Multiple–Scattering Phenomena in the Scattering of Light from Randomly Rough Surfaces

A. A. Maradudin

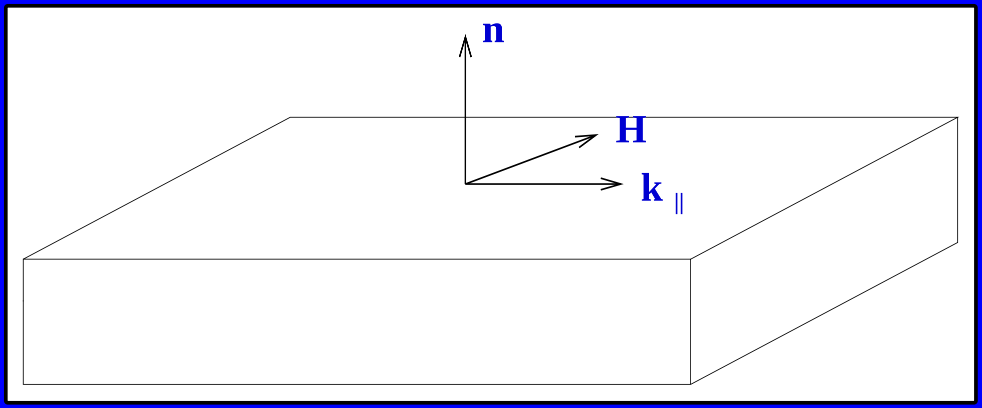
Department of Physics and Astronomy
and Institute for Surface and Interface Science
University of California, Irvine, CA 92697, USA

Collaborators

- Vittorio Celli - University of Virginia
- Valentin Freilikher - Bar–Ilan University, Ramat–Gan, Israel
- Zu–Han Gu - Surface Optics Corporation, San Diego, California
- Tamara A. Leskova - Institute of Spectroscopy, Troitsk, Russia
- Jun Q. Lu - University of California, Irvine
- Arthur R. McGurn - Western Michigan University
- Eugenio R. Méndez - CICESE, Ensenada, B.C., México
- Thierry R. Michel - University of California, Irvine
- Jose A. Sánchez–Gil - Instituto de Estructura de la Materia, Madrid, Spain
- Andrei V. Shchegrov - University of California, Irvine
- Phuc Tran - Naval Air Warfare Center, China lake, California

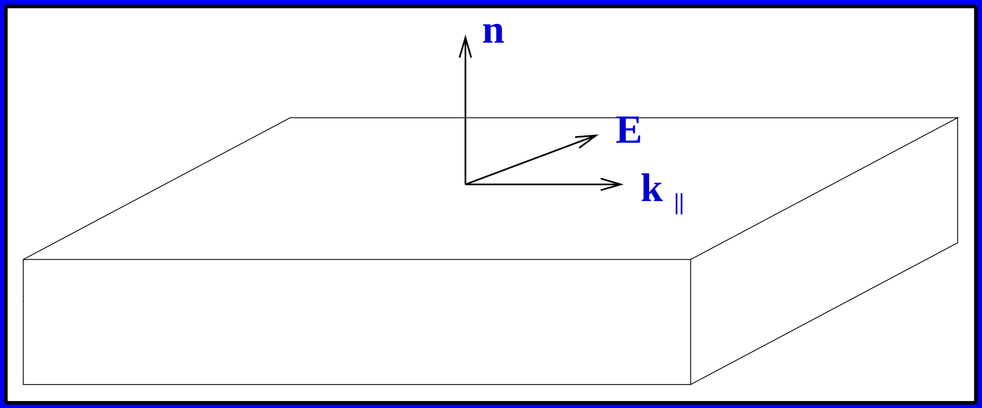
1. *Introduction*
2. *Scattering from One–Dimensional Random Surfaces*
    - 2.A. *Small–Amplitude Perturbation Theory*
    - 2.B. *Many–Body Perturbation Theory*
    - 2.C. *Computer Simulations*
3. *Computer Simulations of the Scattering of Electromagnetic Waves from a Two–Dimensional Random Surface*
4. *Angular Intensity Correlation Functions*
5. *Second Harmonic Generation in Reflection from a Randomly Rough Metal Surface*
6. *Some Directions for Future Research*

