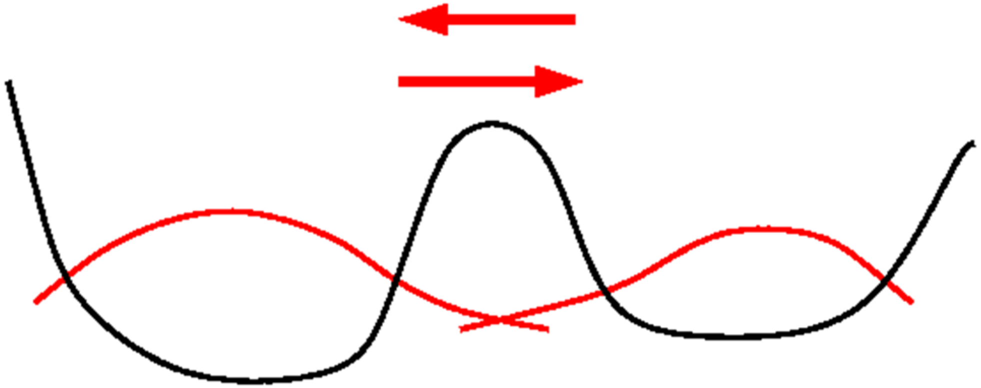
Josephson oscillations between exciton condensates in electrostatic traps



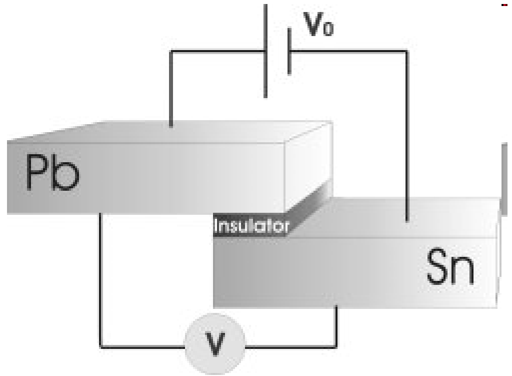
Massimo Rontani

CNR-NANO S3, Modena, Italy

L. J. Sham

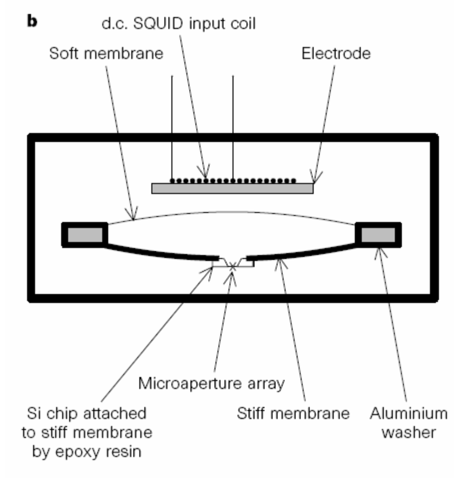
University of California San Diego, California

Josephson effect



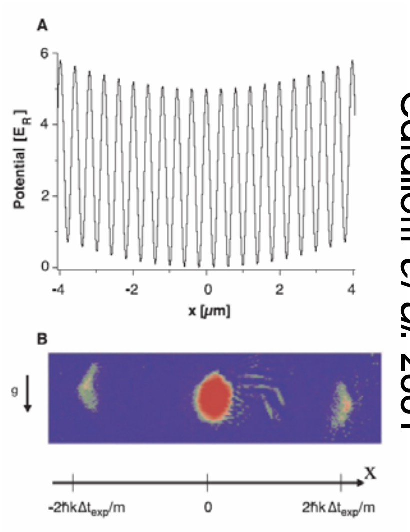
Anderson & Rowell 1963

superconductors



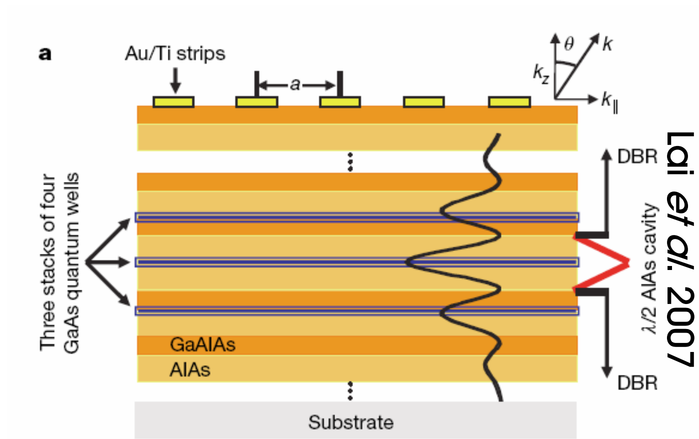
Pereverzev *et al.* 1997

^{3}He



Cataliotti *et al.* 2001

atomic gases

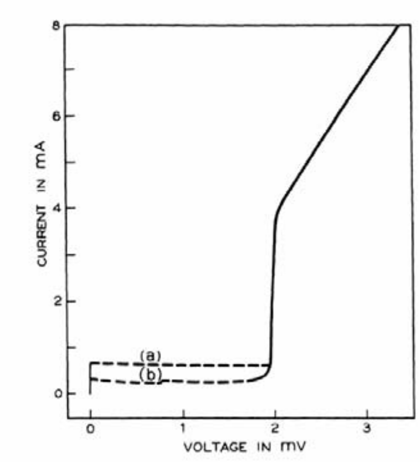


Lai *et al.* 2007

polaritons

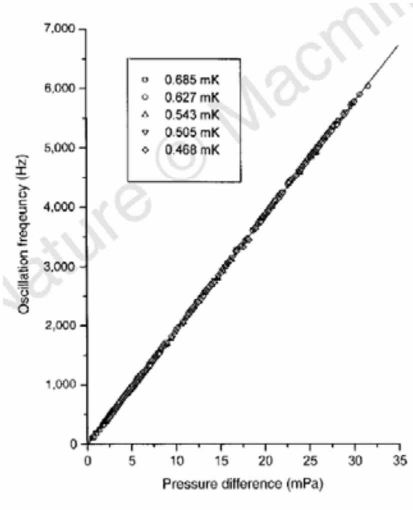
…and excitons?

probing the Josephson effect



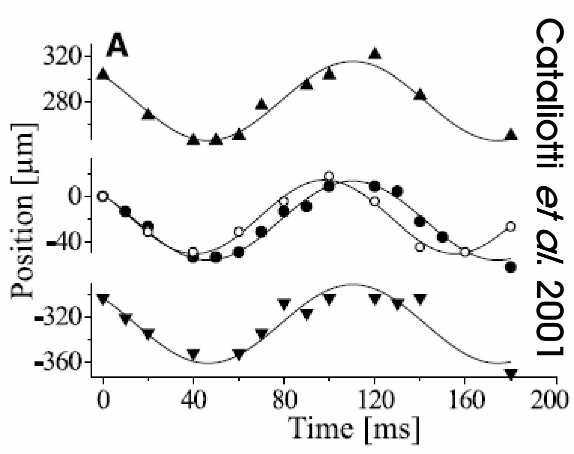
Anderson & Rowell 1963

electric current



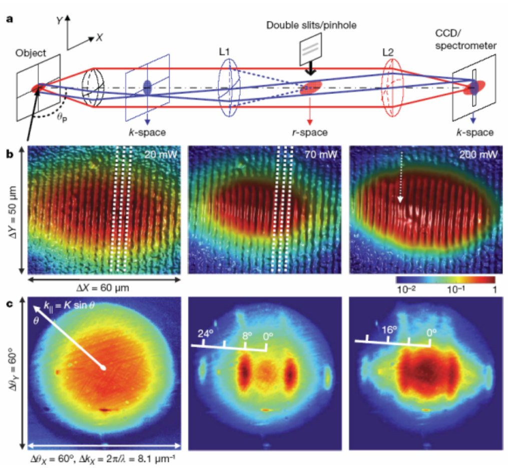
Pereverzev *et al.* 1997

sound waves



Cataliotti *et al.* 2001

interferograms



Lai *et al.* 2007

light

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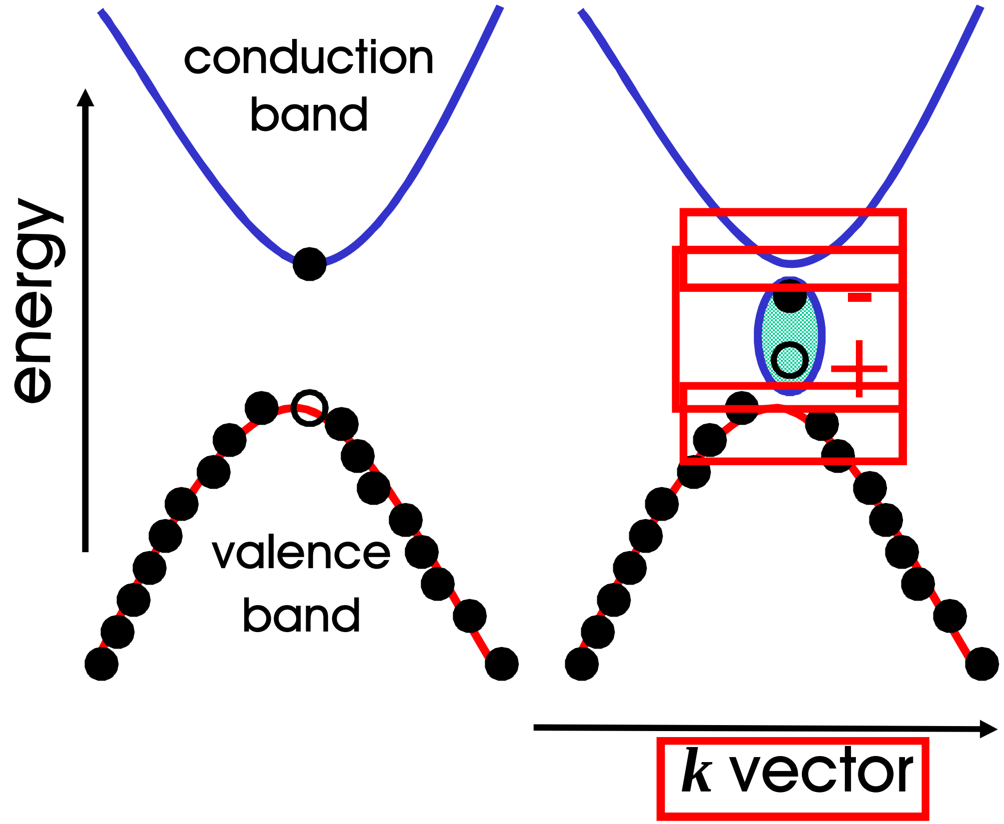
coherent transport of excitons?

indirect evidence

no charge

no real mass

no real momentum



“real” transport: dipole, spin, energy ($T > 0$)

Halperin & Rice 1968

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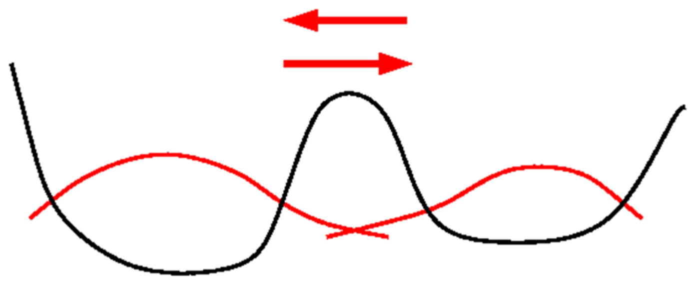
two obvious questions

(1) does the exciton condensate exist?

