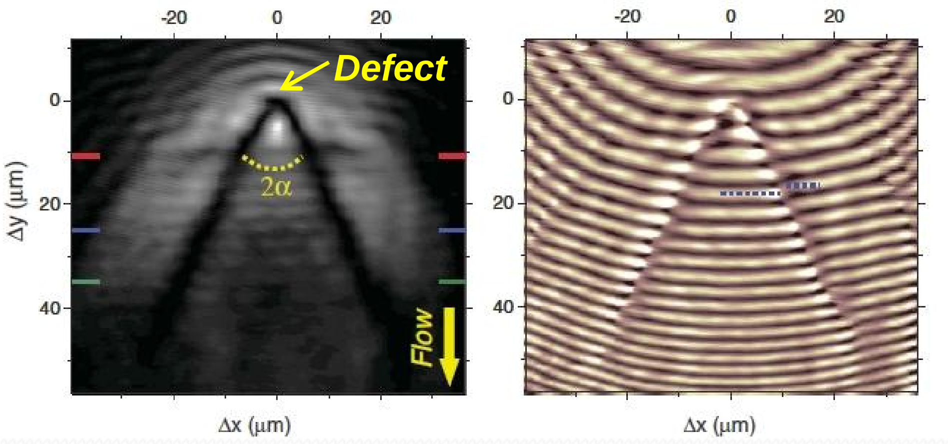
$$\theta = l\phi \Rightarrow \vec{v} = \frac{l\hbar}{mr}\vec{u}_\phi$$

Oblique dark solitons in 2D

- BEC flowing at supersonic velocity against an obstacle
- Pair of oblique solitons in the wake of the obstacle
- 1D objects replicated in space: $x=t$



Theory: El and Kamchatnov, *PRL* 2006



Experiment: Amo et al. *Science* 2011
(polaritons)

Outline

- Integer topological defects in scalar BECs
  - Solitons
  - Vortices
  - Oblique solitons
- Half-integer topological defects in 2 component BECs
  - Half-solitons
  - Half-vortices and oblique half-solitons
- Towards magnetricity in semiconductor microcavities
  - Generation of half-soliton currents
  - Half-vortices injection and propagation

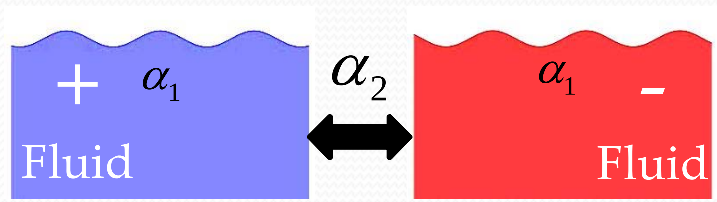
Outline

- Solitons in scalar BECs
  - Solitons (1D)
  - Oblique solitons (2D)
- 2 component spinor BEC
  - Pseudospin dynamics
  - Half-solitons
  - Magnetic charges
- Towards magnetricity
  - Polariton condensate
  - Half-solitons imprinting
  - Experimental evidence
- Summary

Spinor BEC

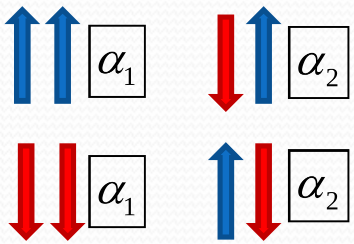
2 component spinor BEC:

$$\vec{\psi} = \begin{pmatrix} \psi_+(\vec{r},t) \\ \psi_-(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \sqrt{n_+} e^{i\theta_+} \\ \sqrt{n_-} e^{i\theta_-} \end{pmatrix}$$



$$i\hbar \frac{\partial \psi_+}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_+ + \alpha_1 |\psi_+|^2 \psi_+ + \alpha_2 |\psi_-|^2 \psi_+$$

$$i\hbar \frac{\partial \psi_-}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_- + \alpha_1 |\psi_-|^2 \psi_- + \alpha_2 |\psi_+|^2 \psi_-$$



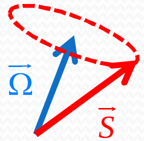
- Vectorial wavefunction
- Intercomponent interaction via α_2
- Richer physics: spin/phase excitations
- Half-integer topological defects: defect in only one component ($\alpha_2 \approx 0$)

Pseudospin representation

- Pseudospin=relevant representation for 2 level spin systems
- 3D Vector on the Poincaré sphere: $\vec{S} = (S_x, S_y, S_z)^T$
- Map to a **magnetic system** and completely defines the polarization states
  - Equator: linear polarization
  - Poles: circular polarization

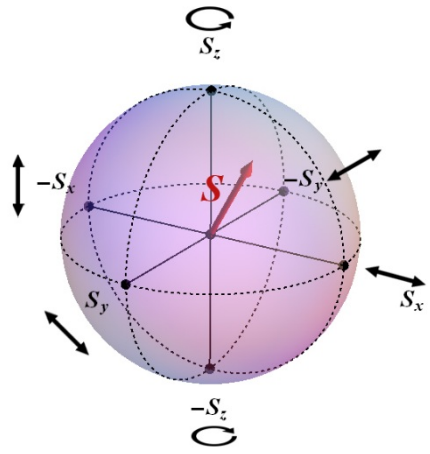
Precession equation:

$$\frac{d\vec{S}}{dt} = \left[\vec{\Omega} \times \vec{S}\right]$$



$\vec{\Omega}$ is a(n) (effective) magnetic field

$$\vec{S} = \begin{cases} S_x = \Re\left(\psi_+ \psi_-^*\right) \\ S_y = \Im\left(\psi_+^* \psi_-\right) \\ S_z = \left(n_+ - n_-\right)/2 \end{cases}$$



Poincaré sphere

$\vec{\Omega}$ in the sGP equations

$$\vec{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)^T$$

$$\vec{\Omega} = \left(\Omega_x, \Omega_y, \Omega_z\right)^T$$

$$\vec{\psi} = \left(\psi_+, \psi_-\right)^T$$

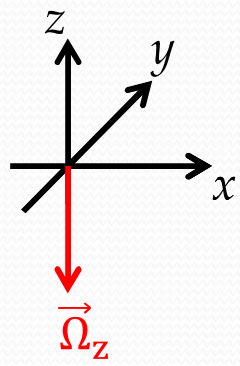
$$i\hbar \frac{\partial \vec{\psi}}{\partial t} = \ldots - \left(\vec{\Omega} \cdot \vec{\sigma}\right)\vec{\psi}$$

Effective magnetic fields

Intrinsic nonlinear effective field – sGP equations can be rewritten as:

$$i\hbar\frac{\partial\psi_+}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_+ + \frac{\alpha_1+\alpha_2}{2}\left(|\psi_+|^2+|\psi_-|^2\right)\psi_+ + \frac{\alpha_1-\alpha_2}{2}\overbrace{\left(|\psi_+|^2-|\psi_-|^2\right)}^{S_z}\psi_+$$

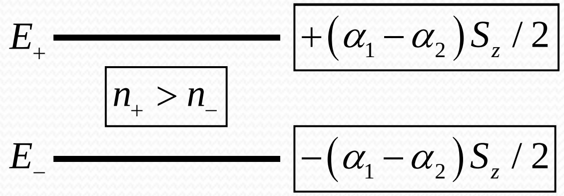
$$i\hbar\frac{\partial\psi_-}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_- + \frac{\alpha_1+\alpha_2}{2}\left(|\psi_+|^2+|\psi_-|^2\right)\psi_- - \frac{\alpha_1-\alpha_2}{2}\underbrace{\left(|\psi_+|^2-|\psi_-|^2\right)}_{S_z}\psi_-$$



$$(\alpha_1-\alpha_2)S_z\underbrace{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}_{\sigma_z}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \Rightarrow \text{ZEEMAN Splitting}$$

Effective field:

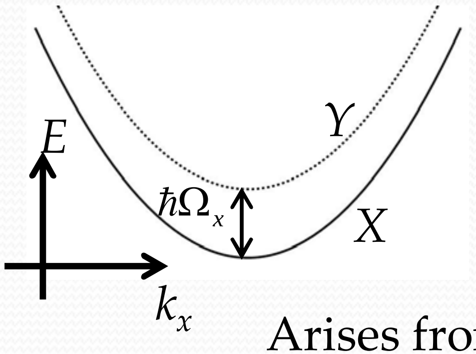
$$\vec{\Omega}_z = -(\alpha_1-\alpha_2)S_z/\hbar\vec{u}_z$$



- $\Omega_z=0$ for $\alpha_1=\alpha_2$ (usual atomic condensates)
- The field becomes stronger as $(\alpha_1-\alpha_2)$ increases (spin anisotropy)

Effective magnetic fields

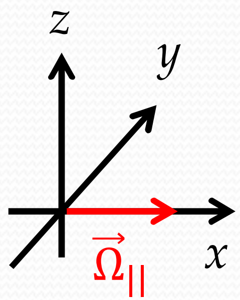
Extrinsic in-plane effective fields: $\vec{\Omega}_{\|} = \Omega_x \vec{u}_x$



$$i\hbar \frac{\partial \psi_+}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_+ + \alpha_1 |\psi_+|^2 \psi_+ + \alpha_2 |\psi_-|^2 \psi_+ - \frac{\hbar\Omega_x}{2}\psi_-$$

$$i\hbar \frac{\partial \psi_-}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_- + \alpha_1 |\psi_-|^2 \psi_- + \alpha_2 |\psi_+|^2 \psi_- - \frac{\hbar\Omega_x}{2}\psi_+$$

Josephson-like coupling

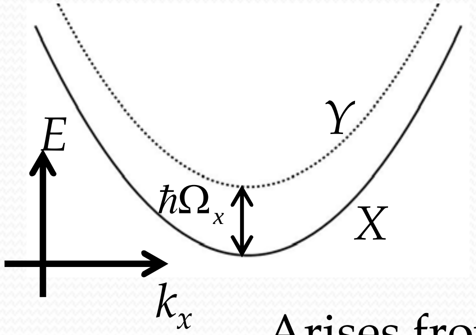


Arises from energy splittings between linearly polarized modes

$$\psi_\pm = \left(\psi_x \pm i\psi_y\right)\sqrt{2}$$

Effective magnetic fields

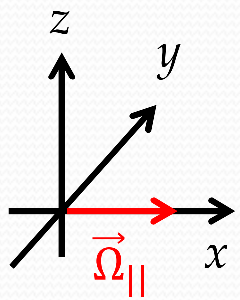
Extrinsic in-plane effective fields: $\vec{\Omega}_{\parallel} = \Omega_x \vec{u}_x$



$$i\hbar \frac{\partial \psi_+}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_+ + \alpha_1 |\psi_+|^2 \psi_+ + \alpha_2 |\psi_-|^2 \psi_+ - \frac{\hbar\Omega_x}{2}\psi_-$$

$$i\hbar \frac{\partial \psi_-}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_- + \alpha_1 |\psi_-|^2 \psi_- + \alpha_2 |\psi_+|^2 \psi_- - \frac{\hbar\Omega_x}{2}\psi_+$$

Josephson-like coupling



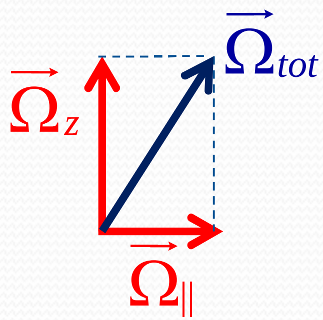
Arises from energy splittings between linearly polarized modes

$$\psi_\pm = \left(\psi_x \pm i\psi_y\right)\sqrt{2}$$

Total effective field: nonlinear dynamics

$$\vec{\Omega}_{tot} = \Omega_x \vec{u}_x + \left(\alpha_1 - \alpha_2\right) S_z / \hbar \vec{u}_z$$

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$



Half-Solitons

- $\boldsymbol{\alpha_2} \ll \boldsymbol{\alpha_1}$: Linearly polarized condensate
- Soliton in **only one** component is a solution of sGP equations
- HALF-SOLITON = Elementary topological excitation

Half-Solitons

- $\boldsymbol{\alpha_2} \ll \boldsymbol{\alpha_1}$: Linearly polarized condensate
- Soliton in **only one** component is a solution of sGP equations
- HALF-SOLITON = Elementary topological excitation

2 representations for the vectorial wave function:

Circular polarization basis (σ^+, σ^-)

$$(\psi_+, \psi_-) = \sqrt{n_0}/2\left(e^{i\theta_+}, e^{i\theta_-}\right)$$

$$(\psi_+^{HS}, \psi_-^{HS}) = \sqrt{n_0/2}\,(\tanh(x), 1)$$

$\boldsymbol{\pi}$ shift of θ_+

Dark half-soliton

Half-Solitons

- $\boldsymbol{\alpha_2} \ll \boldsymbol{\alpha_1}$: Linearly polarized condensate
- Soliton in **only one** component is a solution of sGP equations
- HALF-SOLITON = Elementary topological excitation