1. Introduction



P polarization

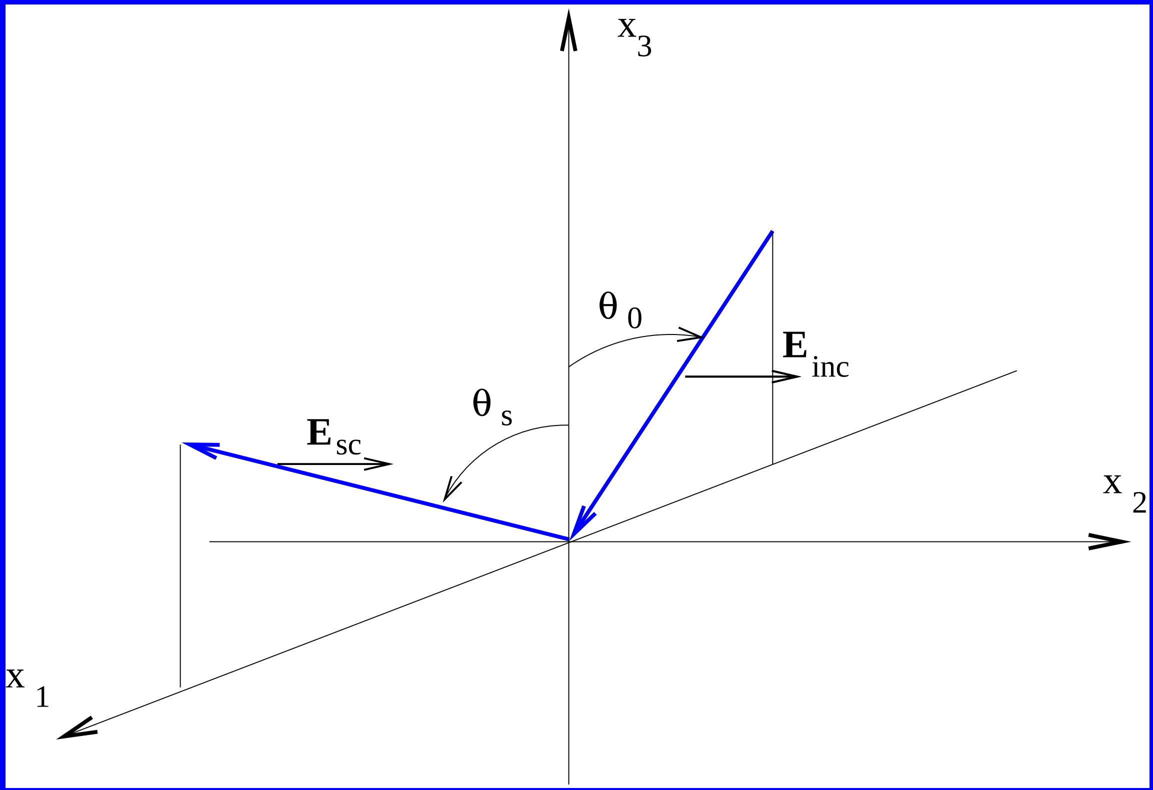
Transverse **M**agnetic polarization



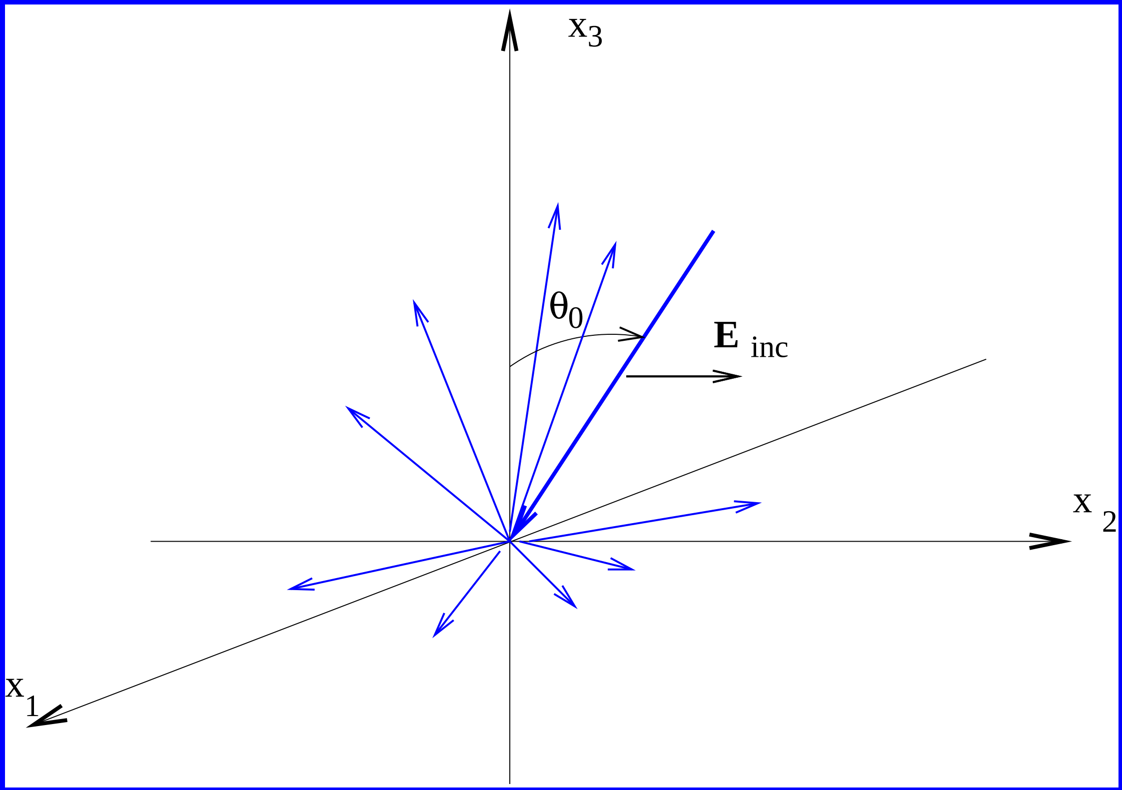
S polarization

Transverse **E**lectric polarization

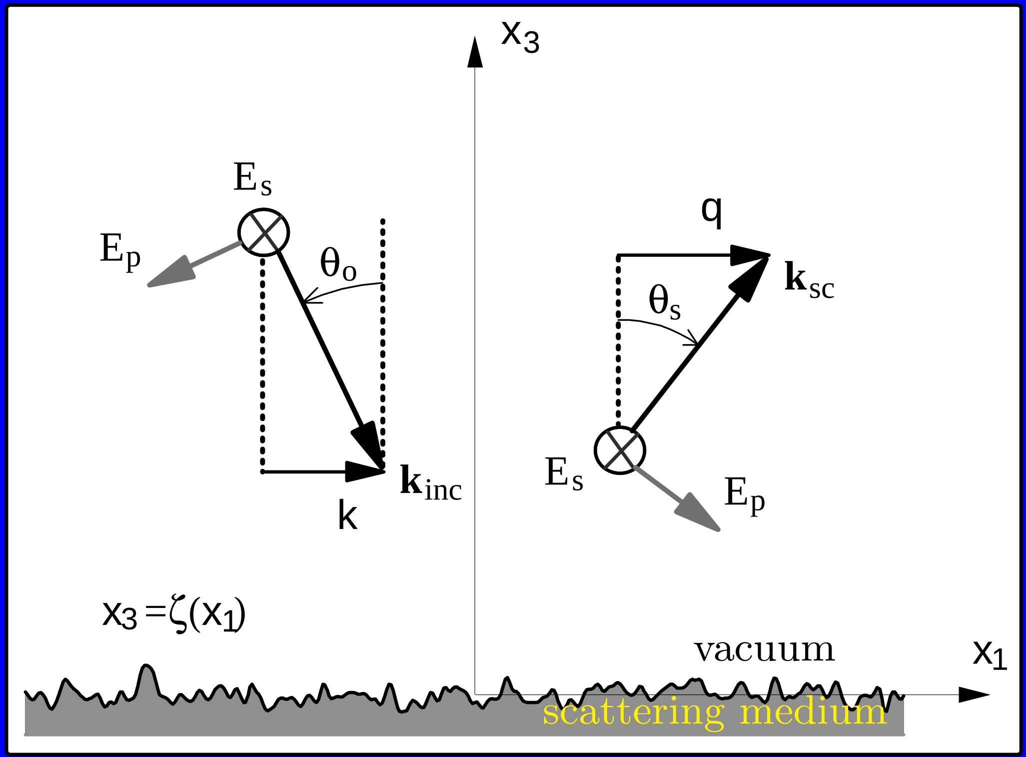
Coherent (Specular) Scattering



Incoherent (Diffuse) Scattering



2. A One–Dimensional Random Surface



The surface profile function $\zeta(x_1)$ is a single–valued function of x_1 that is differentiable as many times as is necessary, and constitutes a stationary, zero-mean, Gaussian random process, defined by

$$\begin{aligned}
\langle \zeta(x_1) \rangle &= 0 \\
\langle \zeta(x_1)\zeta(x_1') \rangle &= \delta^2 W(|x_1 - x_1'|) \\
\delta &= \langle \zeta^2(x_1) \rangle^{\frac{1}{2}}.
\end{aligned}$$

We introduce the Fourier representation

$$\zeta(x_1) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \widehat{\zeta}(q) e^{iqx_1},$$

where

$$\langle \widehat{\zeta}(q) \rangle = 0$$

$$\langle \widehat{\zeta}(q) \widehat{\zeta}(q') \rangle = 2\pi\delta(q+q')\delta^2 g(|q|).$$

$g(|q|)$ is the *power spectrum* of the surface roughness,

$$g(|q|) = \int_{-\infty}^{\infty} dx_1 W(|x_1|) e^{-iqx_1}.$$

A Zero–Mean, Stationary, Gaussian Random Process

All odd–order correlation functions vanish. For example,

$$\langle \zeta(x_1)\zeta(x_1')\zeta(x_1'')\rangle = 0.$$

Even-order correlation functions are sums of products of two-point correlation functions in all possible independent combinations. For example,

$$\begin{aligned}\langle \zeta(x_1)\zeta(x_1')\zeta(x_1'')\zeta(x_1''')\rangle &= \langle \zeta(x_1)\zeta(x_1')\rangle\langle \zeta(x_1'')\zeta(x_1''')\rangle \\ &+ \langle \zeta(x_1)\zeta(x_1'')\rangle\langle \zeta(x_1')\zeta(x_1''')\rangle \\ &+ \langle \zeta(x_1)\zeta(x_1''')\rangle\langle \zeta(x_1')\zeta(x_1'')\rangle\end{aligned}$$

where each contraction is given by

$$\langle \zeta(x_1)\zeta(x_1')\rangle = \delta^2 W(|x_1 - x_1'|).$$

Similarly,

$$\langle\widehat{\zeta}(q)\widehat{\zeta}(q')\widehat{\zeta}(q'')\rangle = 0, \text{etc.}$$

and

$$\begin{aligned}
\langle\widehat{\zeta}(q)\widehat{\zeta}(q')\widehat{\zeta}(q'')\widehat{\zeta}(q''')\rangle &= \langle\widehat{\zeta}(q)\widehat{\zeta}(q')\rangle\langle\widehat{\zeta}(q'')\widehat{\zeta}(q''')\rangle \\
&+ \langle\widehat{\zeta}(q)\widehat{\zeta}(q'')\rangle\langle\widehat{\zeta}(q')\widehat{\zeta}(q''')\rangle \\
&+ \langle\widehat{\zeta}(q)\widehat{\zeta}(q''')\rangle\langle\widehat{\zeta}(q')\widehat{\zeta}(q'')\rangle,
\end{aligned}$$

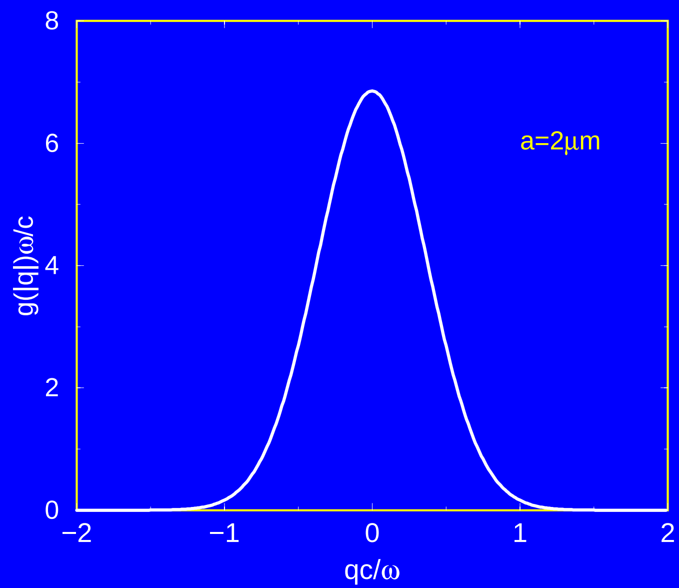
where

$$\langle\widehat{\zeta}(q)\widehat{\zeta}(q')\rangle = 2\pi\delta(q+q')\delta^2 g(|q|).$$

A commonly used power spectrum is the Gaussian power spectrum

$$g(|q|) = \sqrt{\pi} a e^{-a^2 q^2/4}.$$

a is called the transverse correlation length of the surface roughness.