(2) what are the (unambiguous) experimental signatures of coherent transport?

traps of optically pumped cold excitons in bilayers

Josephson oscillations



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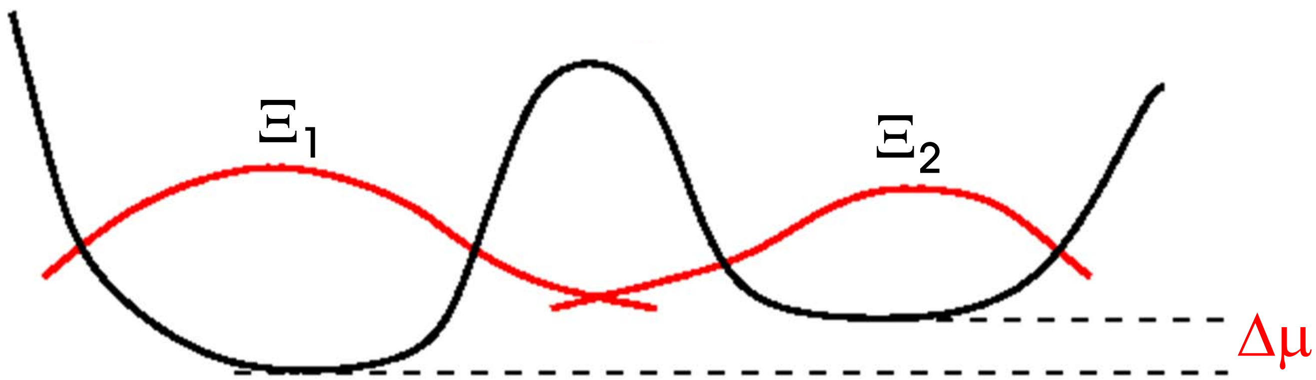
the very essence of Josephson effect

(a) two macroscopic quantum systems (Ξ_1 and Ξ_2)

(b) weak link between Ξ_1 and Ξ_2

(c) well defined phases, φ_1 and φ_2

$$\mathbf{J}_s = \mathbf{J}_{\max} \sin(\varphi_1 - \varphi_2)$$



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(a) two macroscopic quantum systems (Ξ_1 and Ξ_2)

(b) weak link between Ξ_1 and Ξ_2

(c) well defined phases, φ_1 and φ_2

outline

(a) indirect excitons in double quantum wells

(b) coupled electrostatic exciton traps

(c) electric control of the exciton phase

(d) double-slit experiment (time-correlated photon counting)

(e) one-shot vs average measurement

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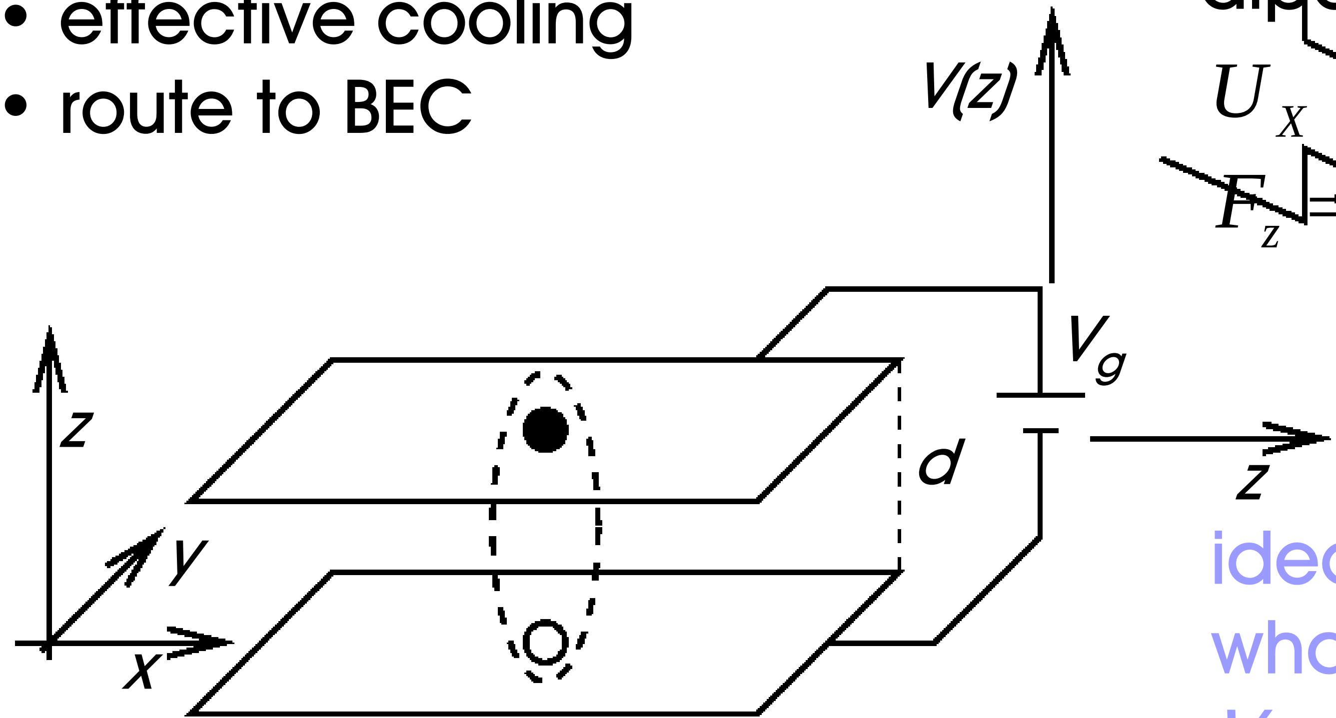
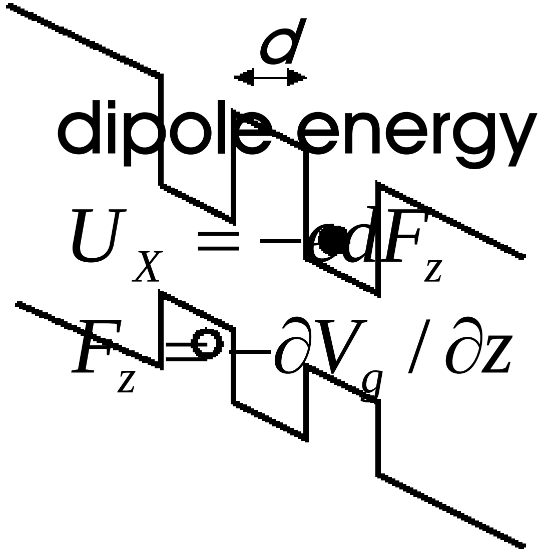
indirect excitons in coupled quantum wells

- optically pumped excitons in coupled QWs
- very long lifetime
- effective cooling
- route to BEC





dipole energy:

$$U_X = -edF_z$$

$$F_z = -\partial V_g / \partial z$$

idea:
what if
$V_g = V_g(x,y)$?

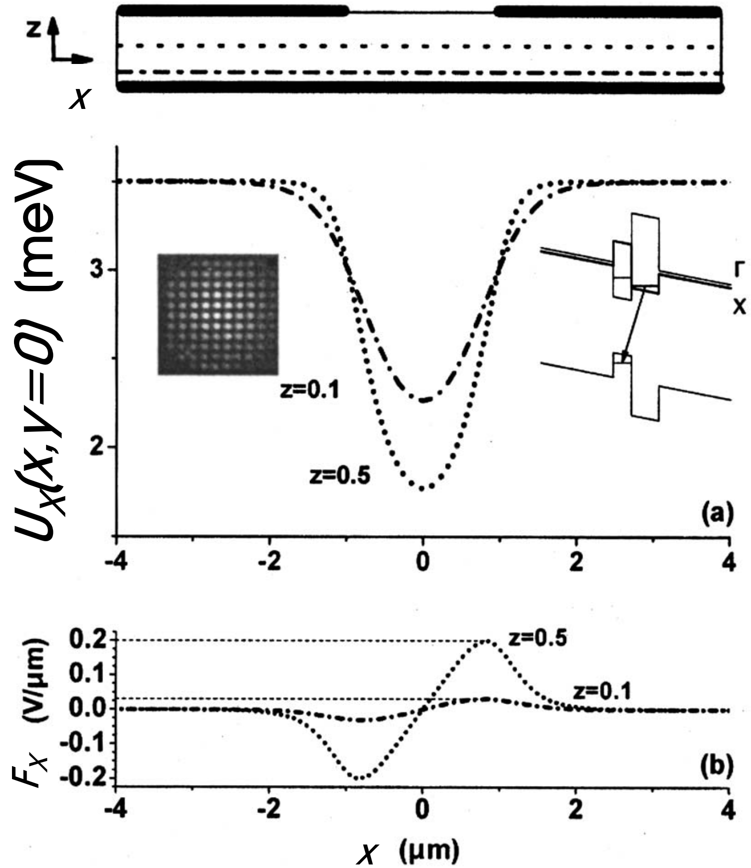
in-plane electrostatic traps

effective potential for excitons:

$U_X(x,y) = - edF_z(x,y)$

S. Zimmermann *et al.*,
PRB **56**, 13414 (1997).

advantages:
trap design
in situ manipulation



Hammack *et al.*,
JAP **99**, 066104 (2006)

Chen *et al.*, PRB **74**, 045309 (2006);
High *et al.*, Science **321**, 229 (2008)

in-plane electrostatic traps

effective potential for excitons:

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S. Zimmermann *et al.*,
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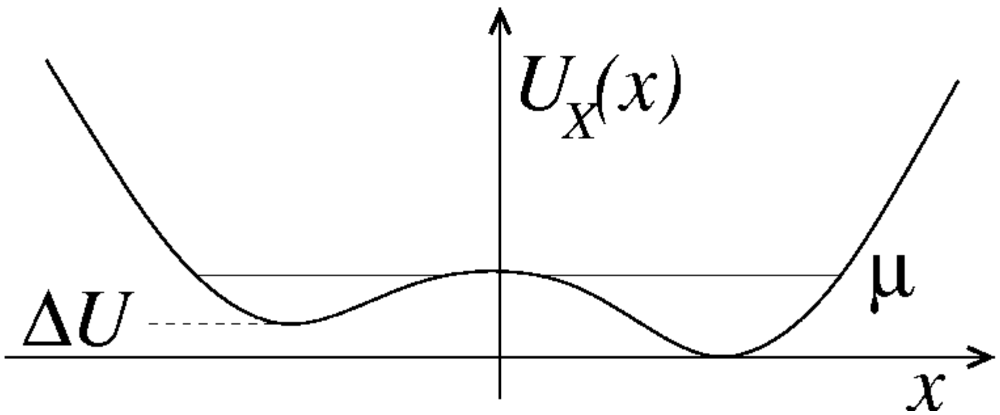

advantages:
trap design
in situ manipulation

coupled exciton traps!

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electrical control of the exciton phase

assume BEC

$$\Xi(x,y) = \left\langle \Psi_v^\dagger(x,y,0)\Psi_c(x,y,d) \right\rangle$$

$$\Xi(x,y) = \sqrt{n_s}\exp(i\varphi)$$

gauge transformation:

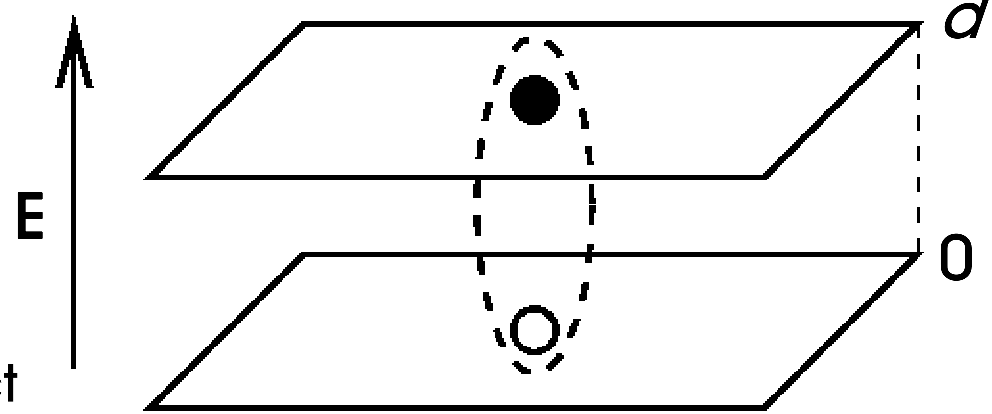
$$V_g \to V_g - c^{-1}\partial\chi(t)/\partial t$$

$$\Psi_v \to \Psi_v \exp[ie\chi(x,y,0)/\hbar c]$$

$$\Psi_c \to \Psi_c \exp[ie\chi(x,y,d)/\hbar c]$$

$$\varphi \to \varphi + e[\chi(x,y,d) - \chi(x,y,0)]/\hbar c$$

e-h phases
not compensated



similar ideas:
Balatsky *et al.*, PRL 2004,
optical Aharonov-Bohm effect

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electrical control of the exciton phase

assume BEC

$$\Xi(x,y) = \left\langle \Psi_v^\dagger(x,y,0)\Psi_c(x,y,d) \right\rangle \qquad \Xi(x,y) = \sqrt{n_s}\exp(i\varphi)$$

gauge invariance requirement

$-U_X = edF_z$

$$\varphi = \varphi^{(0)} + edF_z t/\hbar \qquad \dot{\varphi} = -\mu/\hbar$$

the phase depends on time

$$\mathbf{J}_s = \frac{\hbar}{m} n_s \left(\nabla_{x,y}\varphi - ed\nabla_{x,y} F_z t/\hbar \right)$$

exciton supercurrent

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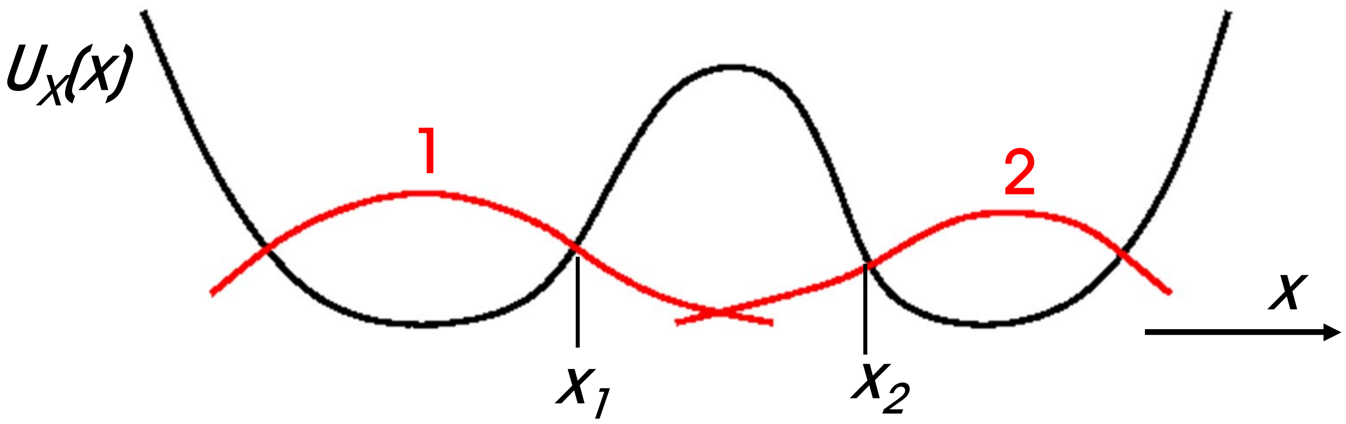
Rontani & Sham PRB **80**, 075309 (2009)

weak link between two traps



Ginzburg-Landau energy functional
boundary conditions for the overlapping case

$$\mathbf{J}_s = \frac{\hbar}{\lambda m} n_s \sin\phi$$

$$\phi = \phi^{(0)} - \frac{t}{\hbar}\left[U_X(x_2) - U_X(x_1)\right]$$

$$\phi = \varphi_1 - \varphi_2$$

λ penetration depth
(evaluated via
Gross-Pitaevskii eq.)

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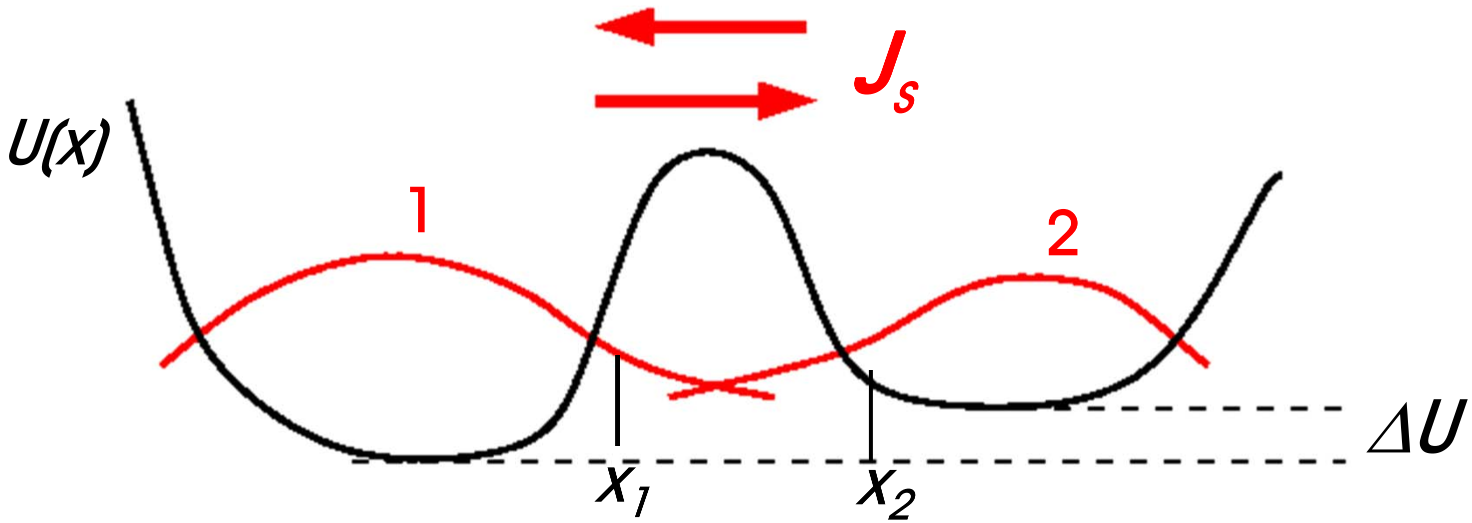
weak link between two traps



$$\mathbf{J}_s = \frac{\hbar}{\lambda m} n_s \sin\phi$$

$$\phi = \phi^{(0)} - \frac{t}{\hbar} \ldots$$

...son oscillation

...n by ΔU

(1) neglecting "charging" effec

(2) how to measure?