2 representations for the vectorial wave function:

Circular polarization basis (σ⁺, σ⁻)

$$(\psi_+, \psi_-) = \sqrt{n_0}/2\left(e^{i\theta_+}, e^{i\theta_-}\right)$$

$$(\psi_+^{HS}, \psi_-^{HS}) = \sqrt{n_0/2}\,(\tanh(x), 1)$$

$\boldsymbol{\pi}$ shift of θ_+

$$\psi_\pm = (\psi_x \mp \psi_y)/\sqrt{2}$$

$$\theta_\pm = \Theta \mp \eta$$

Linear polarization basis (x, y)

$$(\psi_x, \psi_y) = \sqrt{n_0}\left(e^{i\Theta}\cos\eta, e^{i\Theta}\sin\eta\right)$$

$$\left(\psi_x^{HS}, \psi_y^{HS}\right) = \sqrt{n_0}/2\left(1 + \tanh(x), i - i\tanh(x)\right)$$

$\boldsymbol{\pi/2}$ shift of Θ and η

Dark half-soliton

Domain wall

Half-Solitons

- $\boldsymbol{\alpha_2} \ll \boldsymbol{\alpha_1}$: Linearly polarized condensate
- Soliton in **only one** component is a solution of sGP equations
- HALF-SOLITON = Elementary topological excitation

2 representations for the vectorial wave function:

Circular polarization basis (σ⁺, σ⁻)

$$(\psi_+, \psi_-) = \sqrt{n_0}/2\,(e^{i\theta_+}, e^{i\theta_-})$$

$$(\psi_+^{HS}, \psi_-^{HS}) = \sqrt{n_0/2}\,(\tanh(x), 1)$$

$\boldsymbol{\pi}$ shift of θ_+

$$\psi_\pm = (\psi_x \mp \psi_y)/\sqrt{2}$$

$$\theta_\pm = \Theta \mp \eta$$

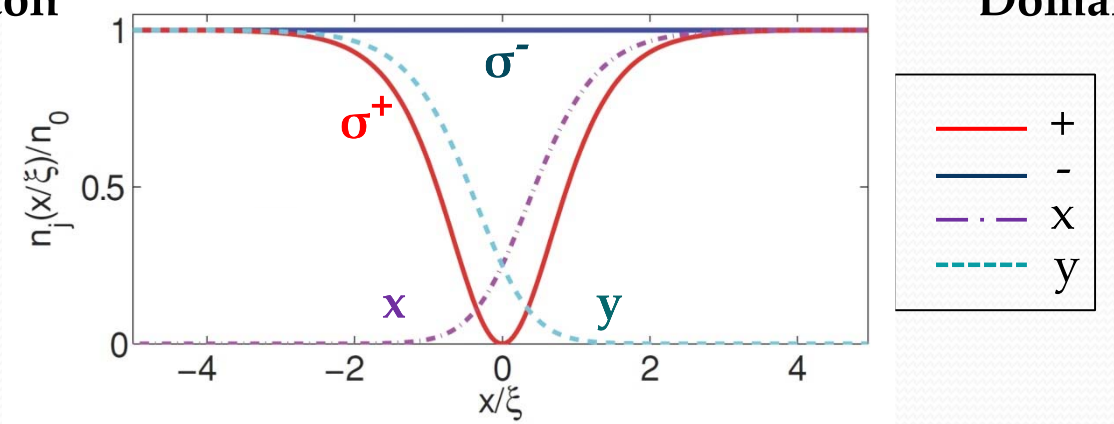
Linear polarization basis (x, y)

$$(\psi_x, \psi_y) = \sqrt{n_0}(e^{i\Theta}\cos\eta\,, e^{i\Theta}\sin\eta)$$

$$(\psi_x^{HS}, \psi_y^{HS}) = \sqrt{n_0}/2(1 + \tanh(x), i - i\tanh(x))$$

$\boldsymbol{\pi/2}$ shift of Θ and η

Dark half-soliton **Domain wall**



Half-soliton pseudospin texture

Dark half-soliton

$$\begin{cases} \psi_+(x) = \sqrt{n_0/2}\tanh(x) \\ \psi_-(x) = \sqrt{n_0/2} \end{cases}$$

→

Pseudospin

$$\vec{S} = \begin{cases} S_x = \Re\left(\psi_+\psi_-^*\right) \\ S_y = \Im\left(\psi_-\psi_+^*\right) \\ S_z = \left(n_+ - n_-\right)/2 \end{cases}$$

→

Pseudospin

$$\begin{cases} S_x = n_0/2\tanh(x) \\ S_y = 0 \\ S_z = n_0\left(\tanh(x)^2 - 1\right)/2 \end{cases}$$

Half-soliton pseudospin texture

Dark half-soliton

$$\begin{cases} \psi_+(x) = \sqrt{n_0/2}\tanh(x) \\ \psi_-(x) = \sqrt{n_0/2} \end{cases}$$

Pseudospin

$$\vec{S} = \begin{cases} S_x = \Re(\psi_+\psi_-^*) \\ S_y = \Im(\psi_-\psi_+^*) \\ S_z = (n_+ - n_-)/2 \end{cases}$$

Pseudospin

$$\begin{cases} S_x = n_0/2\tanh(x) \\ S_y = 0 \\ S_z = n_0\left(\tanh(x)^2 - 1\right)/2 \end{cases}$$

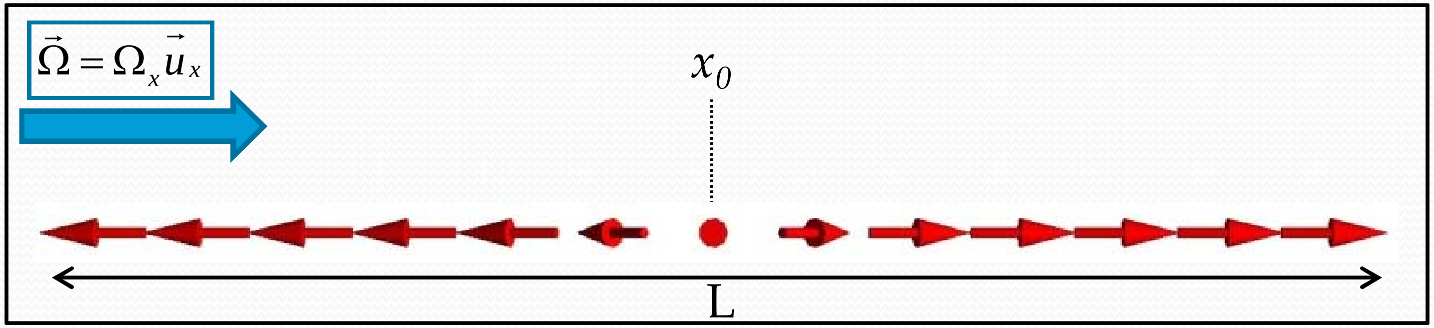
$\vec{S}$

Divergent pseudospin texture:
Field of a point charge

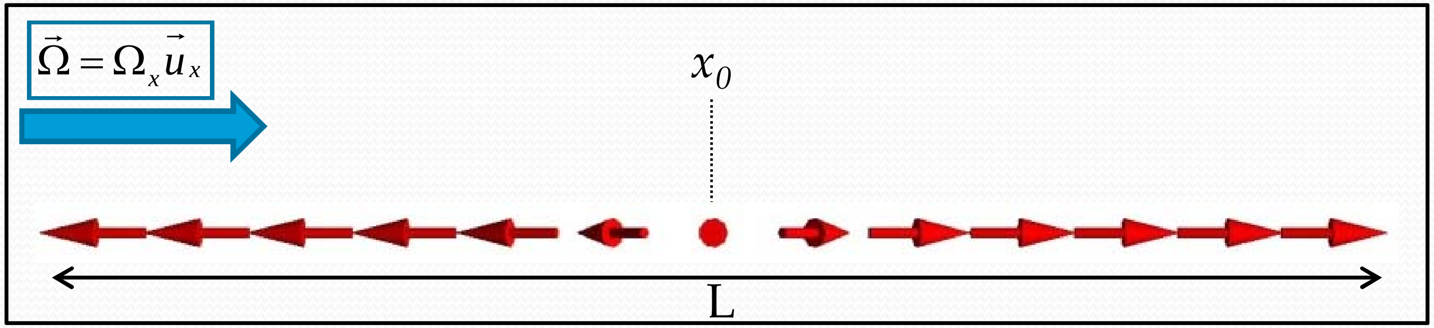
$$\mathrm{div}(\vec{S}) \neq 0$$

Half- soliton = Magnetic charge ?

Half-Soliton acceleration



Half-Soliton acceleration



Magnetic Energy:

$$E_{mag} = -\frac{\hbar}{2}\int \vec{\Omega}\cdot\vec{S}dx$$

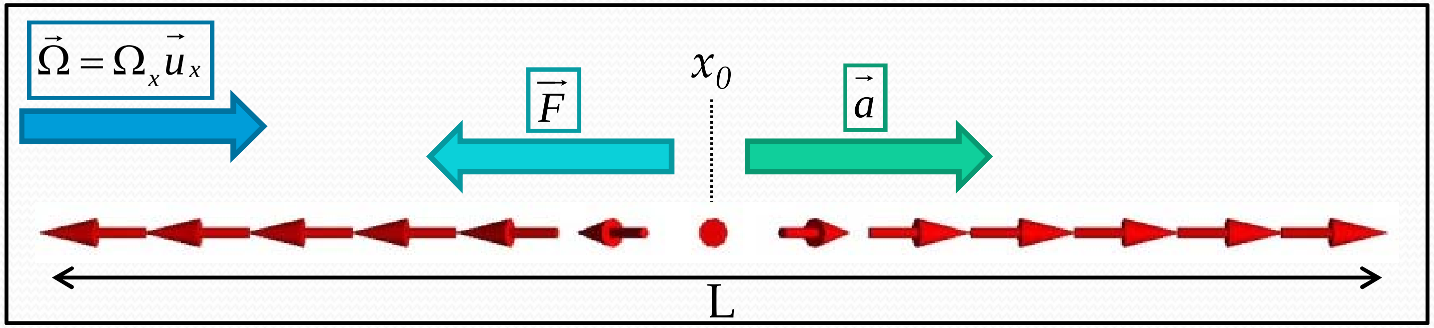
The motion of the soliton is governed by the minimization of E_{mag}

In the limit $L >> \xi$:

$$E_{mag}(x_0) = -n\frac{\hbar\Omega_x}{2}\int \text{sign}(x - x_0)\,dx$$

$$\boxed{E_{mag}(x_0) = n\hbar\Omega_x x_0}$$

Half-Soliton acceleration



Magnetic Energy:

$$E_{mag} = -\frac{\hbar}{2}\int \vec{\Omega}\cdot\vec{S}dx$$

The motion of the soliton is governed by the minimization of E_{mag}

In the limit $L >> \xi$:

$$E_{mag}(x_0) = -n\frac{\hbar\Omega_x}{2}\int \text{sign}(x-x_0)\,dx$$

$$\boxed{E_{mag}(x_0) = n\hbar\Omega_x x_0}$$

Force:

$$\vec{F}_{mag} = -\overrightarrow{grad}_{x_0} E_{mag} = -n\hbar\Omega_x \vec{u}_x$$

Acceleration:

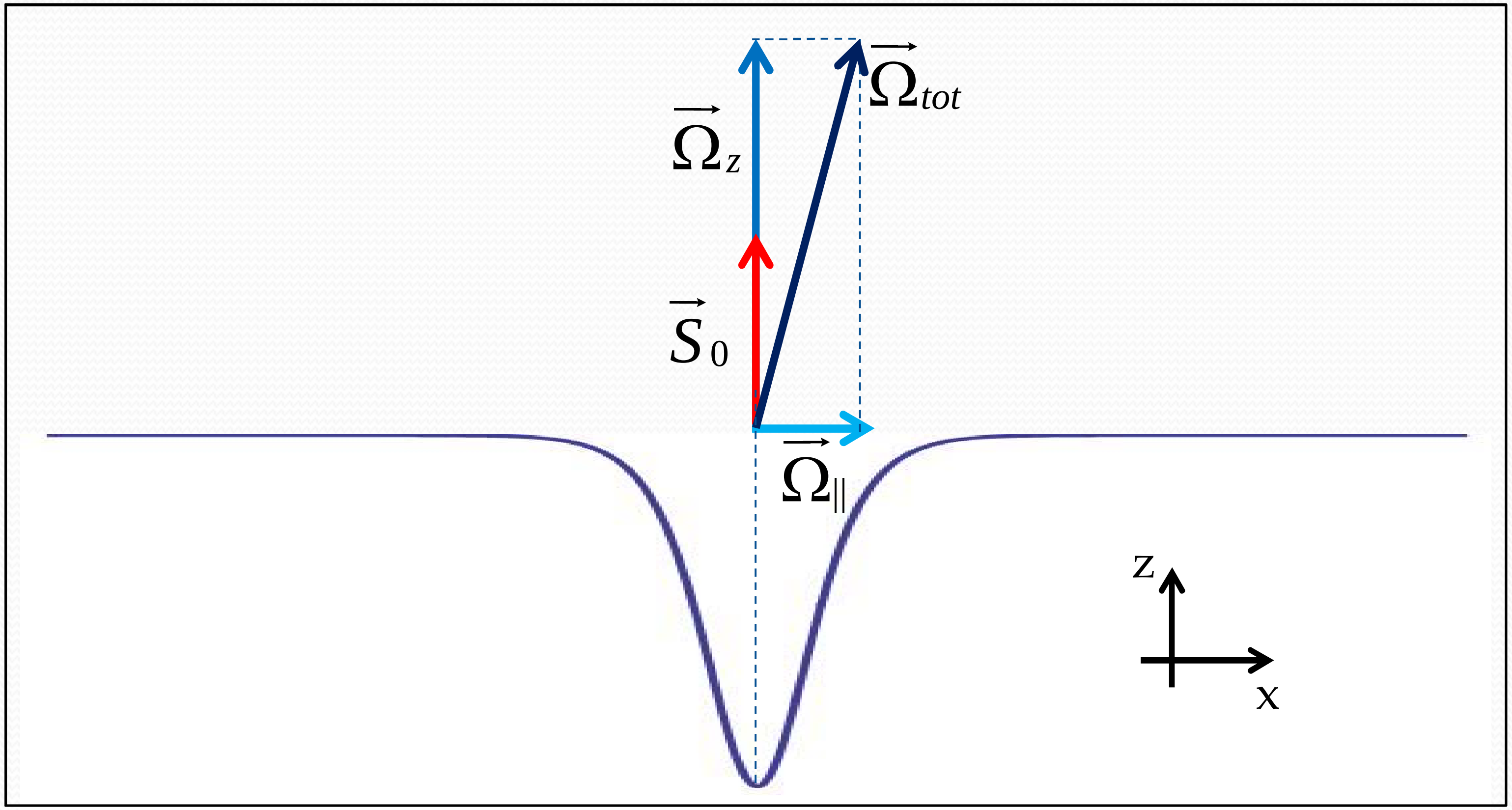
$$\vec{a} = \frac{\vec{F}_{mag}}{m_{HS}} = -n\frac{\hbar\Omega_x}{m_{HS}}\vec{u}_x$$

$m_{HS}<0$

$$\vec{a} = +|a_x|\vec{u}_x$$

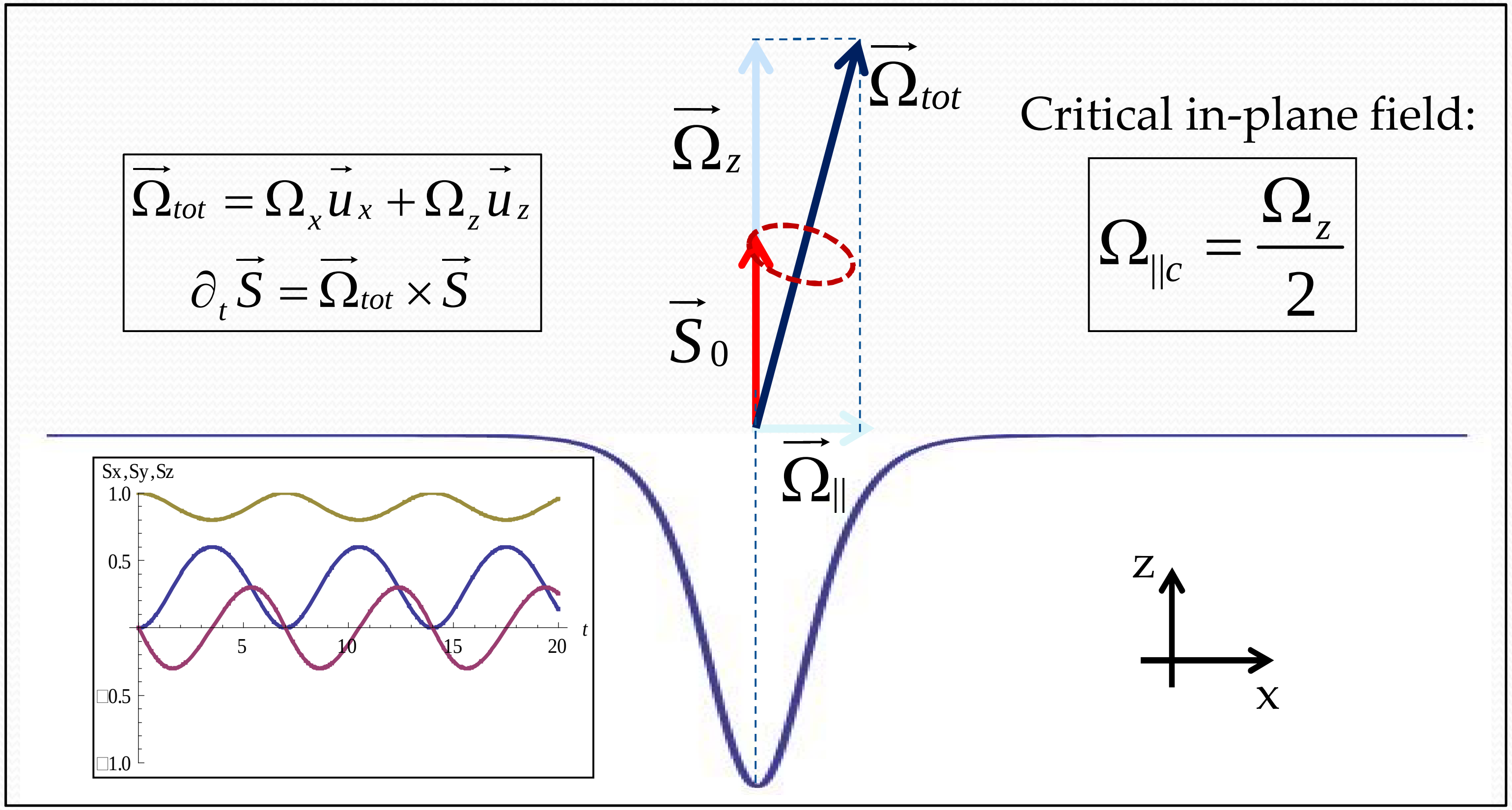
Half-soliton stability

- The half-soliton core is filled by the other component: Strongly circularly polarized
- Intrinsic effective magnetic field $\vec{\Omega}_z$ is strong at the core
- Protects the pseudospin against precession around $\vec{\Omega}_{||}$



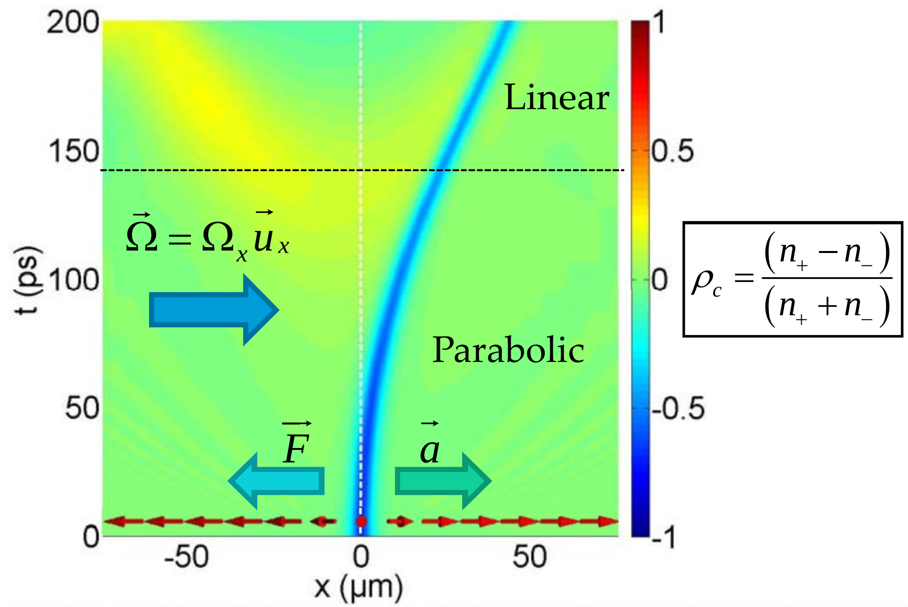
Half-soliton stability

- The half-soliton core is filled by the other component: Strongly circularly polarized
- Intrinsic effective magnetic field $\vec{\Omega}_z$ is strong at the core
- Protects the pseudospin against precession around $\vec{\Omega}_{||}$



Half-Soliton acceleration

Numerical result solving spinor GP equations



Dark soliton gaining speed becomes shallower

$$n_+(0) = v_+ / c_+$$
$$\Delta\theta_+ = 2\arccos(v_+ / c_+)$$

In addition, the **charge** of the soliton is renormalized

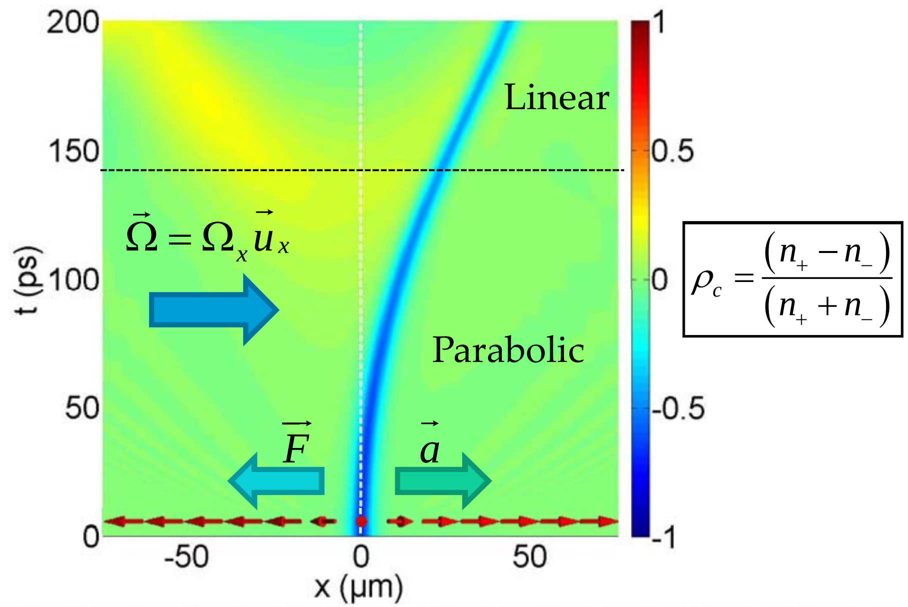
$$q = q_0 \left(1 - v^2 / c^2\right) = q_0 / \gamma^2$$
$$q_0 = (\alpha_1 - \alpha_2) n_0 / 2$$

The acceleration and velocity read

$$\vec{a} = -\frac{q_0}{m_0} \Omega_x \gamma^{-3} \vec{u}_x$$
$$\vec{v}(t) = c \tanh\left(\frac{n q_0 \Omega_x}{c} t\right) \vec{u}_x, \quad v(0) = 0$$

Half-Soliton acceleration

Numerical result solving spinor GP equations



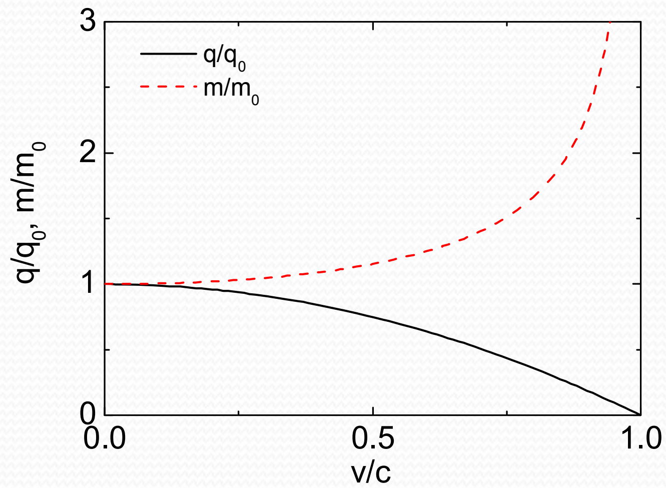
- Accelerated soliton becomes **shallower**
- Phase shift **reduced**

$$n_+(0) = v_+ / c_+$$
$$\Delta\theta_+ = 2\arccos(v_+ / c_+)$$

The magnetic **charge** is **renormalized** !