This power spectrum corresponds to a surface height autocorrelation function

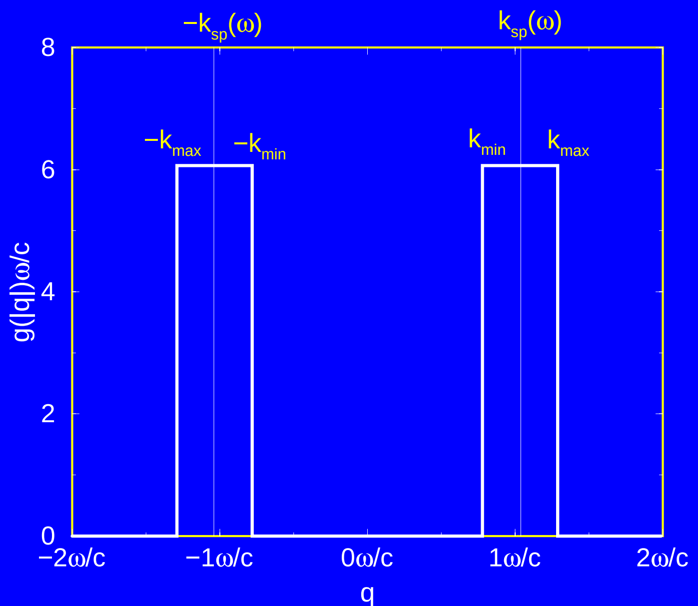
$$\boxed{W(|x_1|) = e^{-x_1^2/a^2}.}$$

A second power spectrum that we will use is the West–O'Donnell power spectrum

$$g(|q|) = \frac{\pi}{k_{max} - k_{min}} [\Theta(q - k_{min})\Theta(k_{max} - q) + \Theta(-q - k_{min})\Theta(k_{max} + q)],$$

where $k_{min} = k_{sp}(\omega) - \frac{\omega}{c} \sin\theta_{max}, \quad k_{max} = k_{sp}(\omega) + \frac{\omega}{c} \sin\theta_{max}$

$$k_{sp}(\omega) = \frac{\omega}{c} \mathrm{Re} \left[\frac{\epsilon(\omega)}{\epsilon(\omega)+1} \right]^{1/2} > \frac{\omega}{c}.$$



This power spectrum corresponds to a surface height autocorrelation function

$$\boxed{W(|x_1|) = \frac{1}{k_{max} - k_{min}} \left[\frac{\sin k_{max} x_1}{x_1} - \frac{\sin k_{min} x_1}{x_1} \right].}$$

P Polarization

$x_3 > \zeta(x_1)_{max}$

$$H_2^>(x_1, x_3|\omega) = e^{ikx_1 - i\alpha_0(k,\omega)x_3} + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k) e^{iqx_1 + i\alpha_0(q,\omega)x_3}$$

where

$$\begin{aligned} \alpha_0(q,\omega) &= \left[\frac{\omega^2}{c^2} - q^2\right]^{1/2} \qquad |q| < \omega/c \\ &= i\left[q^2 - \frac{\omega^2}{c^2}\right]^{1/2} \qquad |q| > \omega/c. \end{aligned}$$

$x_3 < \zeta(x_1)_{min}$

$$H_2^<(x_1, x_3|\omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} T(q|k) e^{iqx_1 - i\alpha(q,\omega)x_3},$$

where

$$\alpha(q,\omega) = \left[\epsilon(\omega)\frac{\omega^2}{c^2} - q^2\right]^{1/2} \mathrm{Re}\alpha(q,\omega) > 0, \mathrm{Im}\alpha(q,\omega) > 0.$$

Differential Reflection Coefficient $\langle \partial R / \partial \theta_s \rangle$:

$$\frac{\partial R}{\partial \theta_s} d\theta_s \quad = \quad \begin{array}{l}\text{fraction of total time-averaged} \\ \text{incident flux scattered into} (\theta_s, \theta_s + d\theta_s)\end{array}$$

$$\frac{\partial R}{\partial \theta_s} = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2 \theta_s}{\cos \theta_0} |R(q|k)|^2,$$

where L_1 is the length of the x_1-axis covered by the random surface, while θ_0 and θ_s are the angles of incidence and scattering, respectively, and are related to the wavenumbers k and q by

$$k = \frac{\omega}{c} \sin \theta_0, \qquad\qquad q = \frac{\omega}{c} \sin \theta_s.$$

Since we are concerned with the scattering of light from a randomly rough surface, it is the mean differential reflection coefficient that we need to calculate. It is given by

$$\boxed{\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2 \theta_s}{\cos \theta_0} \langle |R(q|k)|^2 \rangle.}$$

We can write

$$\boxed{R(q|k) = \langle R(q|k)\rangle + [R(q|k) - \langle R(q|k)\rangle]\,.}$$

Then

$$\begin{aligned}
\langle\frac{\partial R}{\partial\theta_s}\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}|\langle R(q|k)\rangle|^2 \\
&+ \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}\langle|R(q|k) - \langle R(q|k)\rangle|^2\rangle.
\end{aligned}$$

The first term is the contribution from specular or coherent scattering:

$$\langle\frac{\partial R}{\partial\theta_s}\rangle_{coh} = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}|\langle R(q|k)\rangle|^2$$

The second term is the contribution from diffuse or incoherent scattering:

$$\begin{aligned}
\langle\frac{\partial R}{\partial\theta_s}\rangle_{incoh} &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}\langle|R(q|k) - \langle R(q|k)\rangle|^2\rangle \\
&= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}\left[\langle|R(q|k)|^2\rangle - |\langle R(q|k)\rangle|^2\right].
\end{aligned}$$

Reduced Rayleigh Equation

P polarization

$$\int_{-\infty}^{\infty} \frac{dq}{2\pi} M(p|q)R(q|k) = -N(p|k),$$

where

$$M(p|q) = \frac{pq + \alpha(p,\omega)\alpha_0(q,\omega)}{\alpha(p,\omega) - \alpha_0(q,\omega)} I(\alpha(p,\omega) - \alpha_0(q,\omega)|p-q)$$

$$N(p|k) = \frac{pk - \alpha(p,\omega)\alpha_0(k,\omega)}{\alpha(p,\omega) + \alpha_0(k,\omega)} I(\alpha(p,\omega) + \alpha_0(k,\omega)|p-k)$$

$$I(\gamma|Q) = \int_{-\infty}^{\infty} dx_1 e^{-iQx_1} e^{-i\gamma\zeta(x_1)}.$$

2.A. Small-Amplitude Perturbation Theory

We seek a solution of the reduced Rayleigh equation in the form

$$R(q|k) = 2\pi\delta(q-k)R_0(k) + \chi_1(q|k)\widehat{\zeta}(q-k)$$

$$+\frac{1}{2}\int\limits_{-\infty}^{\infty}\frac{dp_1}{2\pi}\chi_2(q|p_1|k)\widehat{\zeta}(q-p_1)\widehat{\zeta}(p_1-k)$$

$$+\frac{1}{6}\int\limits_{-\infty}^{\infty}\frac{dp_1}{2\pi}\int\limits_{-\infty}^{\infty}\frac{dp_2}{2\pi}\chi_3(q|p_1|p_2|k)\widehat{\zeta}(q-p_1)\widehat{\zeta}(p_1-p_2)\widehat{\zeta}(p_2-k) + \cdots,$$

where

$$\boxed{R_0(k) = \frac{\epsilon(\omega)\alpha_0(k,\omega)-\alpha(k,\omega)}{\epsilon(\omega)\alpha_0(k,\omega)+\alpha(k,\omega)},}$$

and the $\{\chi_n\}$ are calculated recursively.

$$\boxed{\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle_{incoh} = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2\theta_s}{\cos\theta_0} \langle |R(q|k) - \langle R(q|k)\rangle|^2 \rangle}$$

$$= \frac{\omega}{2\pi c} \frac{\cos^2\theta_s}{\cos\theta_0} \Big\{ \delta^2 |\chi_1(q|k)|^2 g(|q-k|)$$