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measurement

classical oscillating electric dipole moment $\langle \mathbf{P}(t) \rangle \neq 0$

Ostreich *et al.* 1996, Fernandez-Rossier *et al.* 1998, Olaya-Castro *et al.* 2001.

$$\mathbf{P}(t) = \int \Xi(x,y,t)\left[\mathbf{x} + i\mathbf{y}\right] dx dy$$

$$\Xi(x,y) = \left\langle \Psi_v^\dagger(x,y,0)\Psi_c(x,y,d) \right\rangle$$

$\Xi(x,y) = 0$ normal phase

$$\mathbf{P}(t) = \mathbf{P}\exp[-i(\mu + E_X)t/\hbar]$$

macroscopic noiseless current

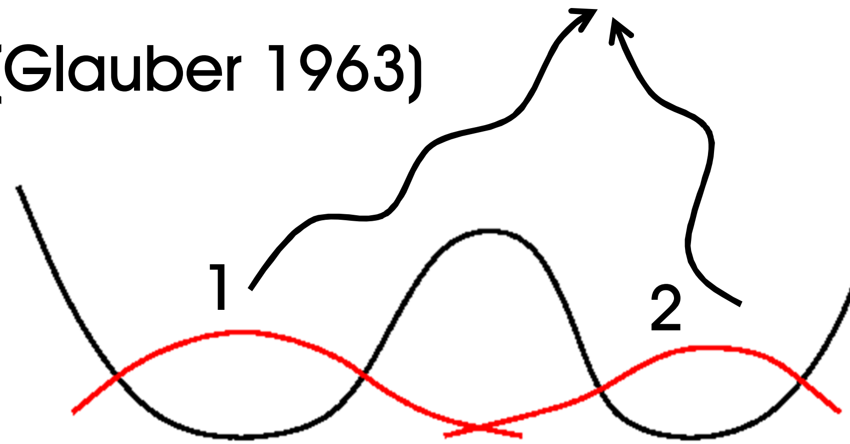
μ = chemical potential

E_X = energy gap

coherent radiation field (Glauber 1963)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

double-slit experiment ?



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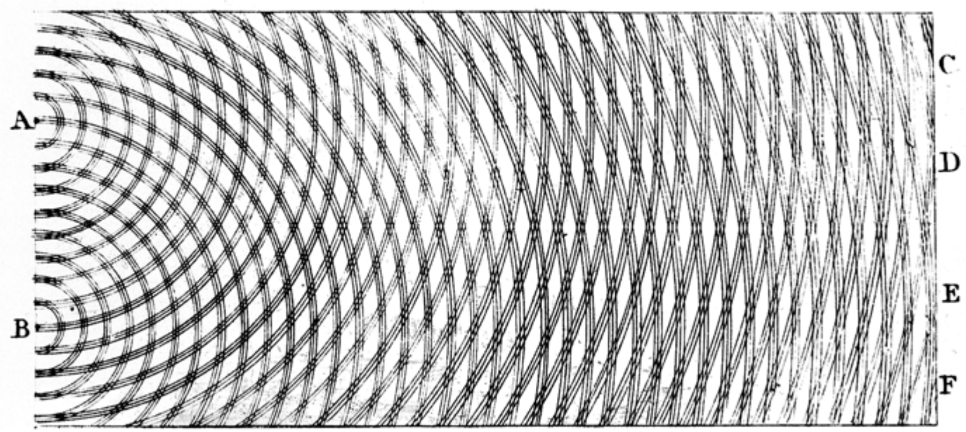
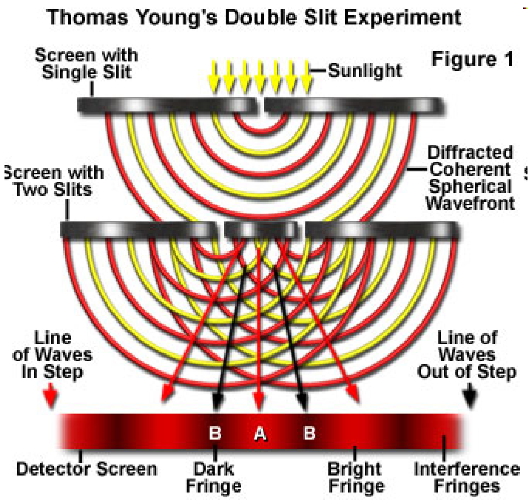
double-slit experiment

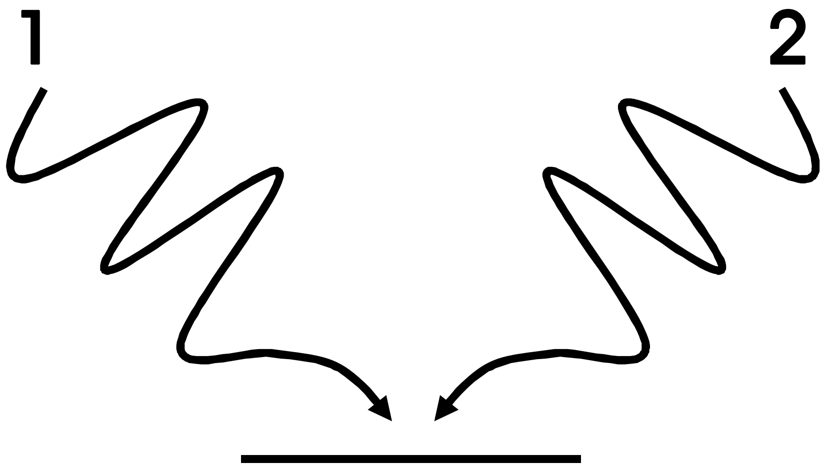


two flavors

"classic"

two independent lasers





locked relative phase
average measure

random relative phase
one-shot measure

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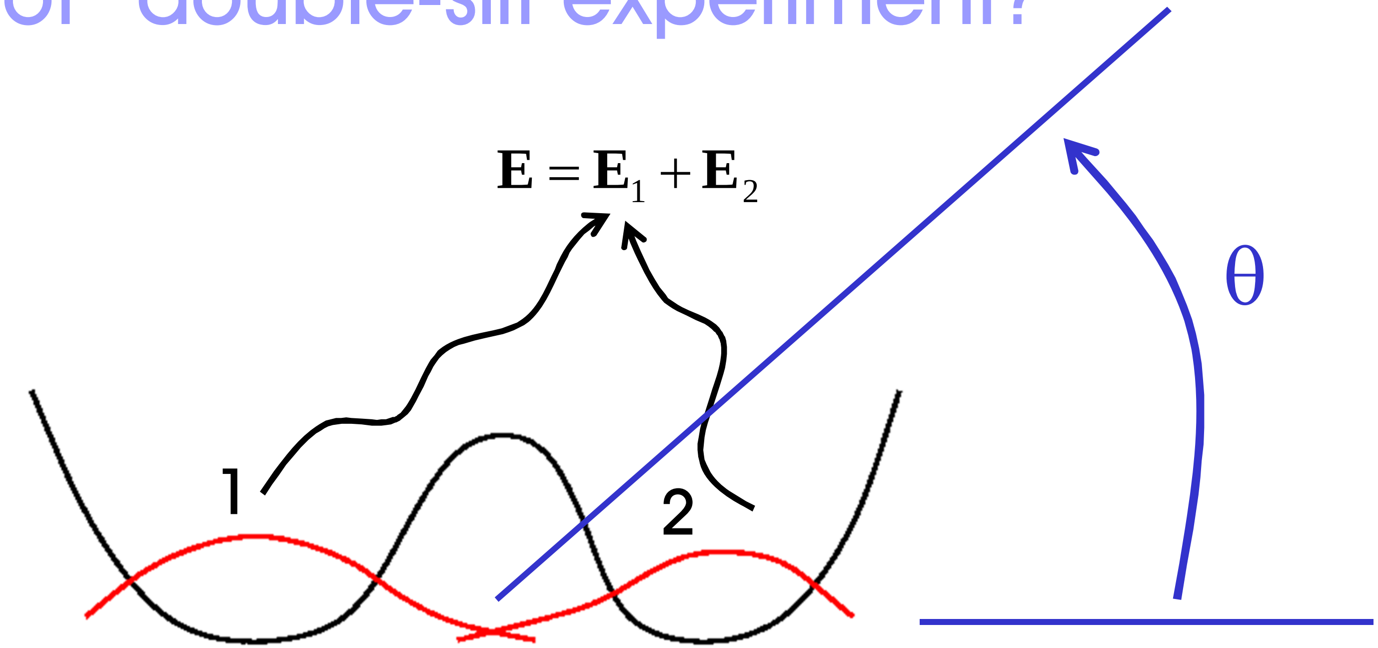
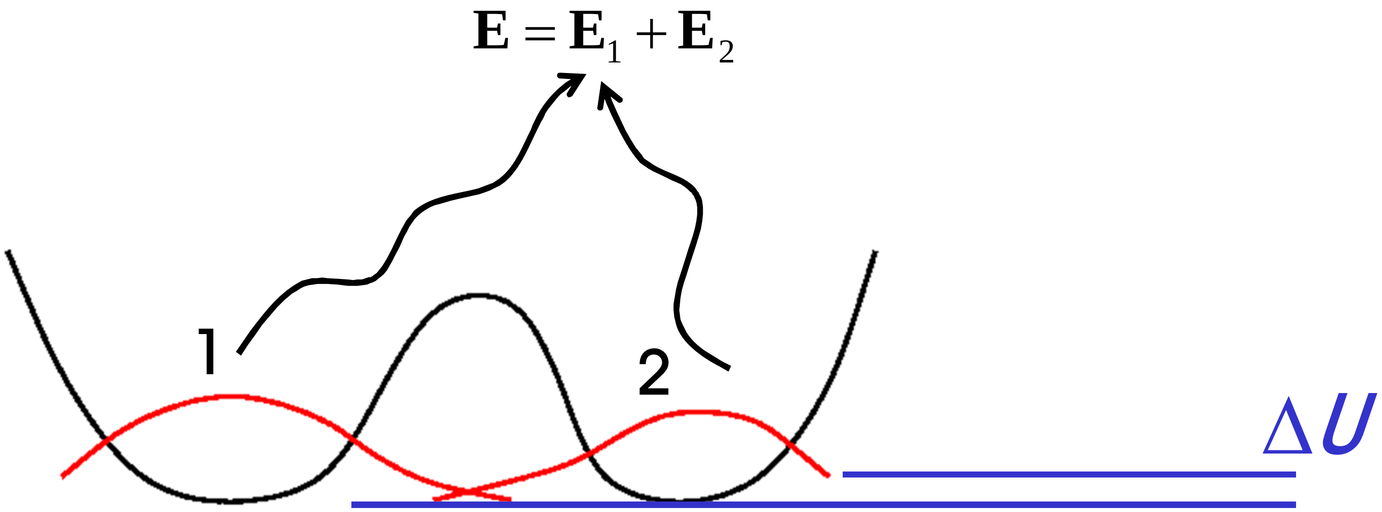
a “one-shot” double-slit experiment?