$$q = q_0\left(1 - v^2 / c^2\right) = q_0 \gamma^{-2}$$
$$q_0 = (\alpha_1 - \alpha_2) n_0 / 2$$

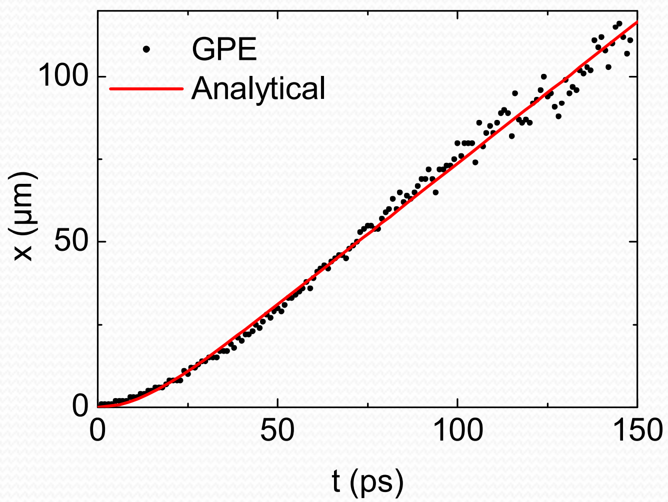
Half-soliton as a relativistic particle



Mass/Charge renormalization

$$q = q_0\left(1 - v^2/c^2\right) = \gamma^{-2} q_0$$

$$m = m_0 / \left(1 - v^2/c^2\right)^{1/2} = \gamma m_0$$



Trajectories

$$x(t) = \frac{c^2}{q_0 \hbar \Omega_x} \log\left[\cosh\left(\frac{q_0 \hbar \Omega_x}{c} t\right)\right]$$

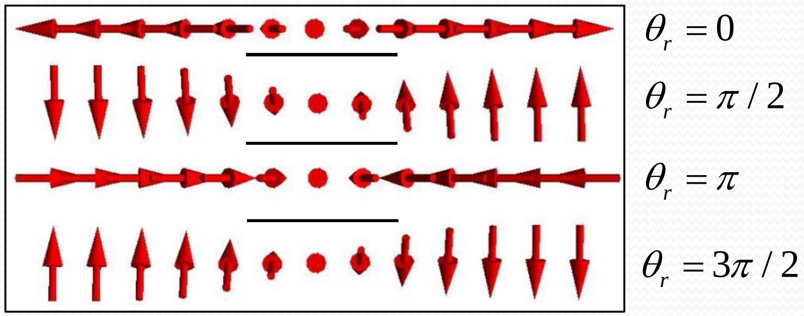
$$x(t \sim 0) = \lambda t^2$$

$$x(t \to +\infty) = \lambda t - \lambda_0$$

Constant relative phase

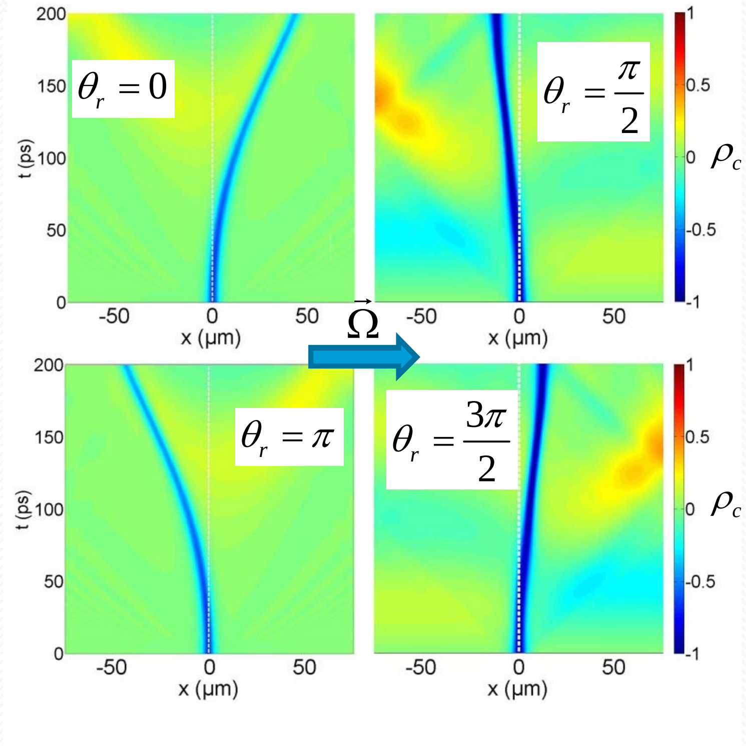
$$\vec{\psi}_{HS} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \sqrt{n_0}\tanh(lx)e^{i\phi_0} \\ \sqrt{n_0} \end{pmatrix}, \begin{cases} l = \pm 1: \text{ sign of the } \pi \text{ phase shift} \\ \phi_0: \text{constant relative phase} \end{cases}$$

Total relative phase: $\boxed{\theta_r = l\phi_0}$



Given: $\vec{\Omega} = \Omega_x \vec{u}_x$

$$q = q_0\left(1 - v^2 / c^2\right)\cos\left(\theta_r\right)$$

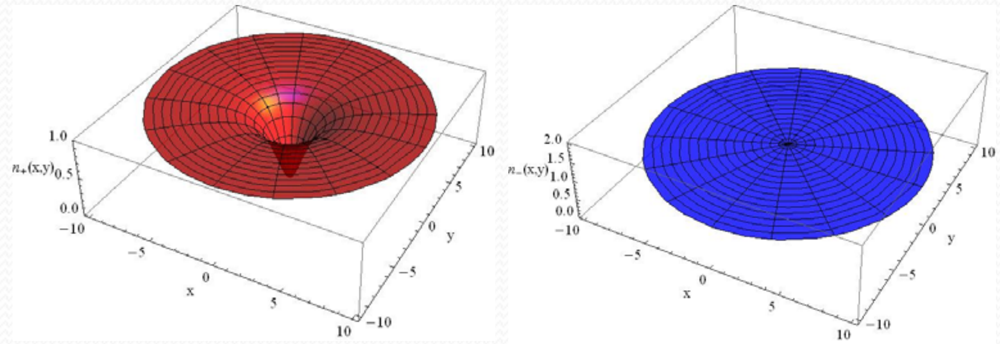


Half-Vortices

A half-vortex wavefunction:

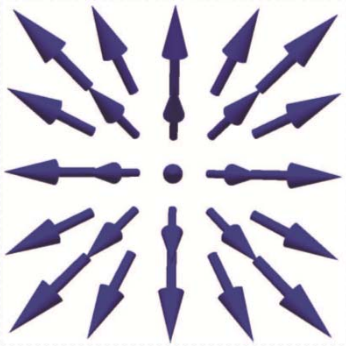
$$\vec{\psi} = \begin{pmatrix} \psi_+(r,\phi) \\ \psi_-(r,\phi) \end{pmatrix} = \begin{pmatrix} \sqrt{n_+(r)}e^{il_+\phi}e^{i\phi_0} \\ \sqrt{n_-} \end{pmatrix}$$

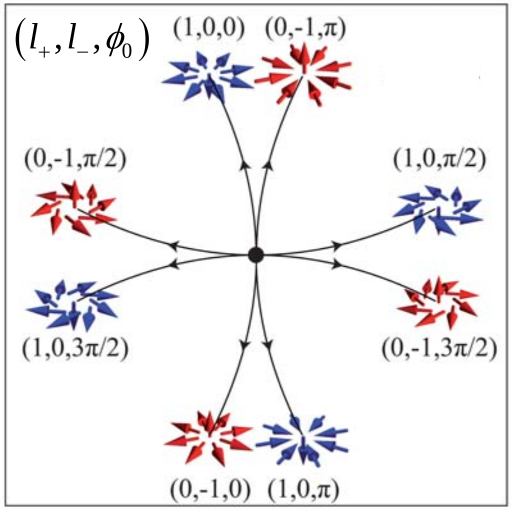
$$n_+(r) = r^2/(r^2+2)$$

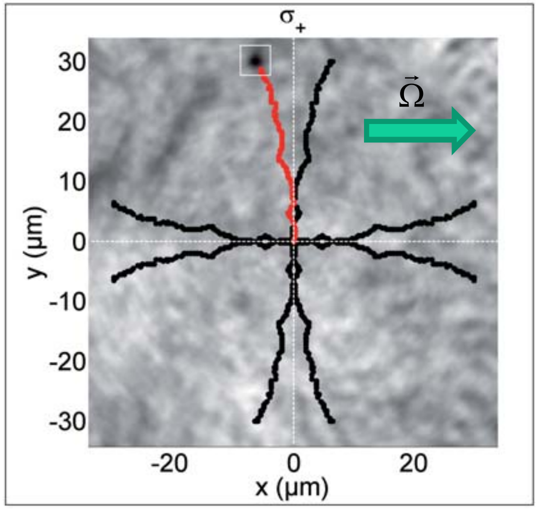


Vortex in σ_+ No vortex in σ_-

Divergent Pseudospin



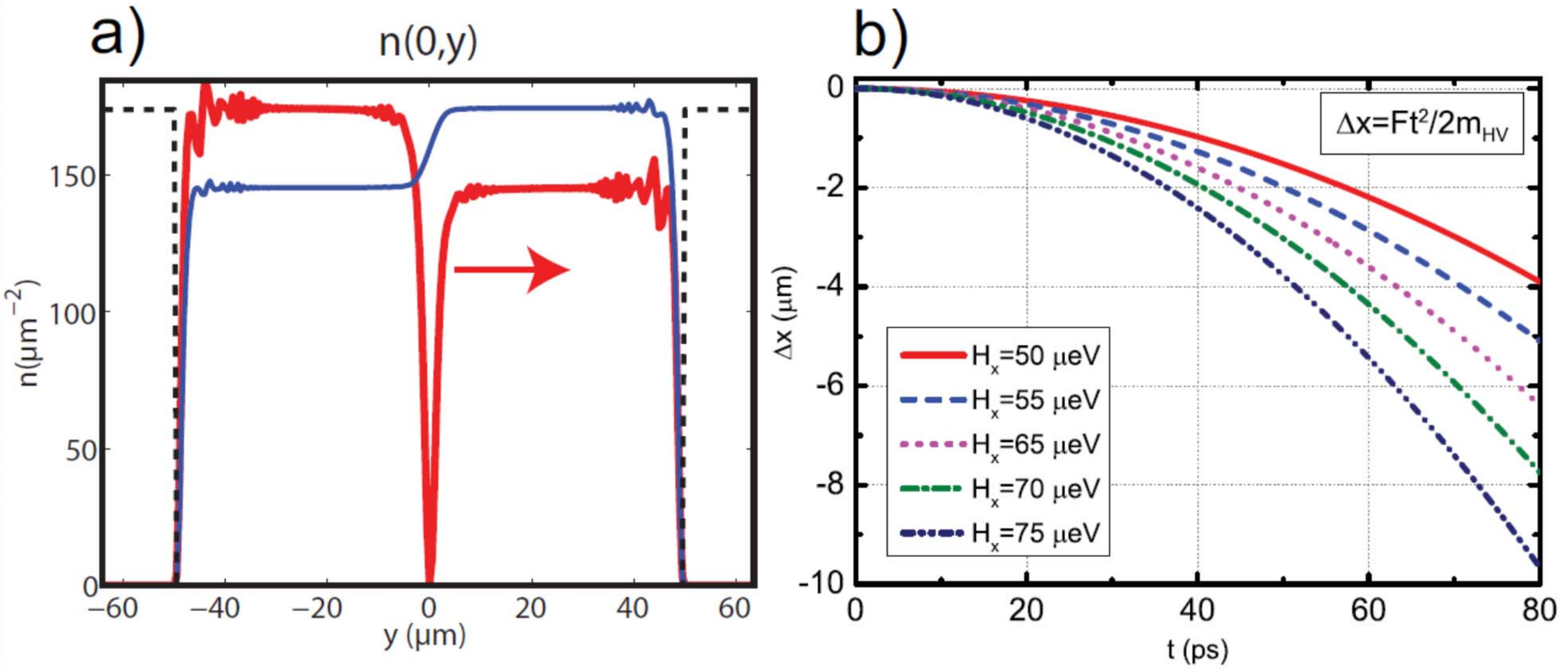




$$q_0 = \begin{pmatrix} +\cos(\alpha) & +\sin(\alpha) \\ +\sin(\alpha) & -\cos(\alpha) \end{pmatrix}, \quad \alpha = \pi - \phi_0$$