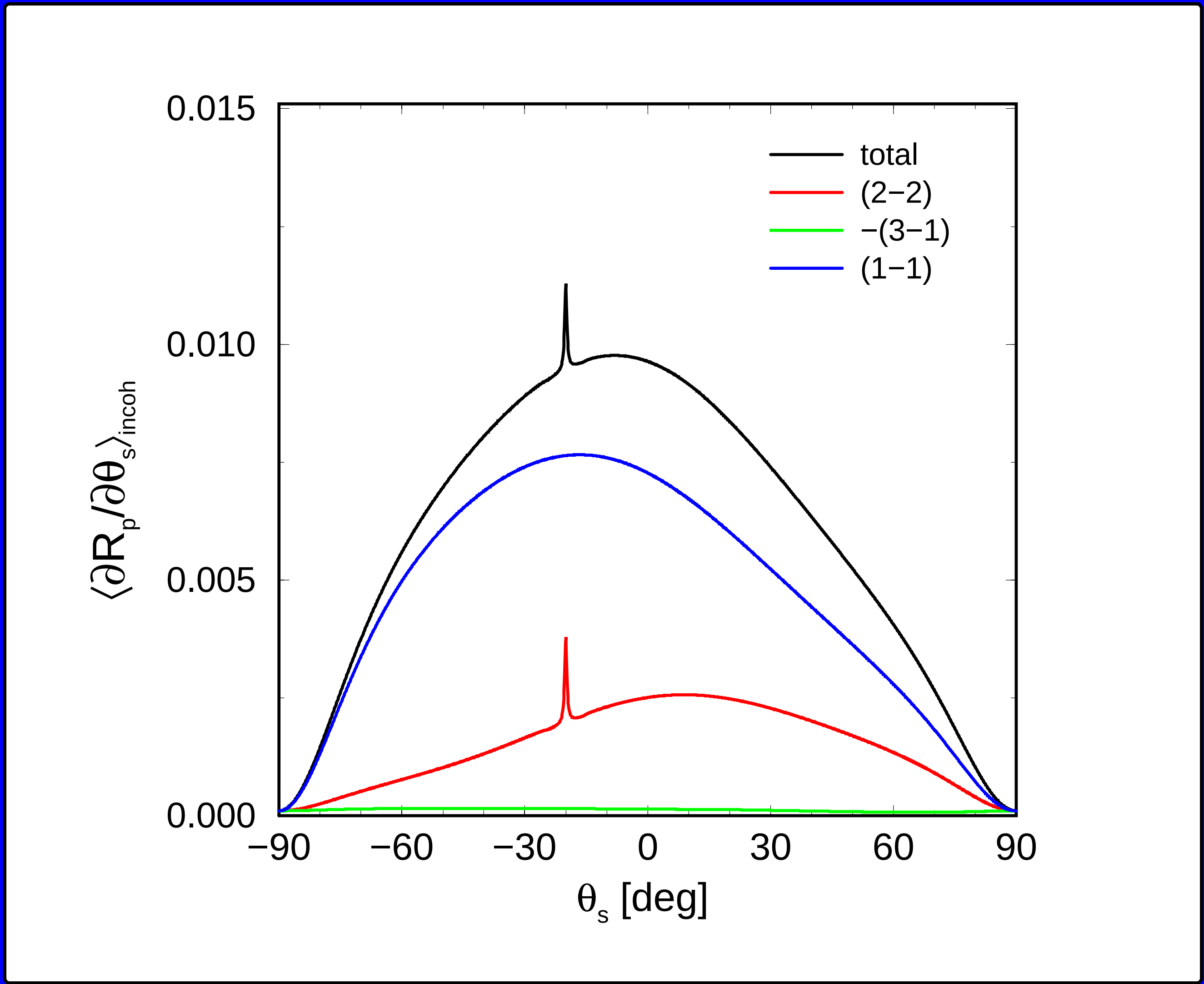
$$+ \frac{1}{4}\delta^4 \int_{-\infty}^{\infty} \frac{dp}{2\pi} g(|q-p|) g(|p-k|) \left[|\chi_2(q|p|k)|^2 + \chi_2(q|p|k)\chi_2^*(q|q+k-p|k) \right]$$

$$+ \frac{1}{3}\delta^4 Re g(|q-k|)\chi_1^*(q|k)$$

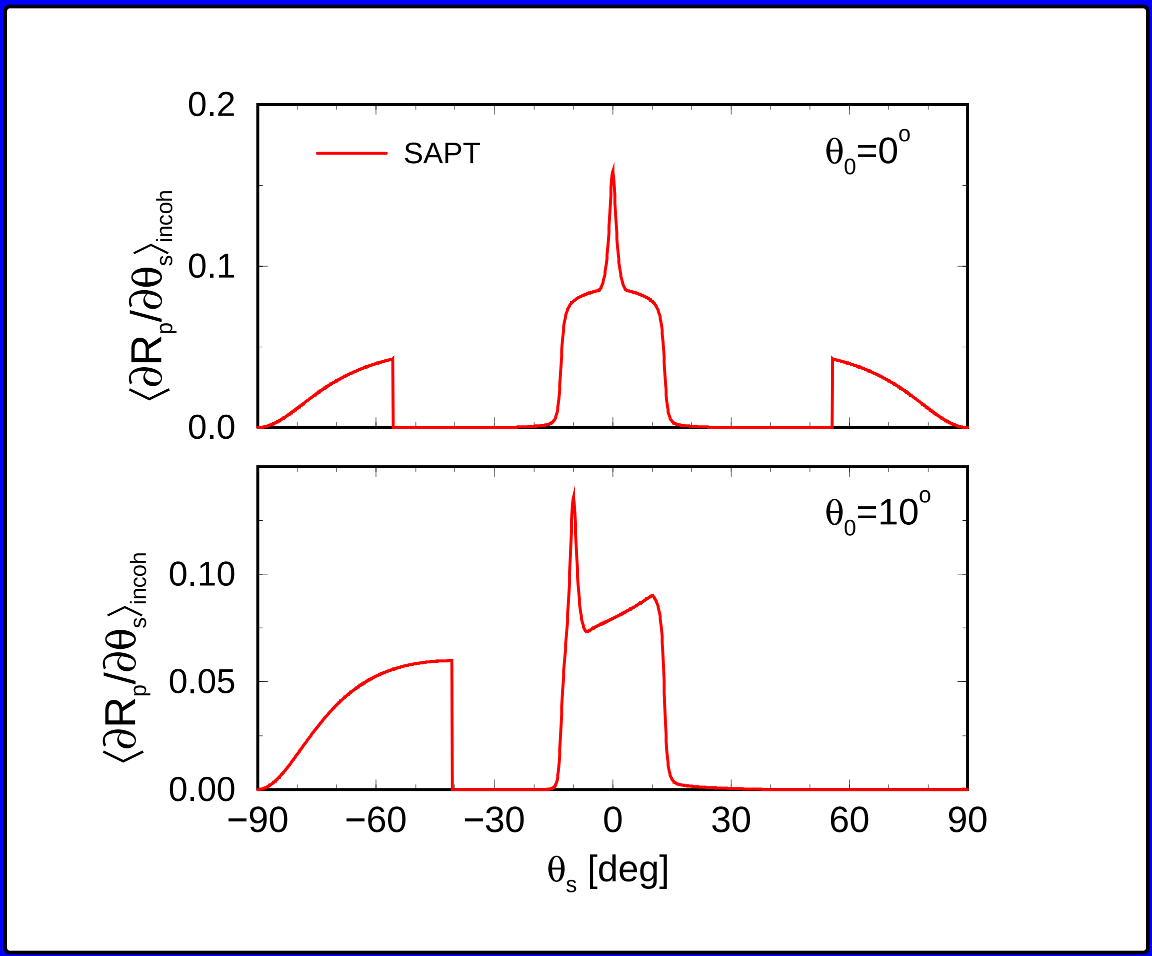
$$\times \int_{-\infty}^{\infty} \frac{dp}{2\pi} \left[\{\chi_3(q|p|q|k) + \chi_3(q|p|p+k-q|k)\} g(|p-q|) + \chi_3(q|k|p|k) g(|p-k|) \right]$$

$$+ 0(\delta^6) \Big\}.$$

Silver: $\epsilon = -7.5 + i0.24$, $\lambda = 457.9\text{nm}$, $\delta = 5\text{nm}$, $a = 100\text{nm}$, $\theta_0 = 20^\circ$



Gold: $\epsilon = -9.0 + i1.29$, $\lambda = 612.0$nm,
$\delta = 10.9$nm, $\theta_{max} = 13.3°$, $k_{min} = 0.827\omega/c$, $k_{max} = 1.288\omega/c$.