measure interference fringes vs

angle θ

a “one-shot” double-slit experiment?



measure interference fringes vs

angle θ

time at fixed θ and ΔU

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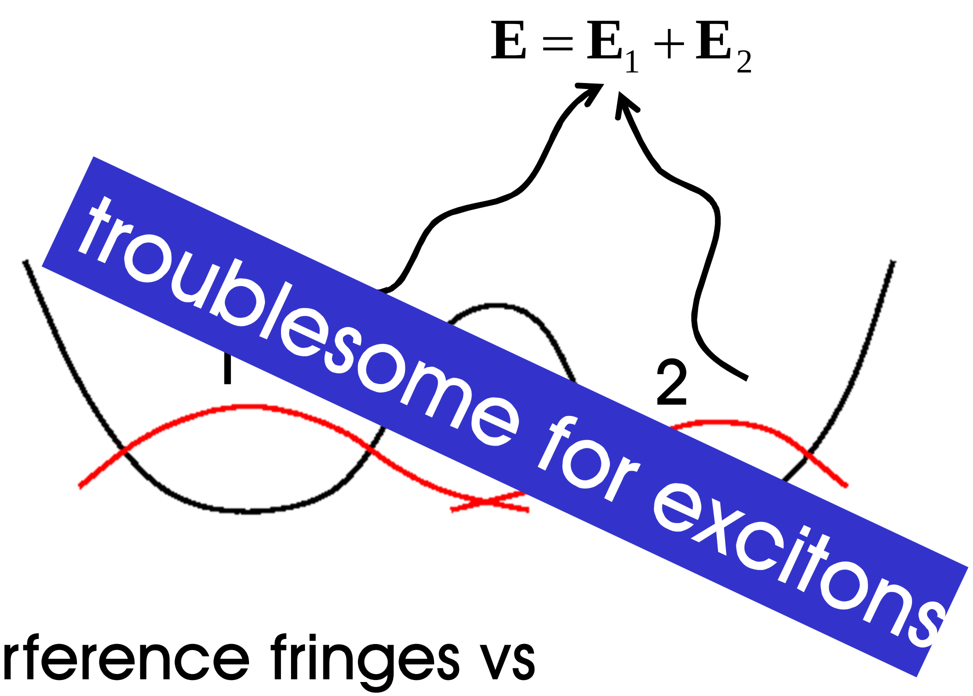
a “one-shot” double-slit experiment?



measure interference fringes vs

angle θ

time at fixed θ and ΔU

polariton condensates

Wouters & Carusotto PRL **99**, 140402 (2007)
Sarchi *et al.* PRB **77**, 125324 (2008)
Shelykh *et al.* PRB **78**, 041302(R) (2008)

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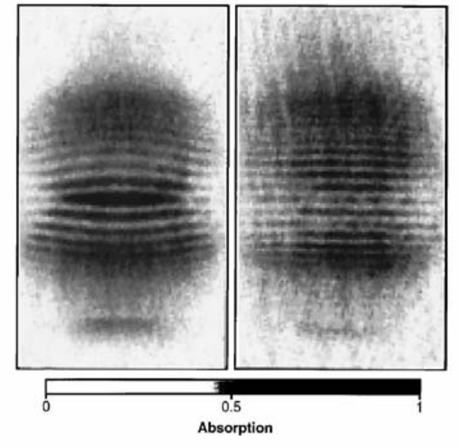
caveats about one-shot measurements

interference even between decoupled traps

independent laser sources (Glauber 1963)

cold atom traps (Andrews *et al.* 1997)

Andrews *et al.* Science 1997

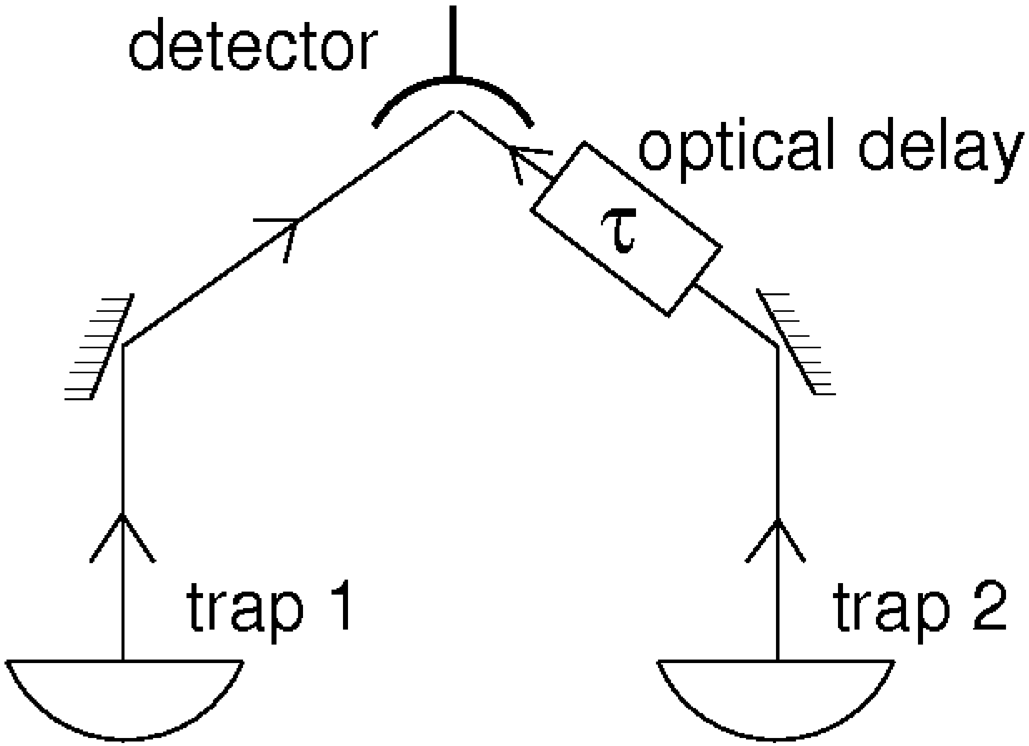


signal-to-noise ratio too much low

exciton recombination = dephasing

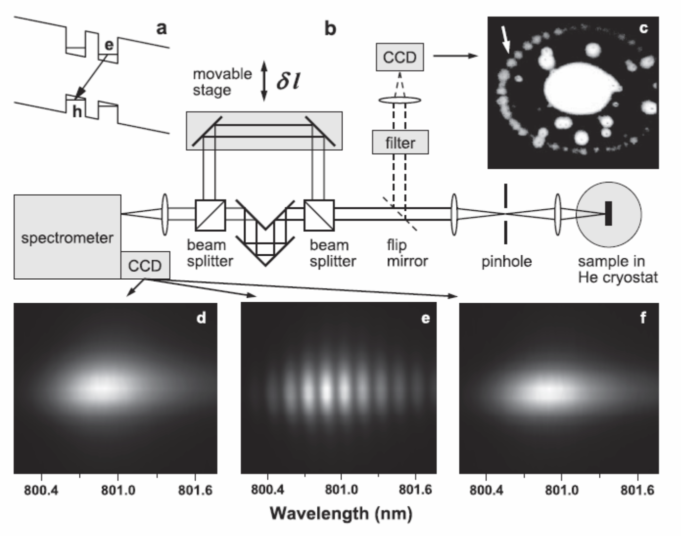
average over experiment replica

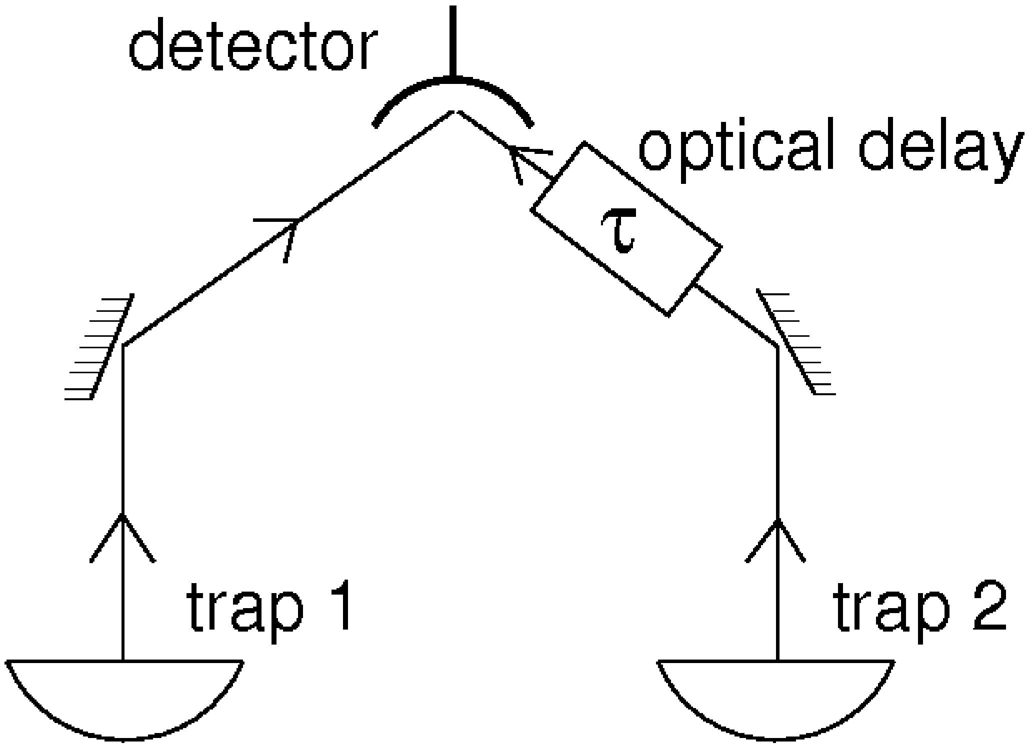
time-correlated photon counting



time-correlated photon counting

Yang *et al.*, PRL 97, 187402 (2006)