Trajectories of half-vortices

Two contributions to the half-vortex motion



Density gradient (because of initial pseudospin rotation)

Magnetic field induced motion

Outline

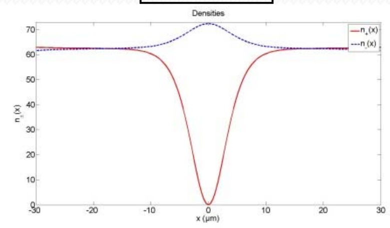
- Integer topological defects in scalar BECs
  - Solitons
  - Vortices
  - Oblique solitons
- Half-integer topological defects in 2 component BECs
  - Half-solitons
  - Half-vortices and oblique half-solitons
- Towards magnetricity in semiconductor microcavities
  - Generation of half-soliton currents
  - Half-vortices injection and propagation

Solitons interactions

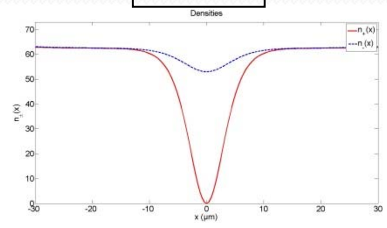
- Dark soliton naturally **repel** each other
- Half-solitons interaction depends on α_2

$$i\hbar\frac{\partial\psi_\pm}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi_\pm+\alpha_1\left|\psi_\pm\right|^2\psi_\pm+\alpha_2\left|\psi_\mp\right|^2\psi_\pm$$