2.B. Many–Body Perturbation Theory

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle_{incoh} = \frac{2}{\pi}\left(\frac{\omega}{c}\right)^3 \cos^2\theta_s \cos\theta_0 |G(q)|^2 \left[X(q|k) + \frac{A(q|k)}{4\Gamma^2} + \frac{A(q|k)}{(q+k)^2 + 4\Gamma^2} \right] |G(k)|^2,$$

where

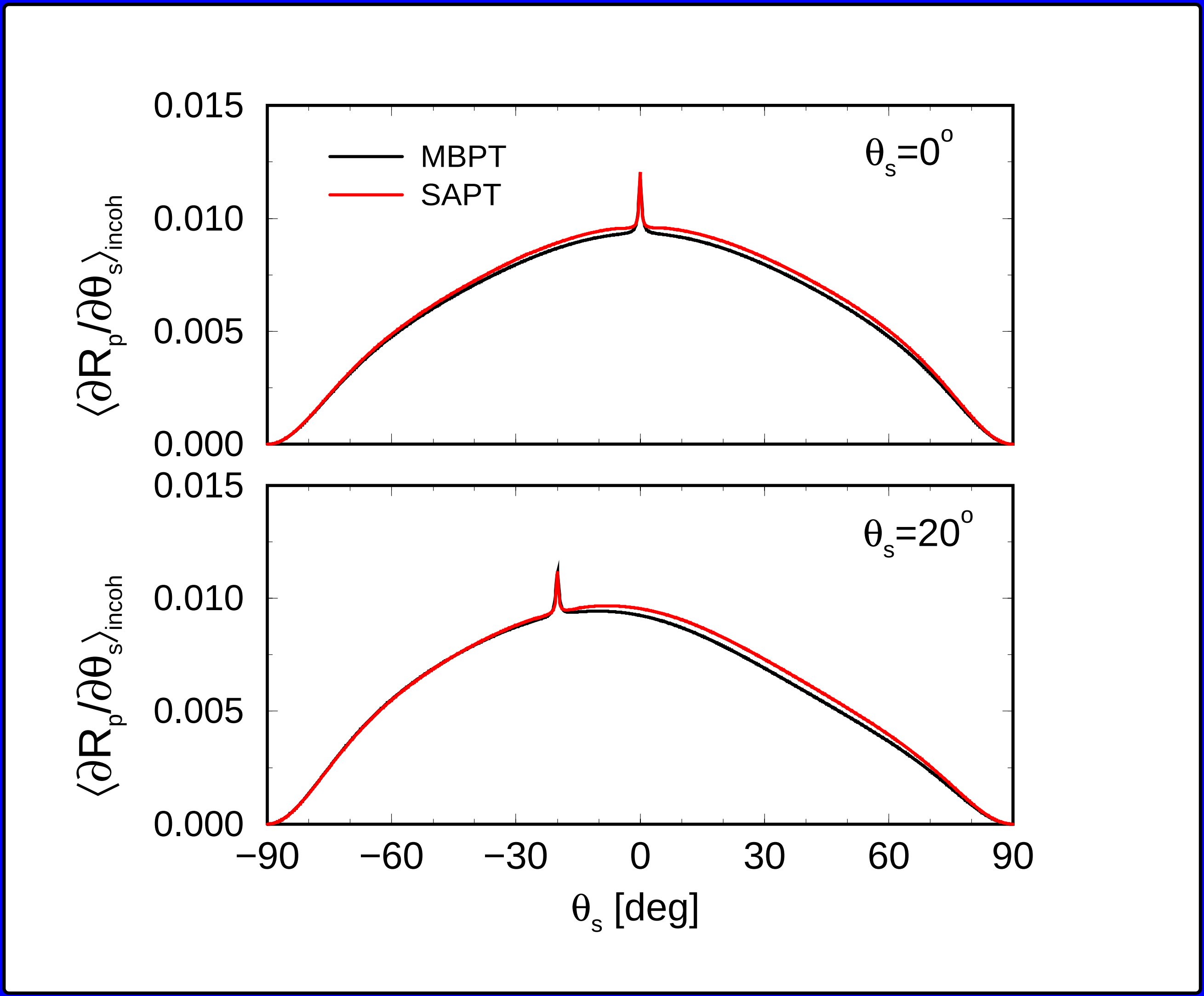
$$X(q|k) = \left[\frac{\epsilon - 1}{\epsilon^2}\right]^2 [\epsilon qk - \alpha(q,\omega)\alpha(k,\omega)]^2 \delta^2 g(|q-k|),$$

$$\begin{aligned} A(q|k) &= 2C^2\Delta[X(q|k_{sp})X(k_{sp}|k) + X(q|-k_{sp})X(-k_{sp}|k)] \\ &+ 2C^2\Delta_{sp}[X(q|k_{sp})X(-k_{sp}|k) + X(q|-k_{sp})X(k_{sp}|k)], \end{aligned}$$

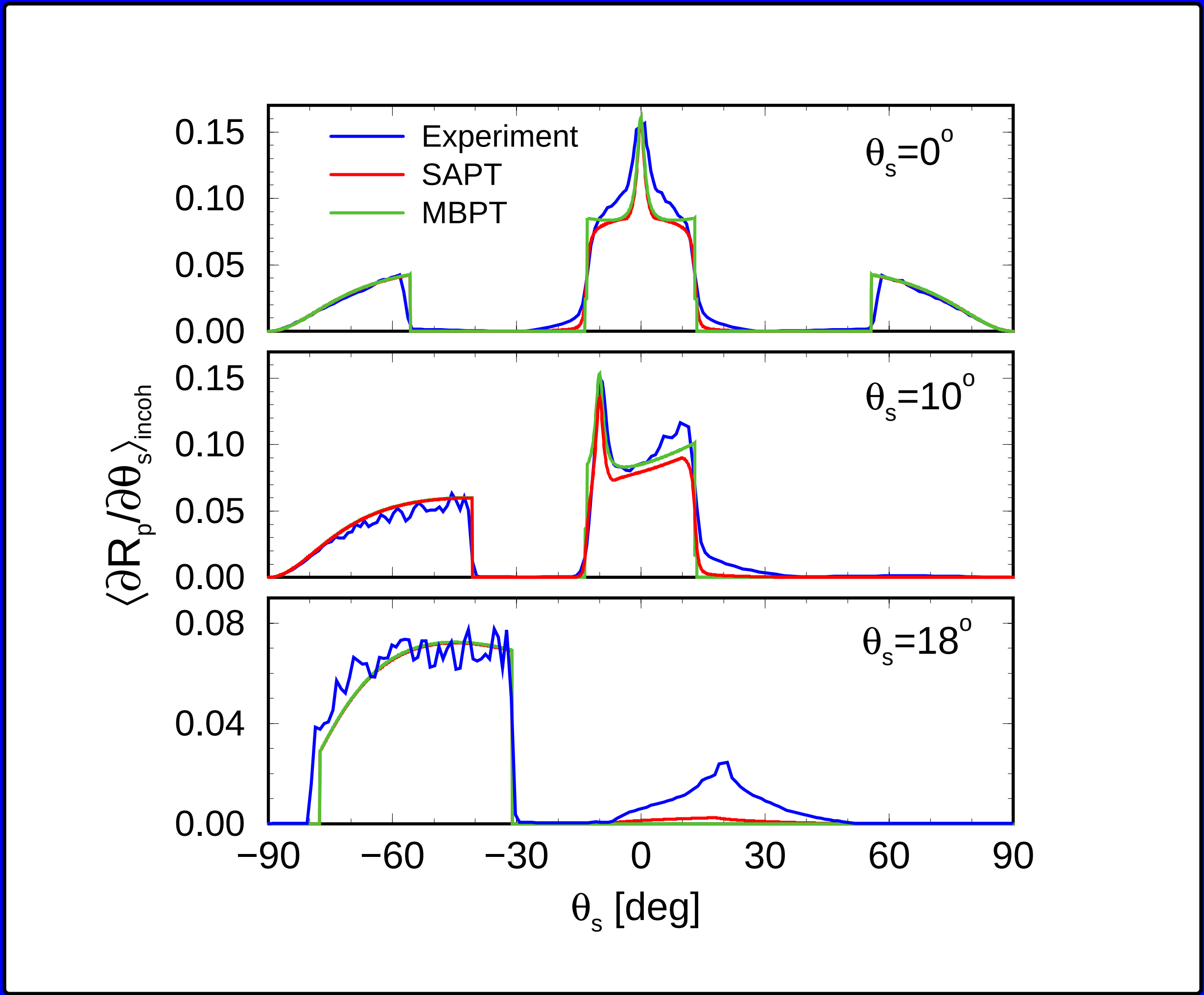
$$k_{sp} = \frac{\omega}{c}\left(\frac{|\epsilon_1|}{|\epsilon_1| - 1}\right)^{\frac{1}{2}}, \qquad C = \frac{|\epsilon_1|^{3/2}}{\epsilon_1^2 - 1}, \qquad \Delta = \Delta_\epsilon + \Delta_{sp},$$

$$\Gamma = [\Delta^2 - \Delta_{sp}^2]^{\frac{1}{2}} = [\Delta_\epsilon(\Delta_\epsilon + 2\Delta_{sp})]^{\frac{1}{2}},$$