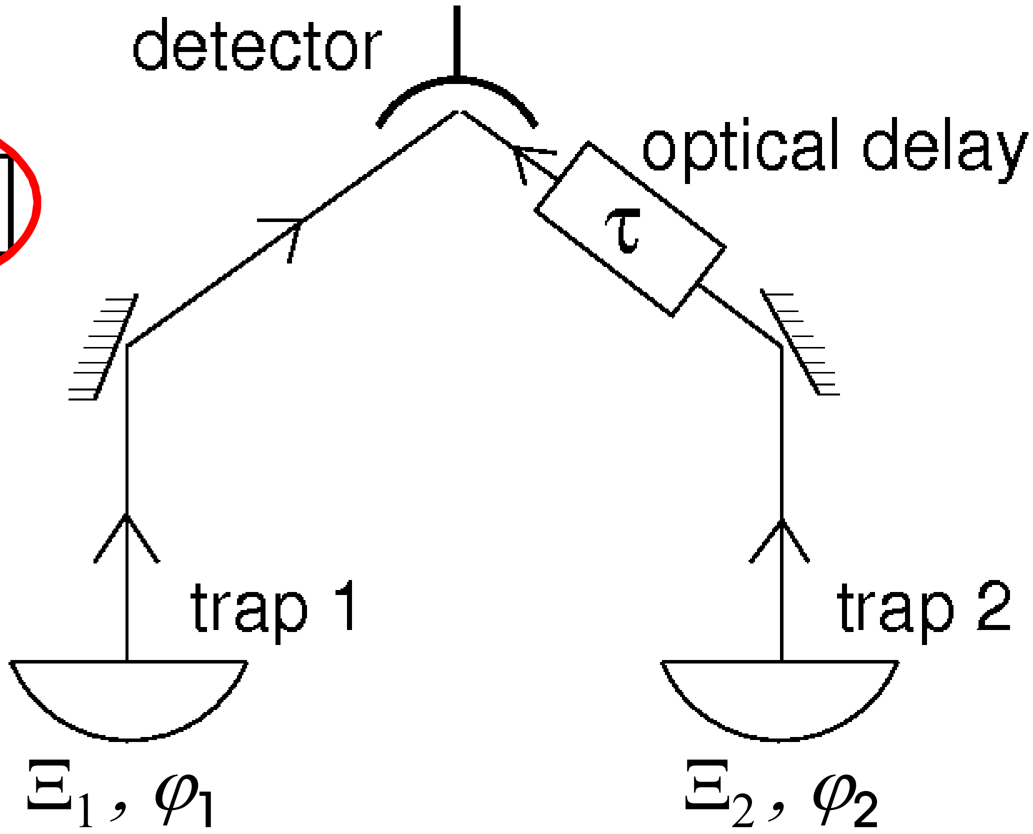




time-correlated photon counting

$$I(\tau) = 2I_0\left[1 + \langle \cos\phi(\tau) \rangle\right]$$

quantum statistical ensemble average



$$\phi = \varphi_1 - \varphi_2$$

$$\Xi_1 \sim \Xi_2$$

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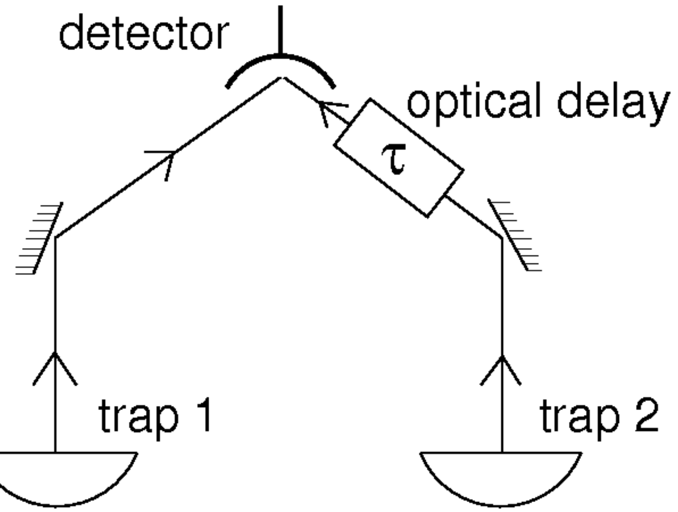
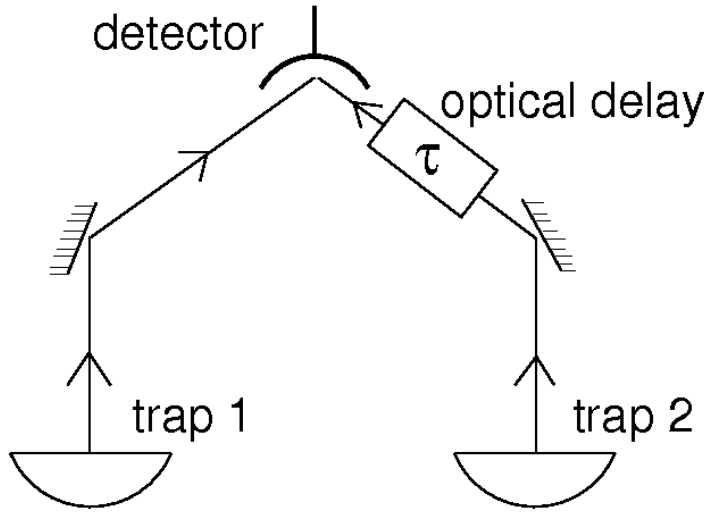
goal: compute $I(\tau)$

$$I(\tau) = 2I_0\left[1 + \langle \cos\phi(\tau) \rangle\right]$$



goal: compute $I(\tau)$



$$I(\tau) = 2I_0\left[1 + \alpha\cos\phi_0(\tau)\right]$$

α fringe visibility

$$\alpha = \left\langle \cos\left[\phi(\tau) - \phi_0(\tau)\right]\right\rangle$$

$$\alpha = \left(I_{\max} - I_{\min}\right)/\left(I_{\max} + I_{\min}\right)$$

$$0 \leq \alpha \leq 1$$

$\phi_0(\tau)$ average phase

$$\left\langle \sin\left[\phi(\tau) - \phi_0(\tau)\right]\right\rangle = 0$$

dephasing mechanisms → $\alpha < 1$

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extrinsic dephasing mechanisms

- exciton recombination
- exciton-phonon scattering
- exciton-exciton scattering

negligible at:

short τ

low T

low density $na_B^2 << 1$

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two-mode approximation (Smerzi *et al.* 1997)

$$\Xi(x,y,t)=\Xi_1(x,y,N_1)\exp(i\varphi_1)+\Xi_2(x,y,N_2)\exp(i\varphi_2)$$

$$\left.\begin{aligned} k &= (N_1-N_2)/2 \\ \phi &= \varphi_1-\varphi_2 \end{aligned}\right\}$$ canonically conjugated

$$H=E_c\frac{k^2}{2}-\frac{\delta_J}{2}\sqrt{N^2-4k^2}\cos\phi+\Delta Uk$$

$$\dot{\phi}=-\frac{\partial H}{\partial(\hbar k)} \qquad \hbar\dot{k}=\frac{\partial H}{\partial\phi}$$

$k = 0$ in superconductors

δ_J = single-exciton tunneling energy

$E_c = 2\, d\mu_1/dN_1$ = exciton "charging" energy $\qquad N = N_1 + N_2$

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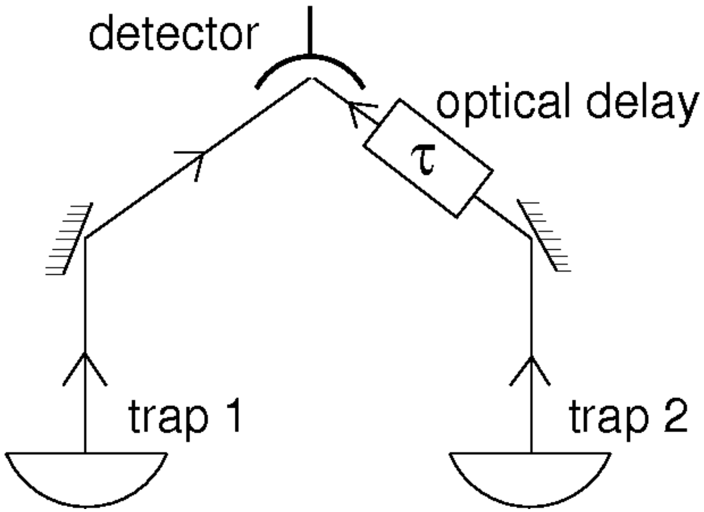
intrinsic dephasing: quantum fluctuations