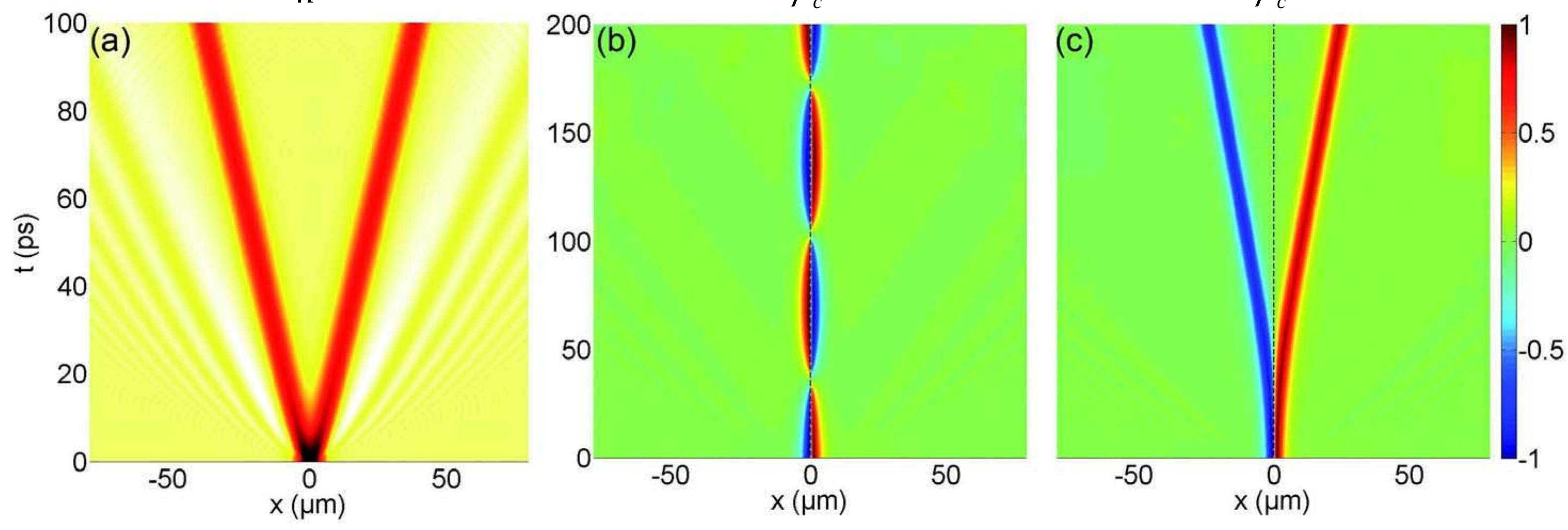
$\alpha_2>0$



$\alpha_2<0$



Short range repulsion | Attraction | Short range repulsion



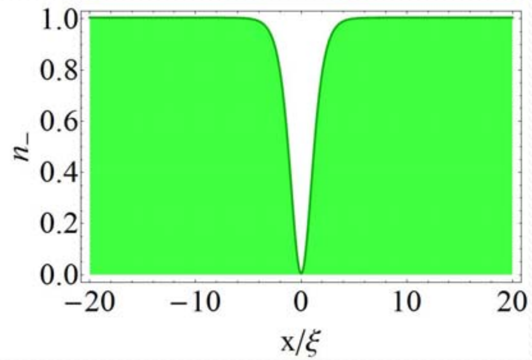
Integer dark solitons

Integer half-solitons $\alpha_2>0$

Integer half-solitons $\alpha_2<0$

Integer soliton in spinor BEC

$n_+ + n_-$

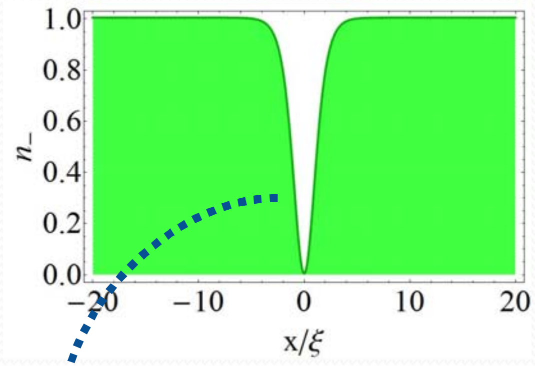


Linear Polarization

Integer Soliton

Integer soliton in spinor BEC

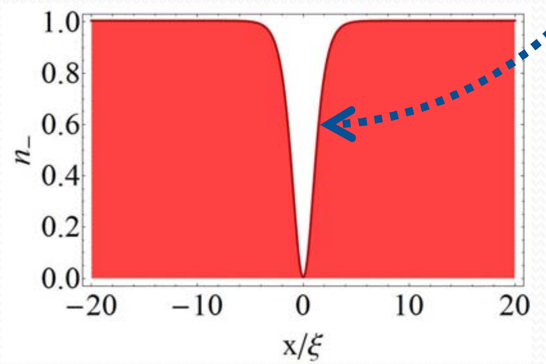
$n_+ + n_-$



Linear Polarization

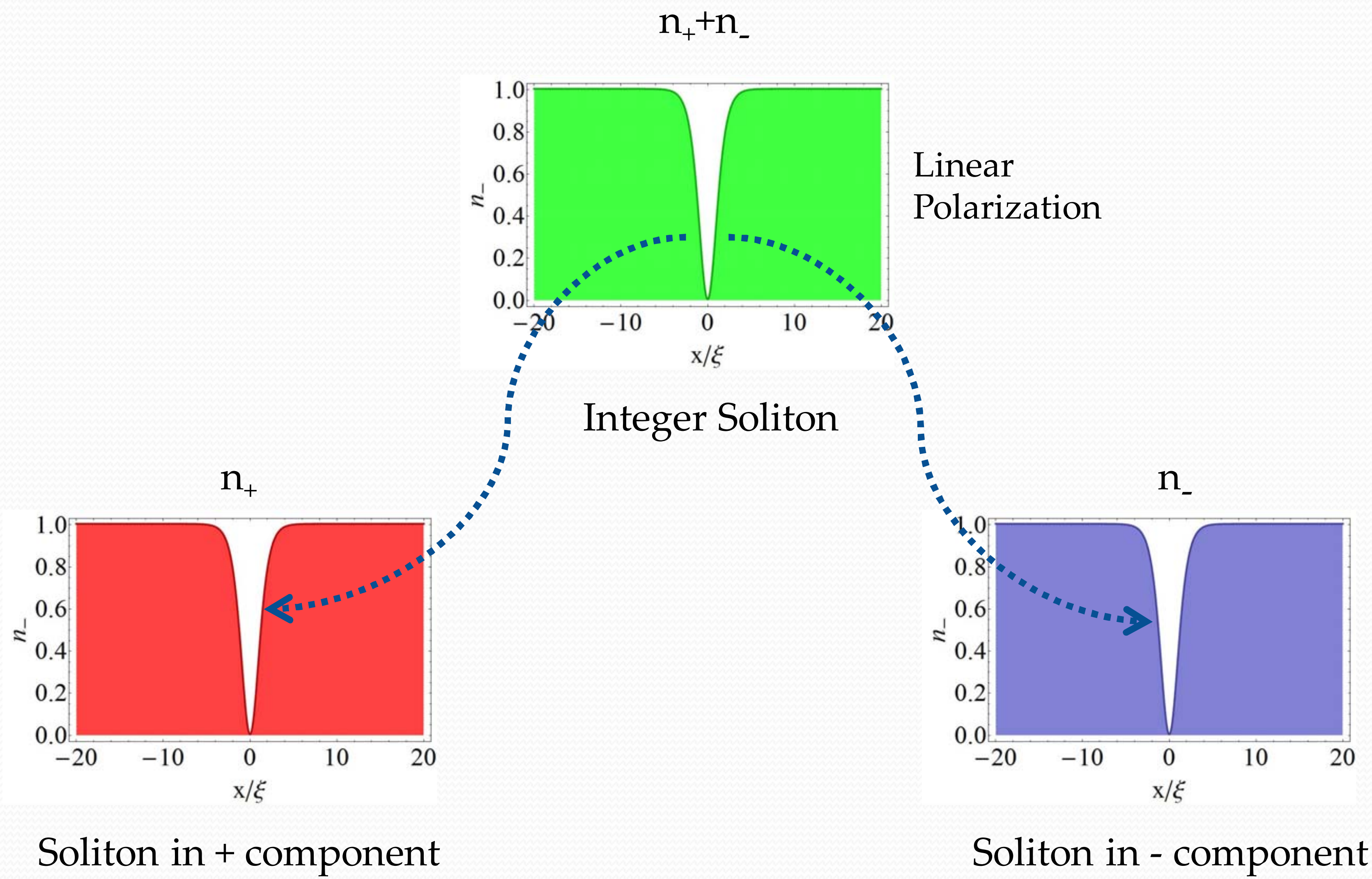
Integer Soliton

n_+



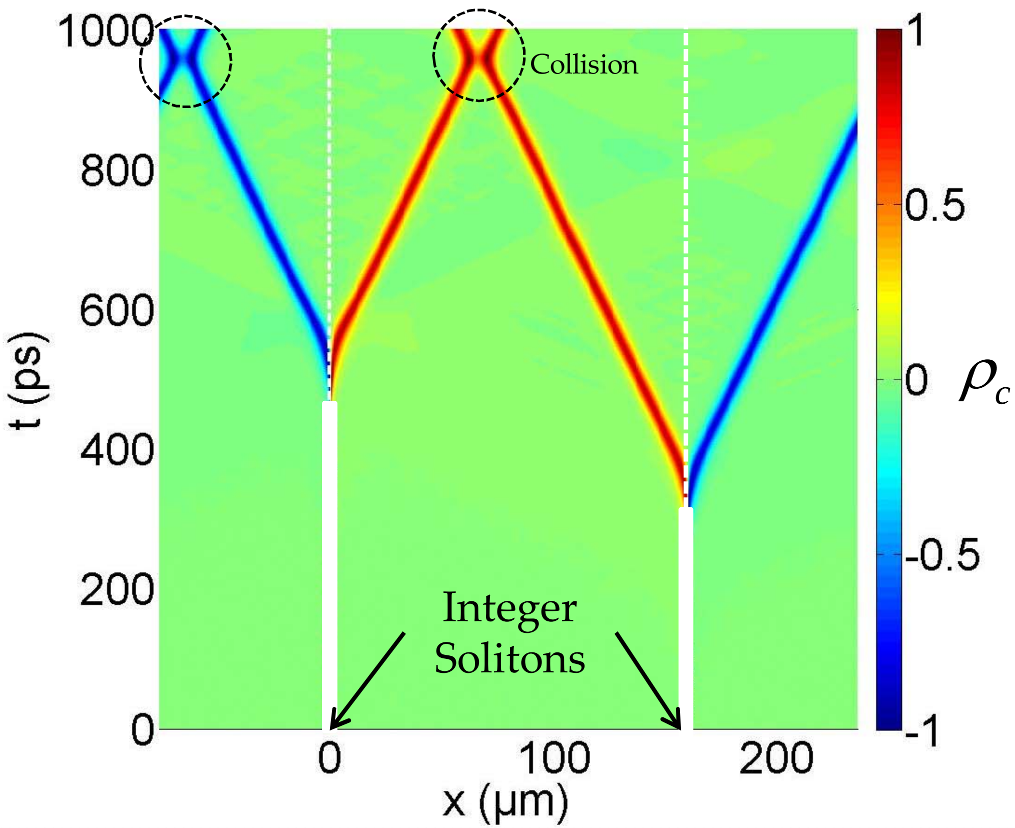
Soliton in + component

Integer soliton in spinor BEC



Noise induced separation

Integer soliton + Noise + $\alpha_2 < 0$ = Decay into half-solitons



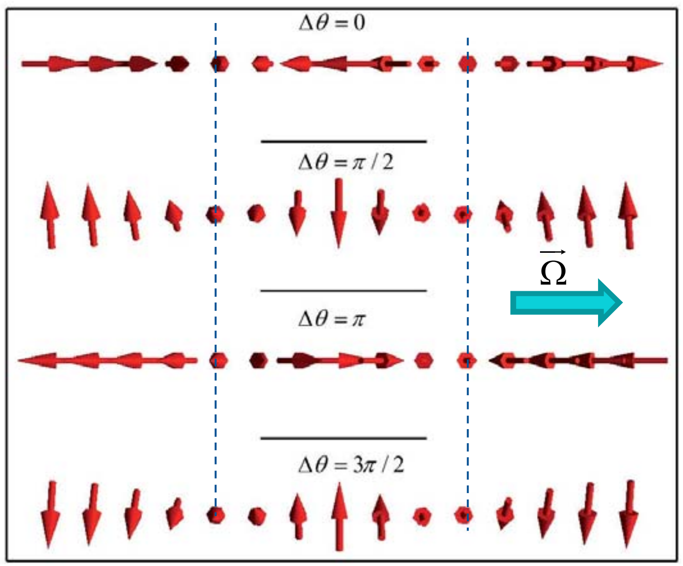
Half-soliton pairs

Half-Solitons separated by d :

$$\vec{\psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \sqrt{\frac{n}{2}} \begin{pmatrix} \tanh\left[l_+ \left(x - d/2 \right) \right] e^{i\,\phi_0} \\ \tanh\left[l_- \left(x + d/2 \right) \right] \end{pmatrix}, \quad \{l_+, l_-\} = \pm 1$$

Total relative phase $\Delta\theta = \mathrm{sign}\left(l_+ l_- \right) \phi_0$

The half-soliton in a pair have opposite charge
They accelerate in opposite directions under Ω_x (for $\Delta\theta = p\pi$)



Separation

No acceleration

Crossing+Repulsion

No acceleration

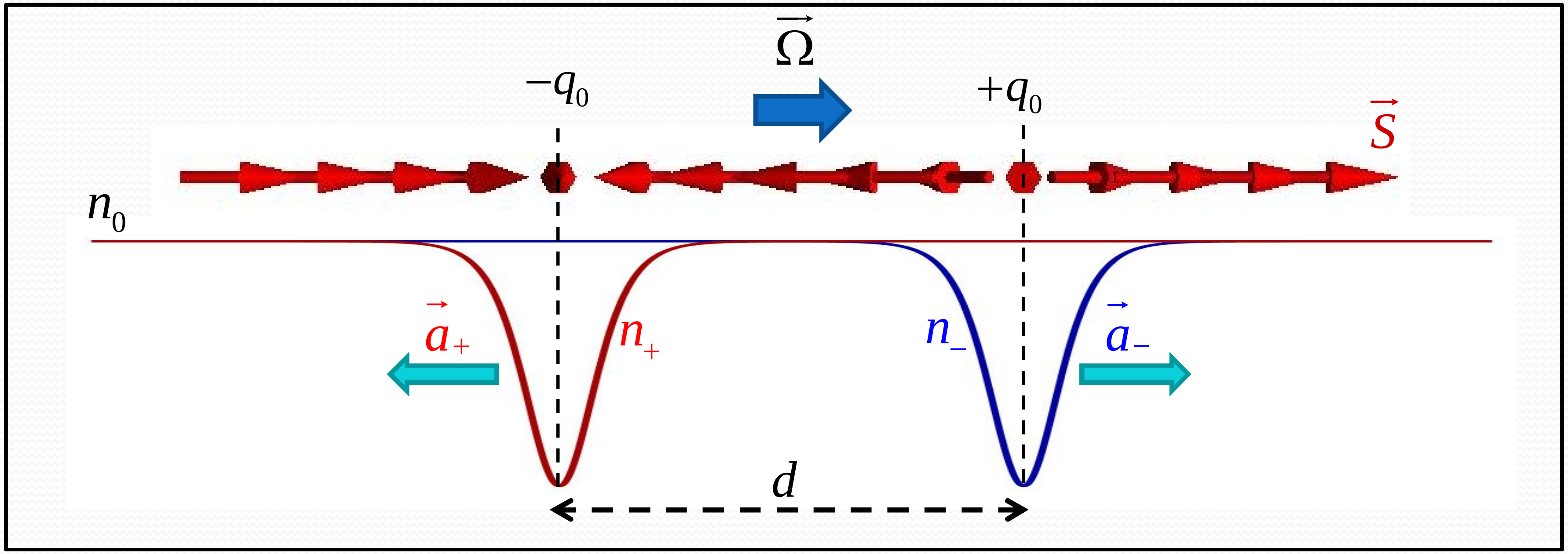
Half-soliton pairs

Half-Solitons separated by d :

$$\vec{\psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \sqrt{\frac{n_0}{2}} \begin{pmatrix} \tanh\left[x - d/2\right] \\ \tanh\left[x + d/2\right] \end{pmatrix}$$

The half-soliton in the pair have *opposite* charges

Acceleration in *opposite* directions



Outline

- Solitons in scalar BECs
  - Solitons (1D)
  - Oblique solitons (2D)
- 2 component spinor BEC
  - Pseudospin dynamics
  - Half-solitons
  - Magnetic charges
- Towards magnetricity
  - Polariton condensate
  - Half-solitons imprinting
  - Experimental evidence
- Summary

Towards magnetricity

Requirements:

- 2 component sBEC
- 1D system
- In-plane field: $\vec{\Omega}$
- $|\alpha_2| << \alpha_1$: spin anisotropy
- $\alpha_2 < 0$: natural separation
- Mean of creating the half-solitons

Towards magnetricity

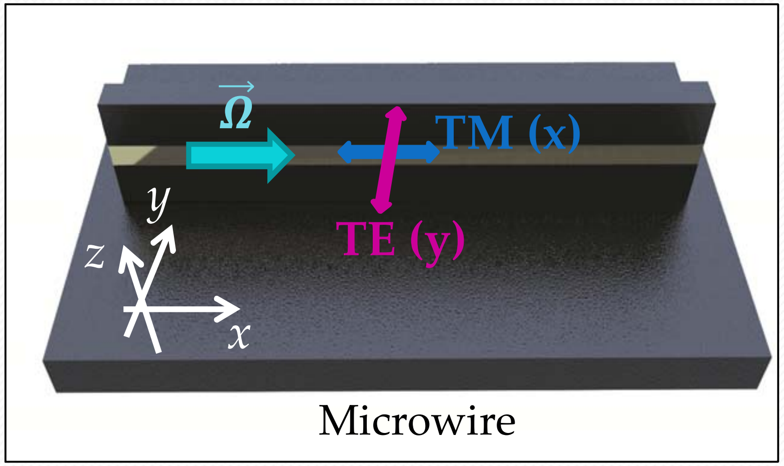
Requirements:

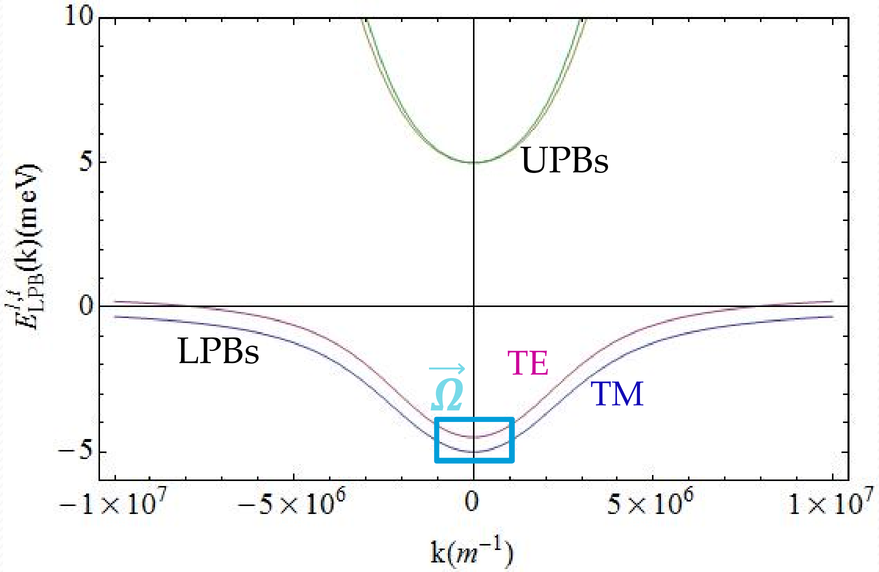
- 2 component sBEC
- 1D system
- In-plane field: $\vec{\Omega}$
- $|\alpha_2| << \alpha_1$: spin anisotropy
- $\alpha_2 < 0$: natural separation
- Mean of creating the half-solitons

Polariton condensate in semiconductor microcavities:

- 2 spin projections ±1 (inherited from excitonic part)
- Wire shaped microcavities
- Polarization splittings at $k=0$: in plane effective magnetic field: $\vec{\Omega}$
- $\alpha_2 \approx -0.1\alpha_1$
- Phase imprinting

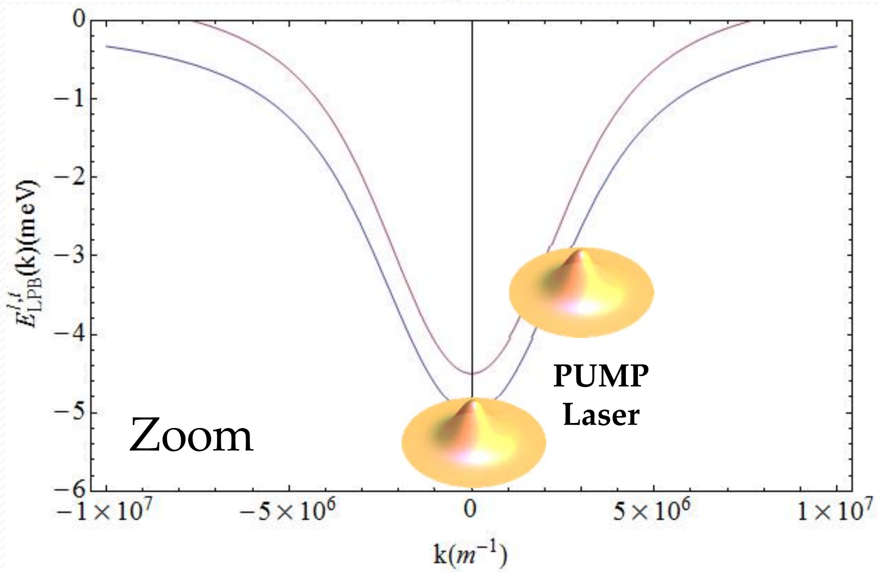
The polariton condensate



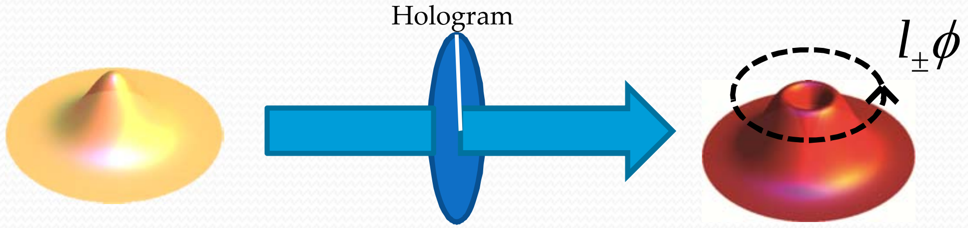


$$\begin{cases}\phi(\vec{r},t): \text{ photonic field}\\ \chi(\vec{r},t): \text{ excitonic field}\end{cases}$$

$$\begin{cases} i\hbar\dfrac{\partial\phi_\pm}{\partial t} = -\dfrac{\hbar^2}{2m_\phi}\Delta\phi_\pm - \hbar\Omega_x\phi_\mp + V_R\chi_\pm - \dfrac{i\hbar}{2\tau_\phi}\phi_\pm + P_{GL}^\pm + U\phi_\pm \\ i\hbar\dfrac{\partial\chi_\pm}{\partial t} = -\dfrac{\hbar^2}{2m_\chi}\Delta\chi_\pm + \alpha_1|\chi_\pm|^2\chi_\pm + \alpha_2|\chi_\mp|^2\chi_\pm + V_R\phi_\pm - \dfrac{i\hbar}{2\tau_\chi}\chi_\pm \end{cases}$$