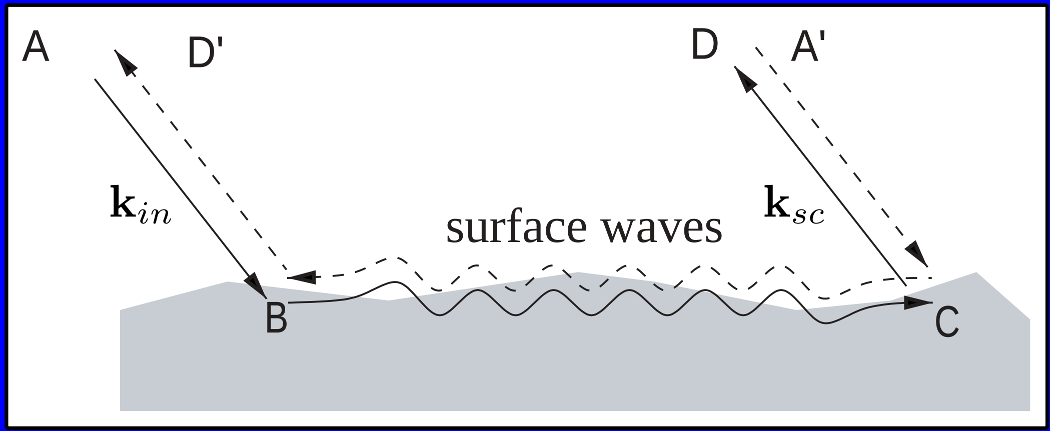
$$\Delta_\epsilon = \frac{1}{2}\frac{\omega}{c}\frac{\epsilon_2}{|\epsilon_1|^{\frac{1}{2}}(|\epsilon_1| - 1)^{3/2}}, \qquad \Delta_{sp} = C\,Im\,M(k_{sp}) = 2\left(\frac{\omega}{c}\right)^2 \frac{|\epsilon_1|^3}{(|\epsilon_1| - 1)^4}\delta^2 g(2k_{sp}).$$



Silver: $\lambda = 457.9\text{nm}$, $\epsilon(\omega) = -7.5 + i0.24$, $\delta = 5\text{nm}$, $a = 100\text{nm}$



Gold: $\lambda = 612.\text{nm}$, $\epsilon(\omega) = -9.0 + i1.29$, $\delta = 10.9\text{nm}$, $k_{min} = 0.827\omega/c$, $k_{max} = 1.288\omega/c$, $\theta_{max} = 13.3°$.

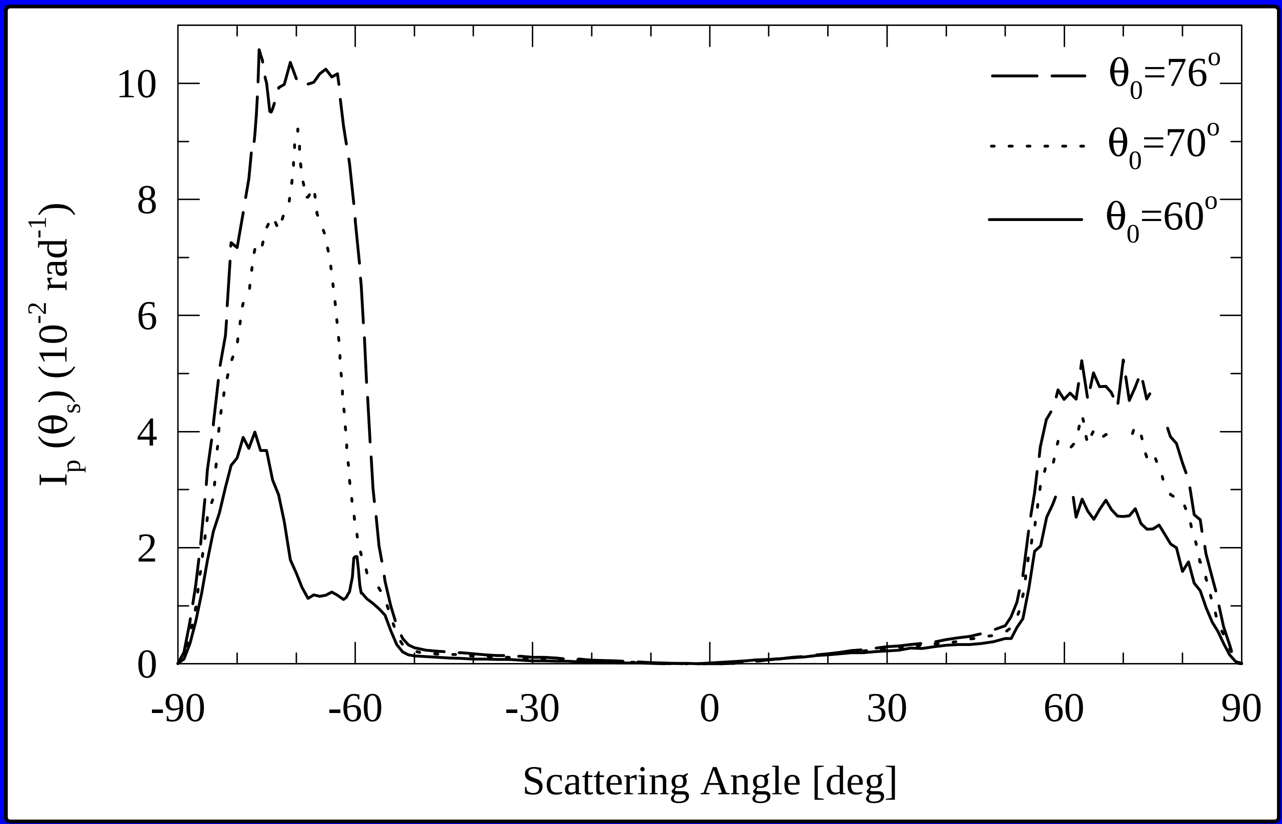


In scattering the phase difference between a path $ABCD$ and its reciprocal partner $A'CBD'$ is

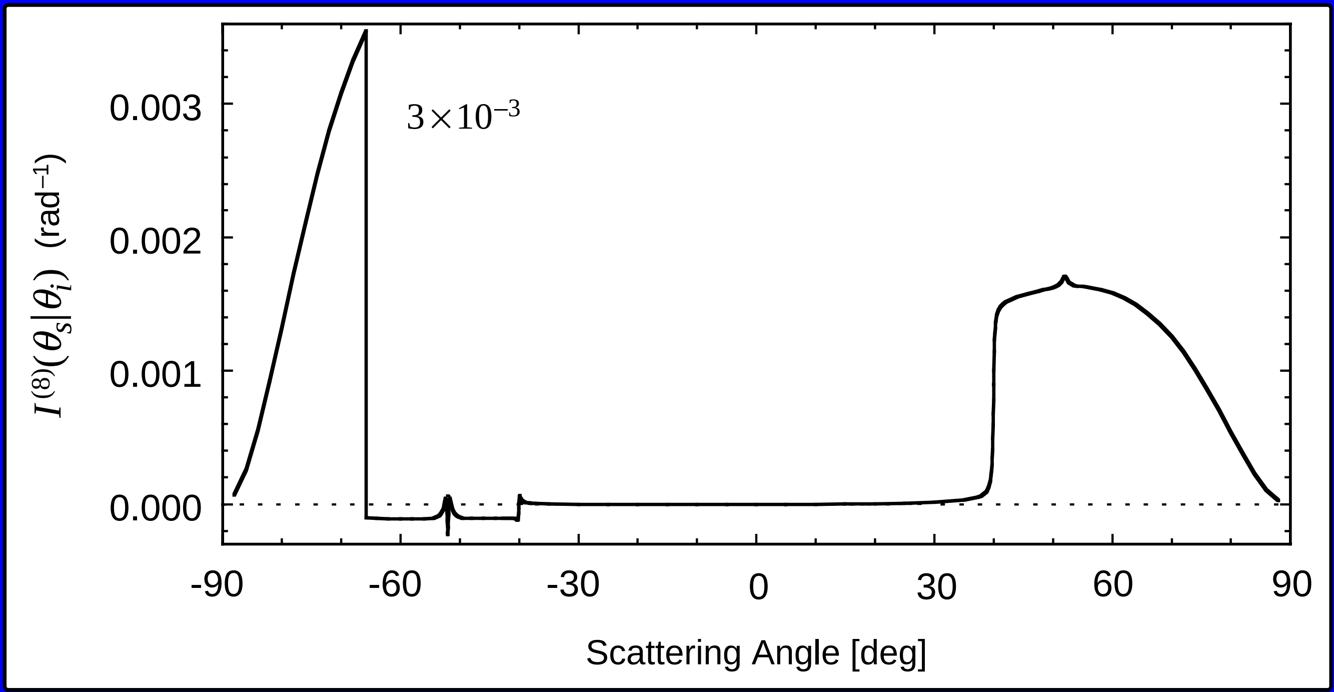
$$\Delta\phi = \mathbf{r}_{BC} \cdot (\mathbf{k}_{in} + \mathbf{k}_{sc})$$