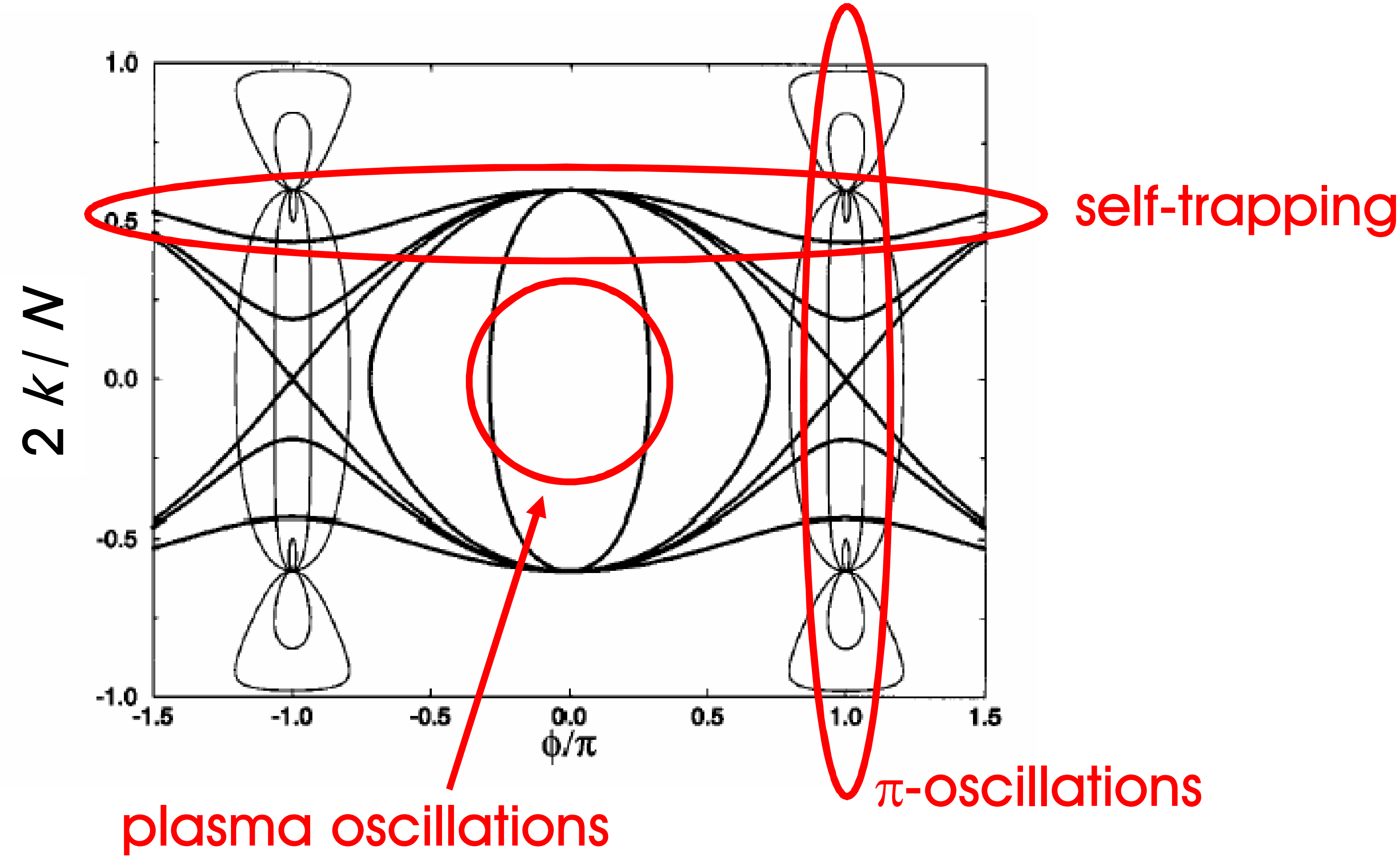
ϕ , *k* "classical" GP order parameters → $\alpha = 1$

ϕ , *k* conjugated quantum variables → $\alpha < 1$

$$I(\tau) = 2I_0\left[1 + \alpha\cos\phi_0(\tau)\right]$$



focus on exciton "plasma" oscillations



Smerzi *et al.*, PRL 79, 4950 (1997)

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exciton plasma oscillations

small oscillations

$$H_{\mathrm{HO}} \approx \frac{k^2}{2}\left(2\frac{\delta_J}{N} + E_C\right) + \frac{1}{4}\delta_J N \phi^2 + \Delta U k - \frac{\delta_J N}{2}$$

$$\left.\begin{array}{l} k = (N_1 - N_2)/2 \\ \phi = \varphi_1 - \varphi_2 \end{array}\right\}$$ canonically conjugated

δ_J = single-exciton tunneling energy

$E_c = 2\, d\mu_1/dN_1$ = exciton "charging" energy $\quad N = N_1 + N_2$

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exciton “plasma” oscillations

small oscillations

$$\omega_J = \hbar^{-1}\left[\delta_J\left(NE_c/2+\delta_J\right)\right]^{1/2}$$ plasma frequency

$$\left(k_0,\phi_0\right)=\left(-\Delta UN\delta_J/(\hbar\omega_J)^2,0\right)$$ equilibrium position

$$\left.\begin{array}{l} k=(N_1-N_2)/2 \\ \phi=\varphi_1-\varphi_2 \end{array}\right\}$$ canonically conjugated

δ_J = single-exciton tunneling energy

$E_c = 2\, d\mu_1/dN_1$ = exciton “charging” energy $N = N_1 + N_2$

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quantize H_{HO}

$$k \to -i\partial / \partial\phi$$

$$\hat{H}_{\text{HO}} = -\frac{1}{2}\left(2\frac{\delta_J}{N} + E_C\right)\frac{\partial^2}{\partial\phi^2} + \frac{1}{4}\delta_J N\phi^2 + \Delta U \frac{\partial}{i\partial\phi} - \frac{\delta_J N}{2}$$

wave functions: periodical functions of ϕ

ground state = coherent state

$$\alpha = 1 - \frac{1}{2}\left\langle (\Delta\phi)^2 \right\rangle \neq 0$$

$$\phi_0(\tau) \neq 0$$

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intrinsic dephasing: quantum fluctuations

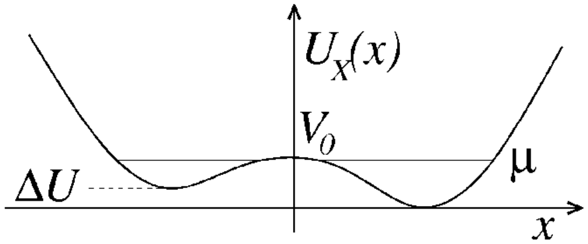
$$\alpha = 1 - \left(\frac{E_C}{8\delta_J N} \right)^{1/2}$$

tunneling ($\delta_J N$) favors coherence

E_c favors number states

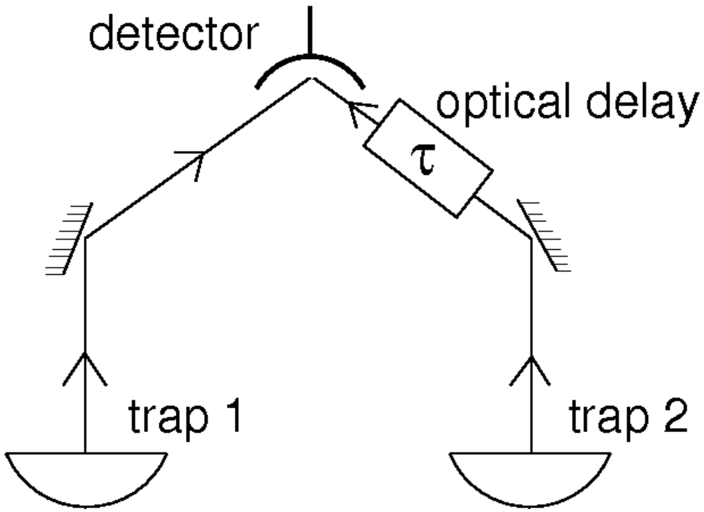
$$\phi_0(\tau) = -\frac{\Delta U}{\hbar\omega_J} \sin(\omega_J \tau)$$

τ-dependence via ΔU



Rontani & Sham, PRB **80**, 075309 (2009)