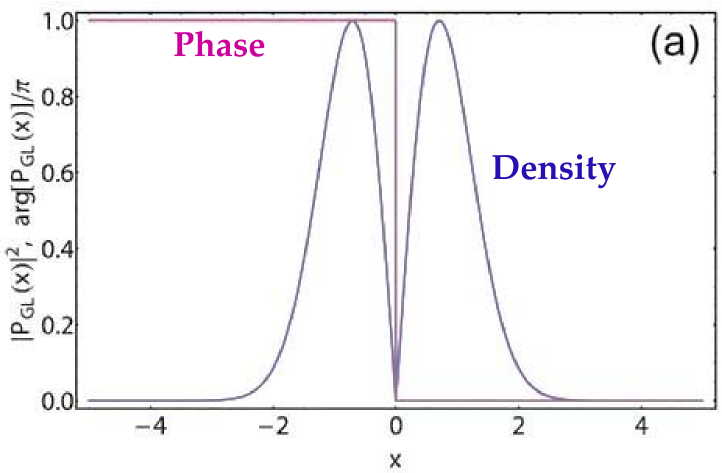
Gauss Laguerre Beam



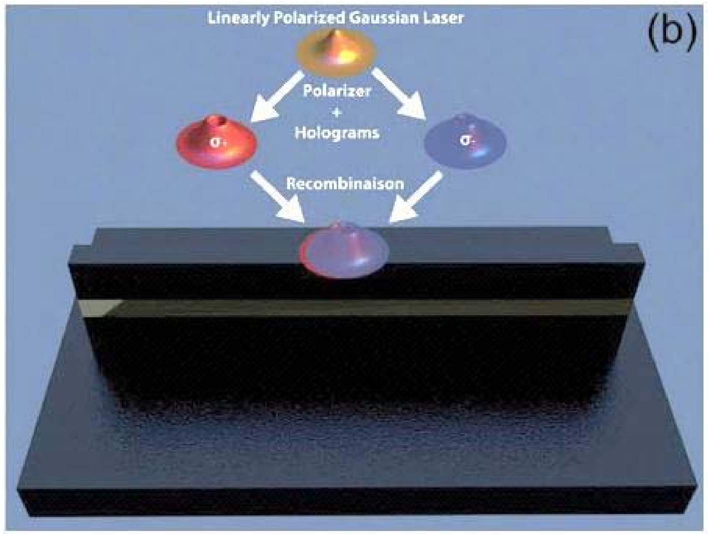
Already used experimentally to imprint vortices in a polariton condensate

See e.g. Sanvitto et al. *Nature Physics* **6**, 527 (2010)

$$P_{GL}\left(\vec{r},t,l_{\pm}\right) = rA_{GL}e^{-r^2/\sigma_r^2}e^{-t^2/\sigma_t^2}e^{-i\omega_{GL}t}e^{il\phi}$$

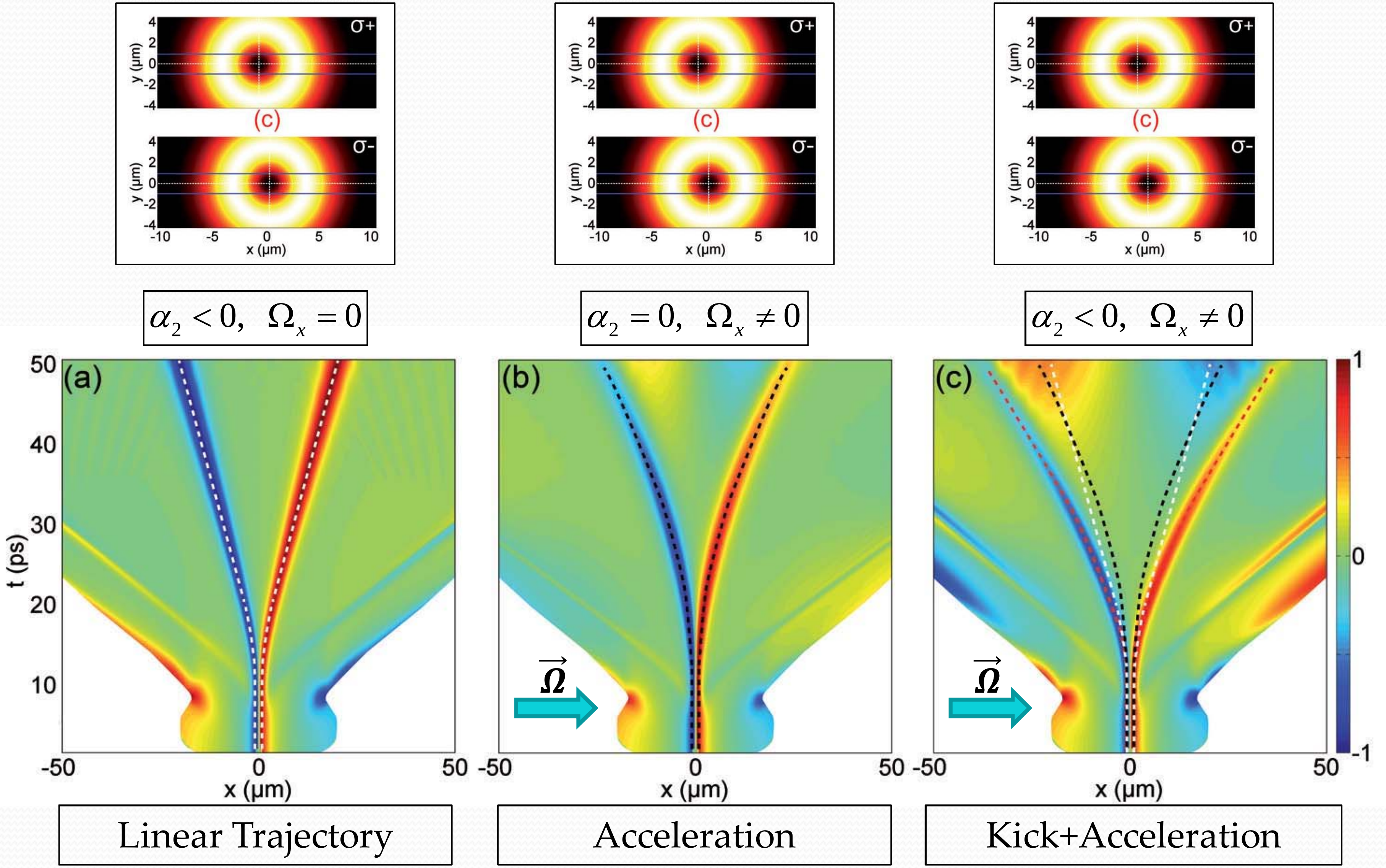


GL beam as seen by a 1D system



Soliton imprinting

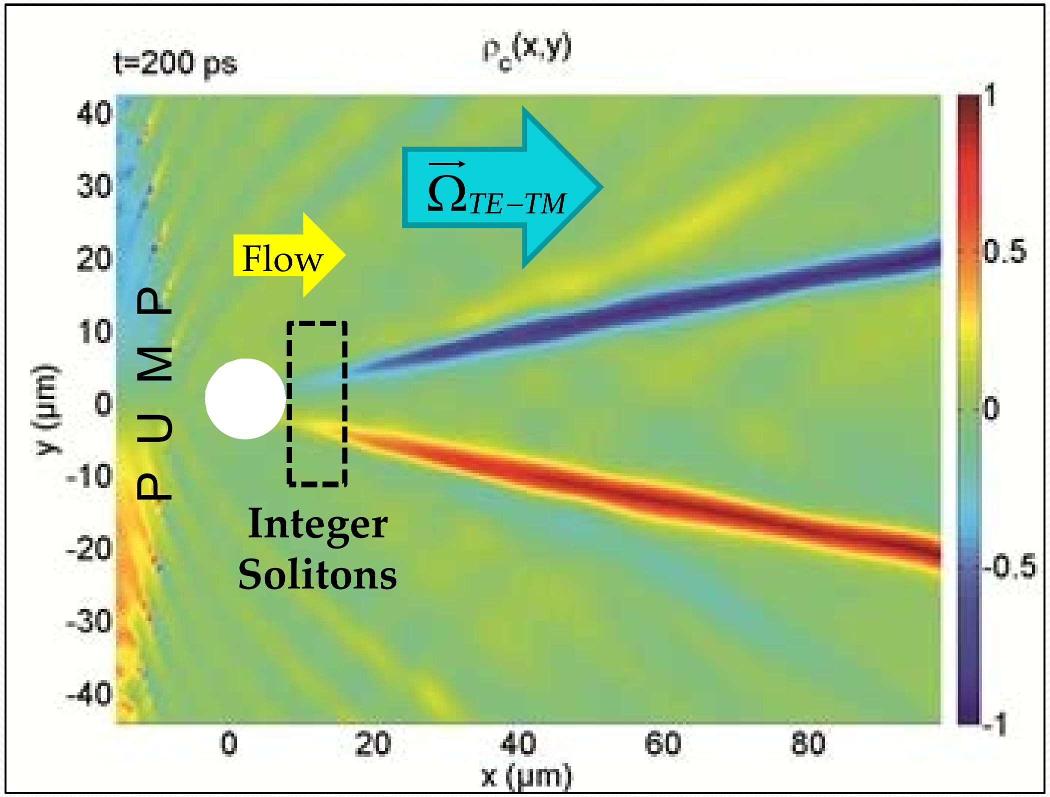
Half-soliton imprinting



Oblique Half-Solitons

Oblique Half-Solitons

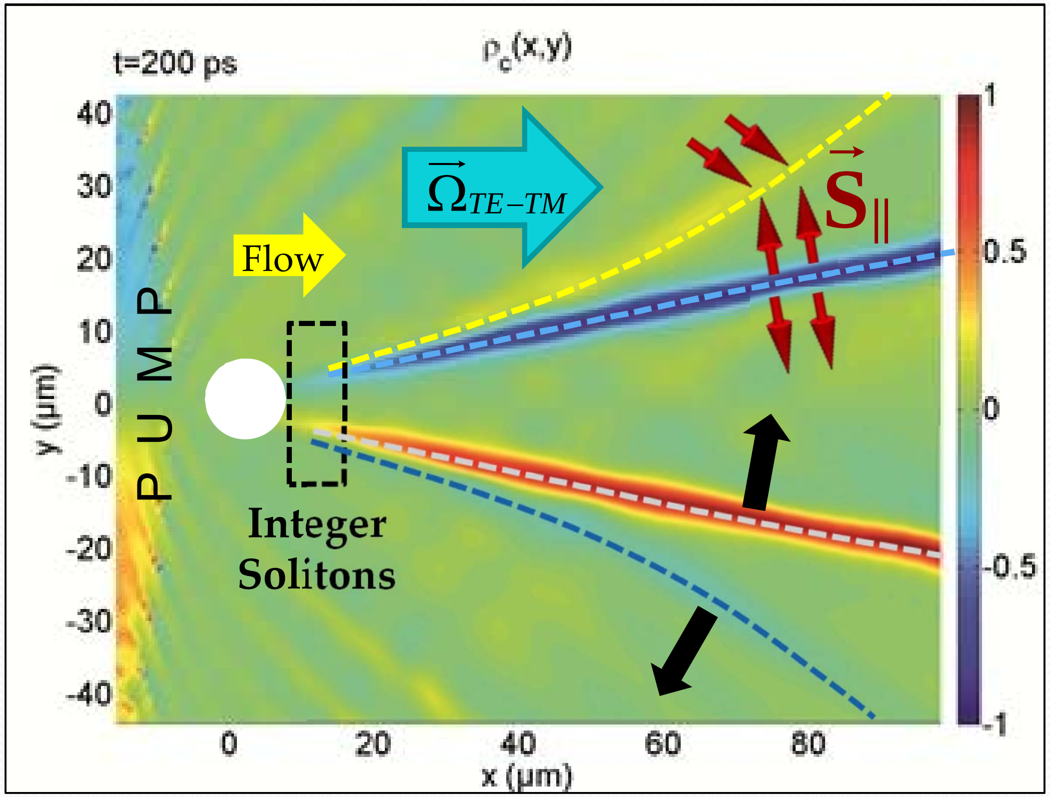
Oblique half-solitons are accelerated as well !