Constructive interference ($\Delta\phi = 0$) occurs when $\mathbf{k}_{in} + \mathbf{k}_{sc} = 0$, i.e. when

$$\mathbf{k}_{sc,\parallel} = -\mathbf{k}_{in,\parallel} \quad \text{and} \quad \mathbf{k}_{sc,\perp} = -\mathbf{k}_{in,\perp}$$



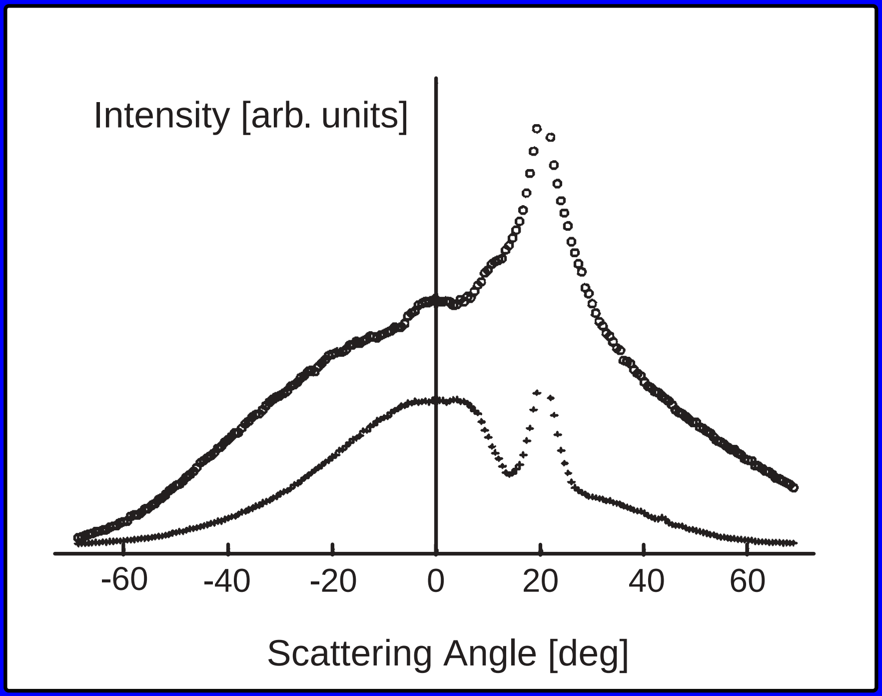
Silver: $\lambda = 1152$nm, $\epsilon(\omega) = -61.0 + i6.2$, $\delta = 11.1$nm
$k_{min} = 0.724\omega/c$, $k_{max} = 1.086\omega/c$, $\theta_{max} = 10.4°$.



$\theta_0 = 52°$

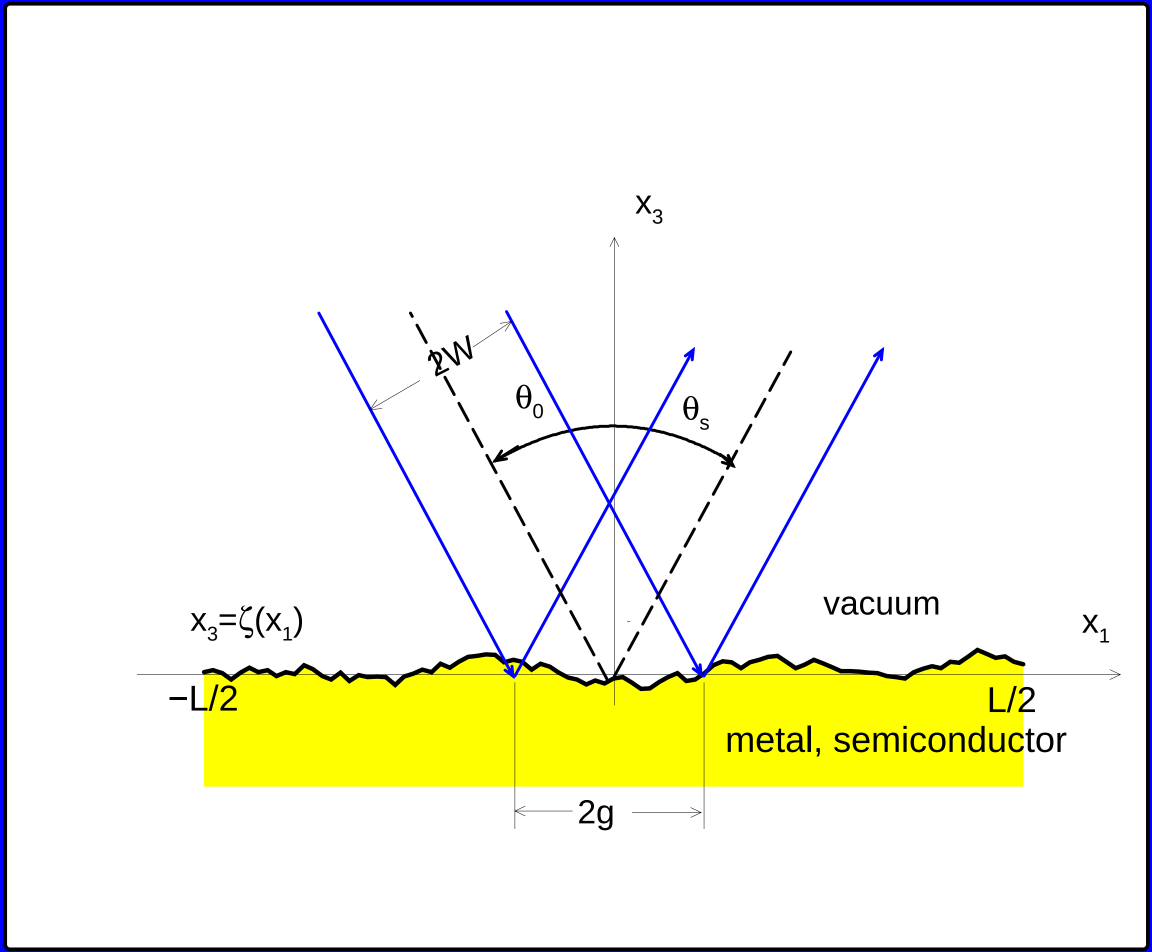
Silver: $\lambda = 457.9$nm, $\epsilon(\omega) = -7.5 + i0.24$, $\delta = 13.74$nm

$k_{min} = 1.7\omega/c$, $k_{max} = 2.25\omega/c$, $\theta_{max} = 15.9°$.



O'Donnell and Méndez (1987)
The intensity of light scattered in plane from a considerably rough two–dimensional random surface measured as a function of the polar scattering angle.
The incident light is $s-$polarized, its wavelength is $\lambda = 633$nm, and the polar angle of incidence is $\theta_0 = -20°$.
(o) $s \longrightarrow s$ scattering data; (+) $s \longrightarrow p$ scattering data.

2.C. Computer Simulations



$$L/g = 4 - 5$$

The scattered field is

$$H_2^{>}(x_1, x_3|\omega)_{sc} = \int\limits_{-\infty}^{\infty} \frac{dq}{2\pi} R(q,\omega) \exp[iqx_1 + i\alpha_0(q,\omega)x_3]$$

where

$$\begin{aligned} R(q,\omega) &= \frac{i}{2\alpha_0(q,\omega)} \int\limits_{-\infty}^{\infty} dx_1 \exp[-iqx_1 - i\alpha_0(q,\omega)\zeta(x_1)] \\ &\quad \times \{i[q\zeta'(x_1) - \alpha_0(q,\omega)]H(x_1|\omega) - L(x_1|\omega)\} \end{aligned}$$

where

$$H(x_1|\omega) = H_2^{>}(x_1, x_3|\omega)\Big|_{x_3=\zeta(x_1)} \qquad L(x_1|\omega) = \frac{\partial}{\partial N} H_2^{>}(x_1, x_3|\omega)\Big|_{x_3=\zeta(x_1)}$$