$$I(\tau) = 2I_0 \left[1 + \alpha \cos\phi_0(\tau) \right]$$



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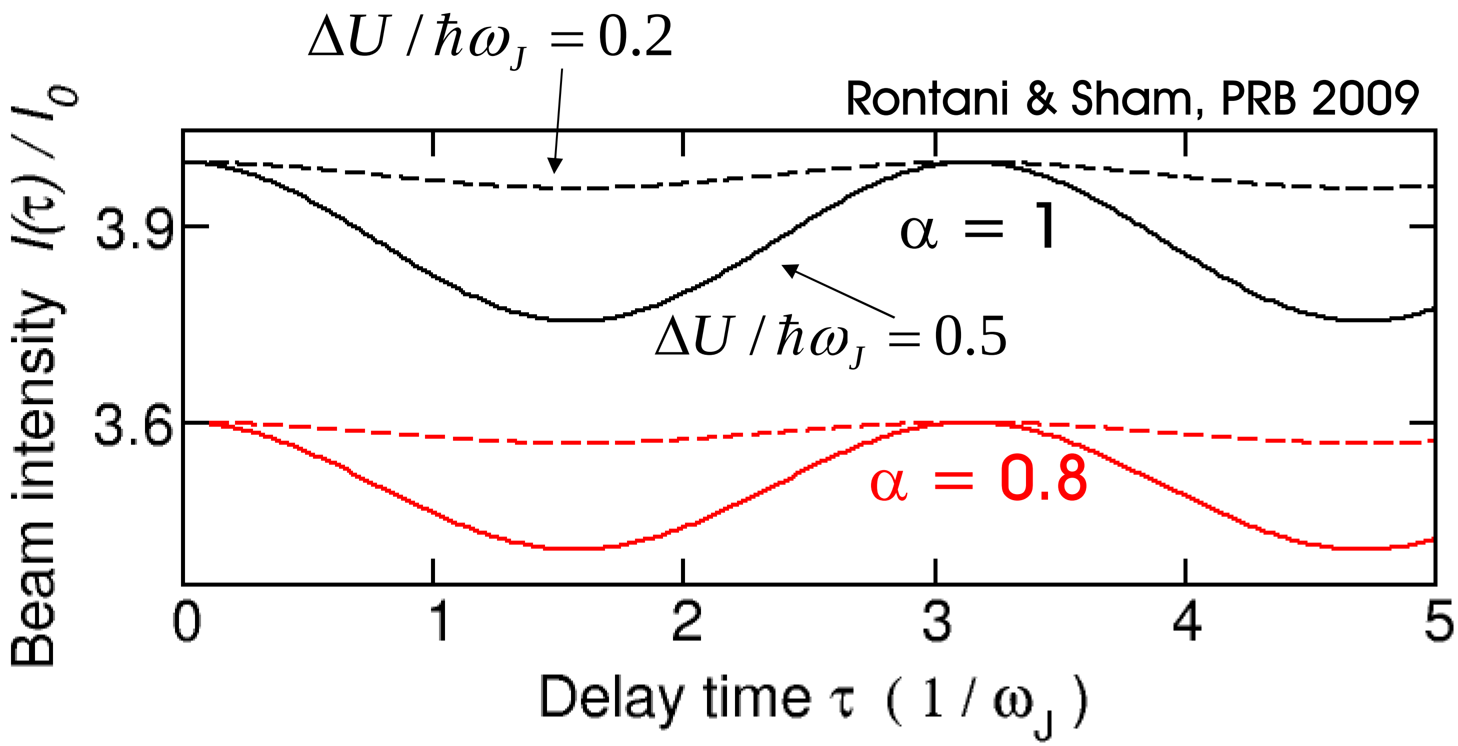
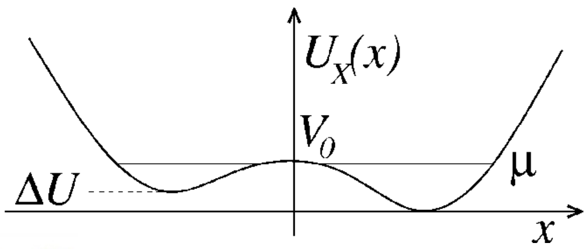
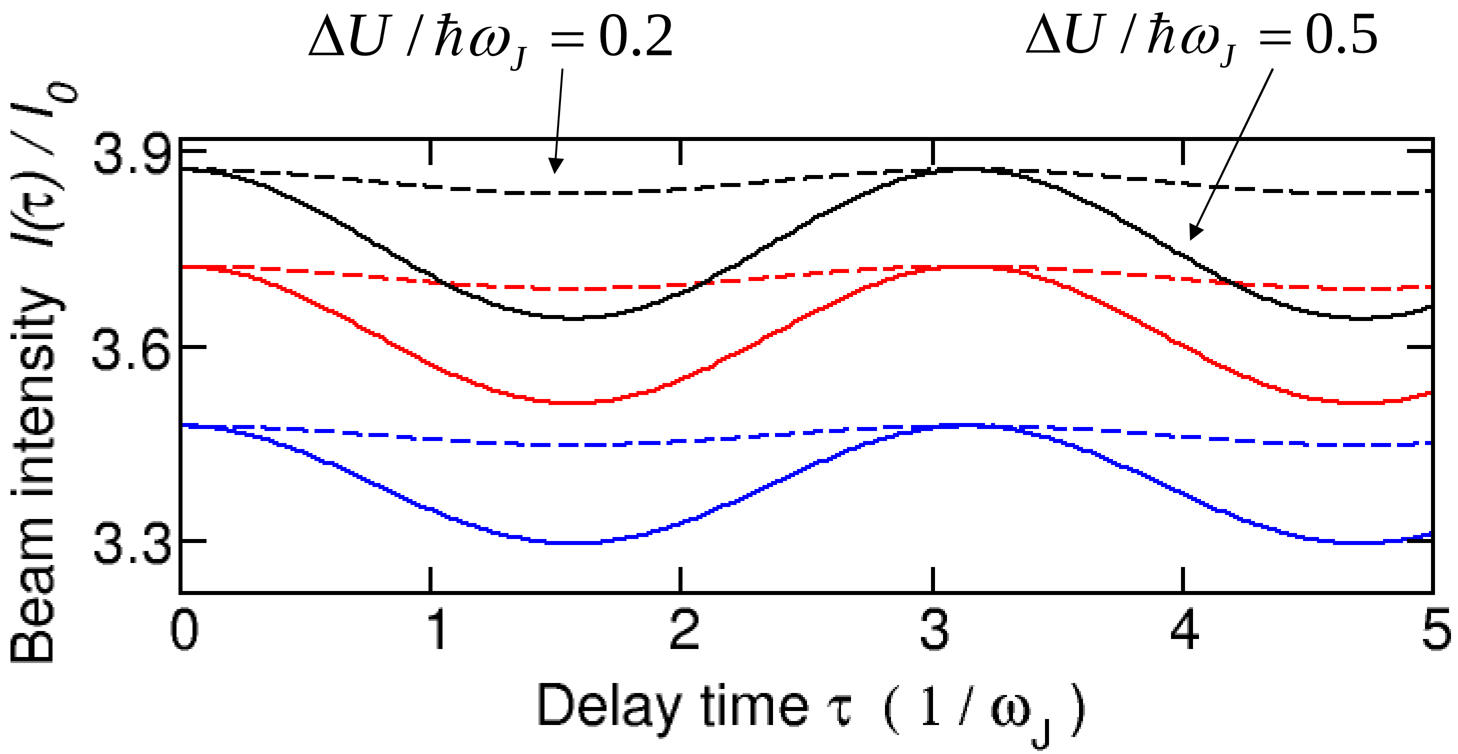
dipole energy ΔU crucial for detection





$$\left(\frac{\Delta U}{\hbar\omega_J}\right)^2$$ modulates visibility

thermal smearing



$k_B T / \hbar\omega_J = 0$

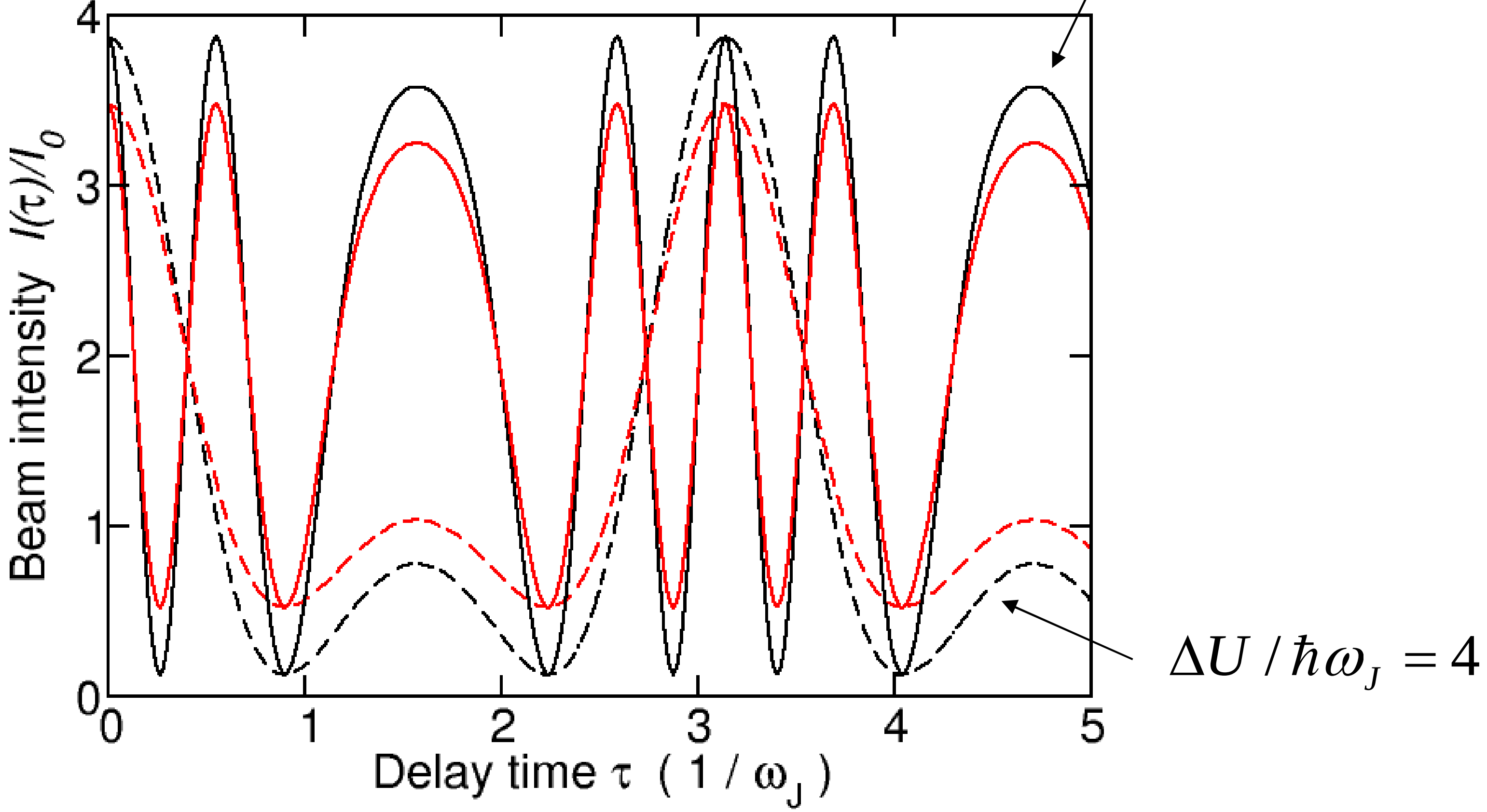
$\alpha(T=0) = 0.94$

$k_B T / \hbar\omega_J = 1$

$k_B T / \hbar\omega_J = 2$

Rontani & Sham, PRB 80, 075309 (2009)

large amplitudes



$k_B T / \hbar\omega_J = 0$

$k_B T / \hbar\omega_J = 2$

$\alpha(T=0) = 0.94$

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estimate ω_J

Thomas-Fermi approximation for 2D traps

↓

compute E_c and δ_J ⇒ compute α and ω_J

N_1 = 1000 $\quad n = 2.5\times 10^{10}\,\mathrm{cm}^{-2} \quad na_B^2 << 1$

α = 0.94 $\quad 2\pi / \omega_J$ = 2 ns << exciton lifetime

$$\hbar\omega_J / k_B = 20\mathrm{mK}$$

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Rontani & Sham, PRB **80**, 075309 (2009)

conclusions

- exciton Josephson effect
- plasma oscillations observable in ensemble measurements
- correlated photon counting setup
- ΔU handle for detection

M. Rontani and L. J. Sham, PRB **80**, 075309 (2009)

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