Separation of an integer defect into its half-integer constituents

Oblique Half-Solitons

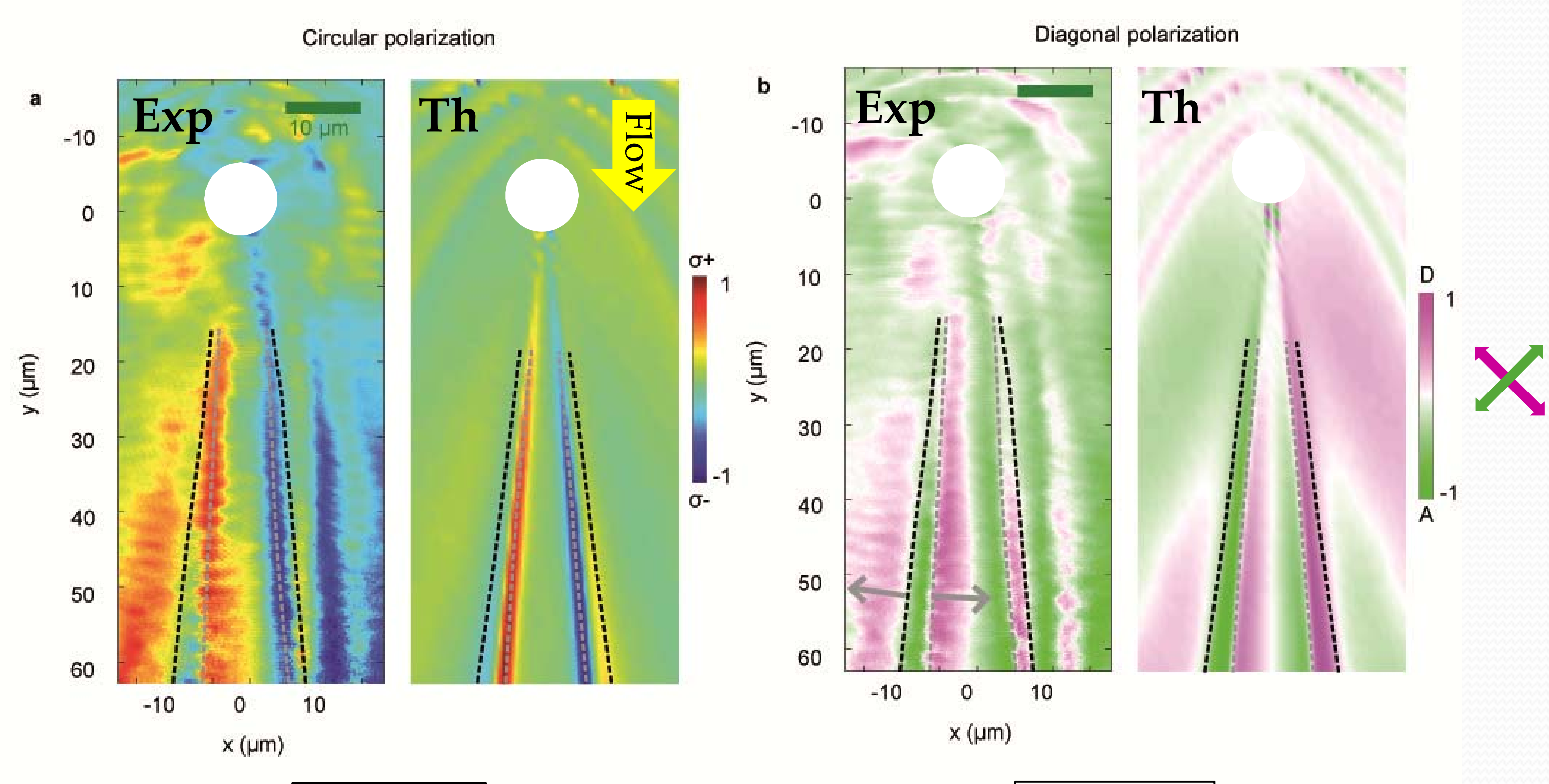
Oblique half-solitons are accelerated as well !



Separation of an integer defect into its half-integer constituents

Oblique Half-Solitons

Observation of the separation of oblique half-solitons
R. Hivet et al. (2012), to appear in *Nature Physics*



$$\rho_c = \frac{I_+ - I_-}{I_+ + I_-}$$

$$\rho_c = \frac{I_D - I_A}{I_D + I_A}$$

Avoiding the beam separation

Slightly Elliptic Pumping

$P_{GL}^{+} = 0.99 P_{GL}^{-}$

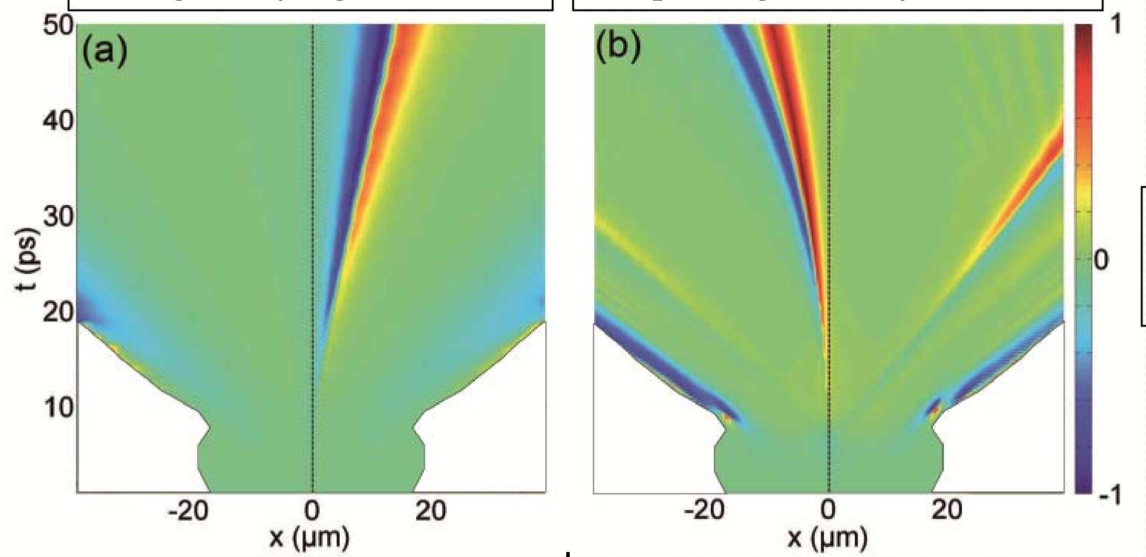
$n_{+} \neq n_{-} \Rightarrow m_{+} \neq m_{-}$

The motion is mass-dependent

Wedge/Varying Thickness

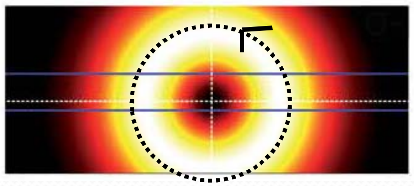
Displacing latteraly the beam

$\alpha_2 \neq 0$
$\Omega_x = 0$



$\alpha_2 \neq 0$
$\Omega_x = 0$

Varying confinement: Potential ramp



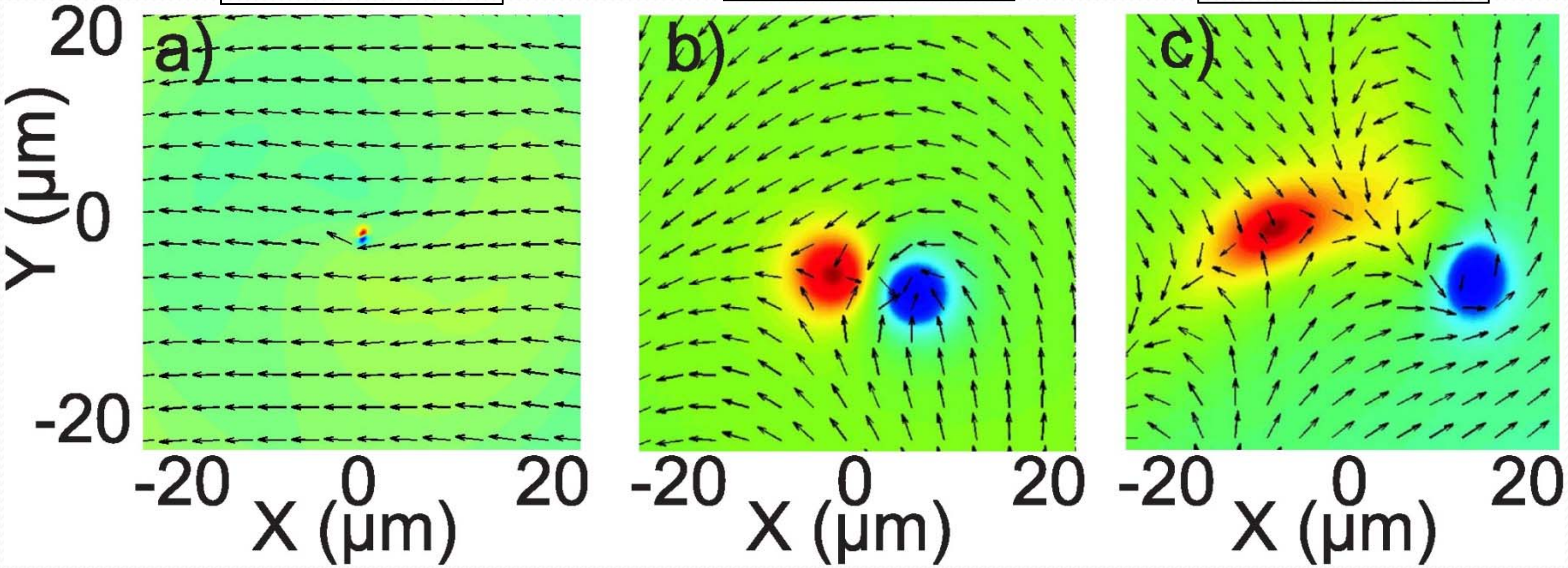
$\downarrow \Delta y$

Gradient of angular velocity

Vortex separation

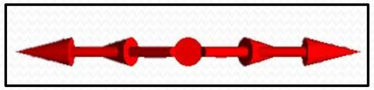
Half-vortices imprinted by Gauss-Laguerre beams

Initial State | $\alpha_2 \neq 0, \quad \Omega_x = 0$ | $\alpha_2 \neq 0, \quad \Omega_x \neq 0$

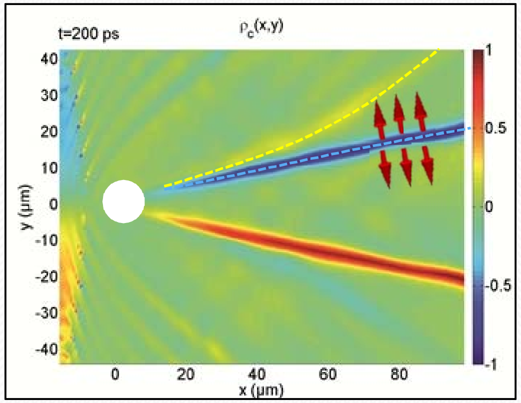


Magnetic monopole analogues

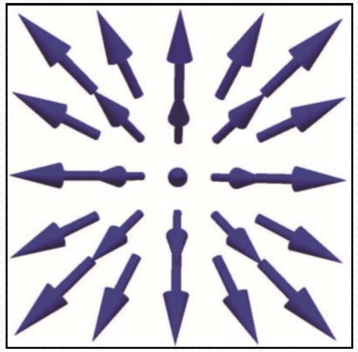
Half-solitons



Oblique half-solitons



Half-vortices



Divergent Pseudospin $div\vec{S}_{||} \neq 0$

MAGNETIC MONOPOLE

Acceleration Along $\vec{\Omega}_{||}$

Stable Single Particle

Summary

- Half integer excitations are *analogues* of *Dirac's magnetic monopoles*
- Behave as relativistic particles (mass, size and charge are *velocity* dependent)
- Naturally separate for $\alpha_2 < 0$
- Stable for $|\alpha_2| \ll \alpha_1$ (spin anisotropy)
- Polariton condensate is well suited for their observation
- Imprinting of half-soliton/vortices with Gauss-Laguerre beams
- Polariton lifetime up to 30 ps in modern structures
- Magnetricity with large velocities in the range of $\mu m/ps$

Thank you for your attention

Papers on the topic:

- H. Flayac et al., *Phys. Rev. B* **83,** 193305 (2011).
- D. Solnyshkov et al., *Phys. Rev. B* **85**, 073105 (2012).
- H. Flayac et al., *arXiv*:1203.0885v1 to appear in *New. J. Phys.* (2012).
- R. Hivet et al., *arXiv*:1204.3564, to appear in *Nature Physics* (2012)