The equations satisfied by $H(x_1|\omega)$ and $L(x_1|\omega)$ are

$$\begin{aligned} H(x_1|\omega) &= H(x_1|\omega)_{inc} + \int_{-\infty}^{\infty} dx_1' [H^{(0)}(x_1|x_1')H(x_1'|\omega) - L^{(0)}(x_1|x_1')L(x_1'|\omega)] \\ 0 &= -\int_{-\infty}^{\infty} dx_1' [H^{(\epsilon)}(x_1|x_1')H(x_1'|\omega) - \epsilon L^{(\epsilon)}(x_1|x_1')L(x_1'|\omega)], \end{aligned}$$

where η is a positive infinitesimal,

$$H(x_1|\omega)_{inc} = H_2^{>}(x_1, \zeta(x_1)|\omega)_{inc},$$

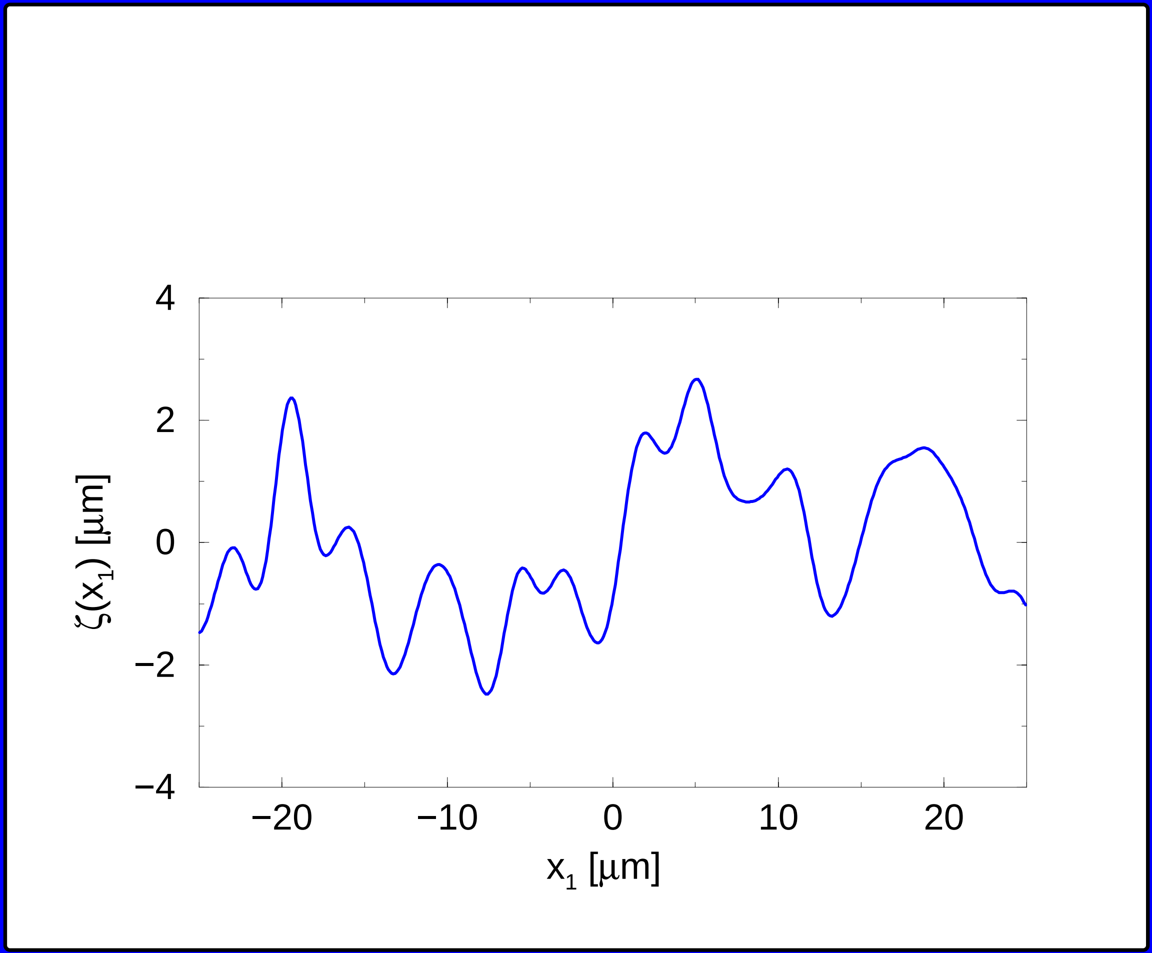
and

$$\begin{aligned} H^{(\epsilon)}(x_1|x_1') &= \left(-\frac{i}{4}\right) n_c^2 \frac{\omega^2}{c^2} \frac{H_1^{(1)}(n_c \frac{\omega}{c}[(x_1 - x_1')^2 + (\zeta(x_1) - \zeta(x_1') + \eta)^2]^{\frac{1}{2}})}{n_c \frac{\omega}{c}[(x_1 - x_1')^2 + (\zeta(x_1) - \zeta(x_1') + \eta)^2]^{\frac{1}{2}}} \\ &\times \quad [(x_1 - x_1')\zeta'(x_1') - (\zeta(x_1) - \zeta(x_1') + \eta)] \end{aligned}$$

$$L^{(\epsilon)}(x_1|x_1') = \frac{i}{4} H_0^{(1)} \left(n_c \frac{\omega}{c} [(x_1 - x_1')^2 + (\zeta(x_1) - \zeta(x_1') + \eta)^2]^{\frac{1}{2}} \right).$$

To obtain $H^{(0)}(x_1|x_1')$ and $L^{(0)}(x_1|x_1')$ set $n_c = 1$ in $H^{(\epsilon)}(x_1|x_1')$ and $L^{(\epsilon)}(x_1|x_1')$.

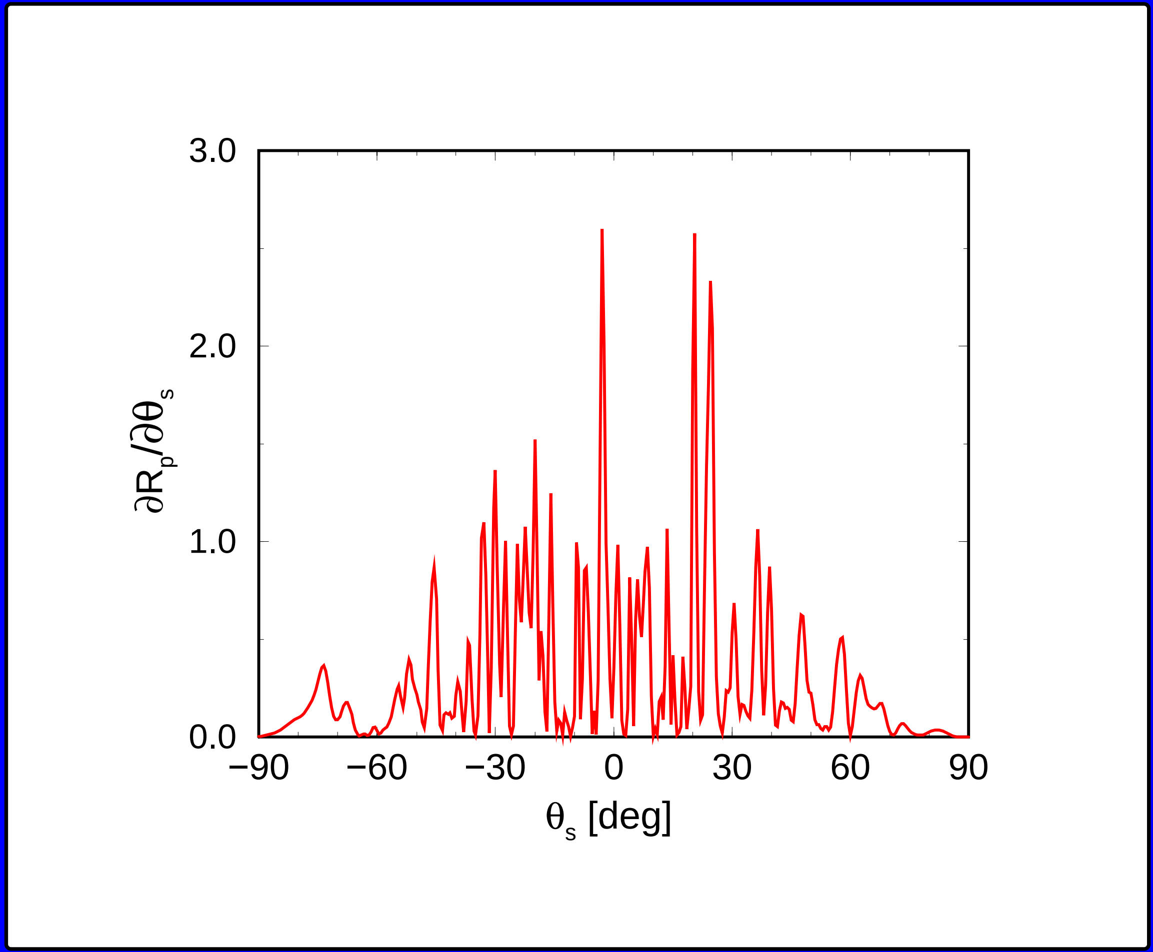
A single realization of $\zeta(x_1)$



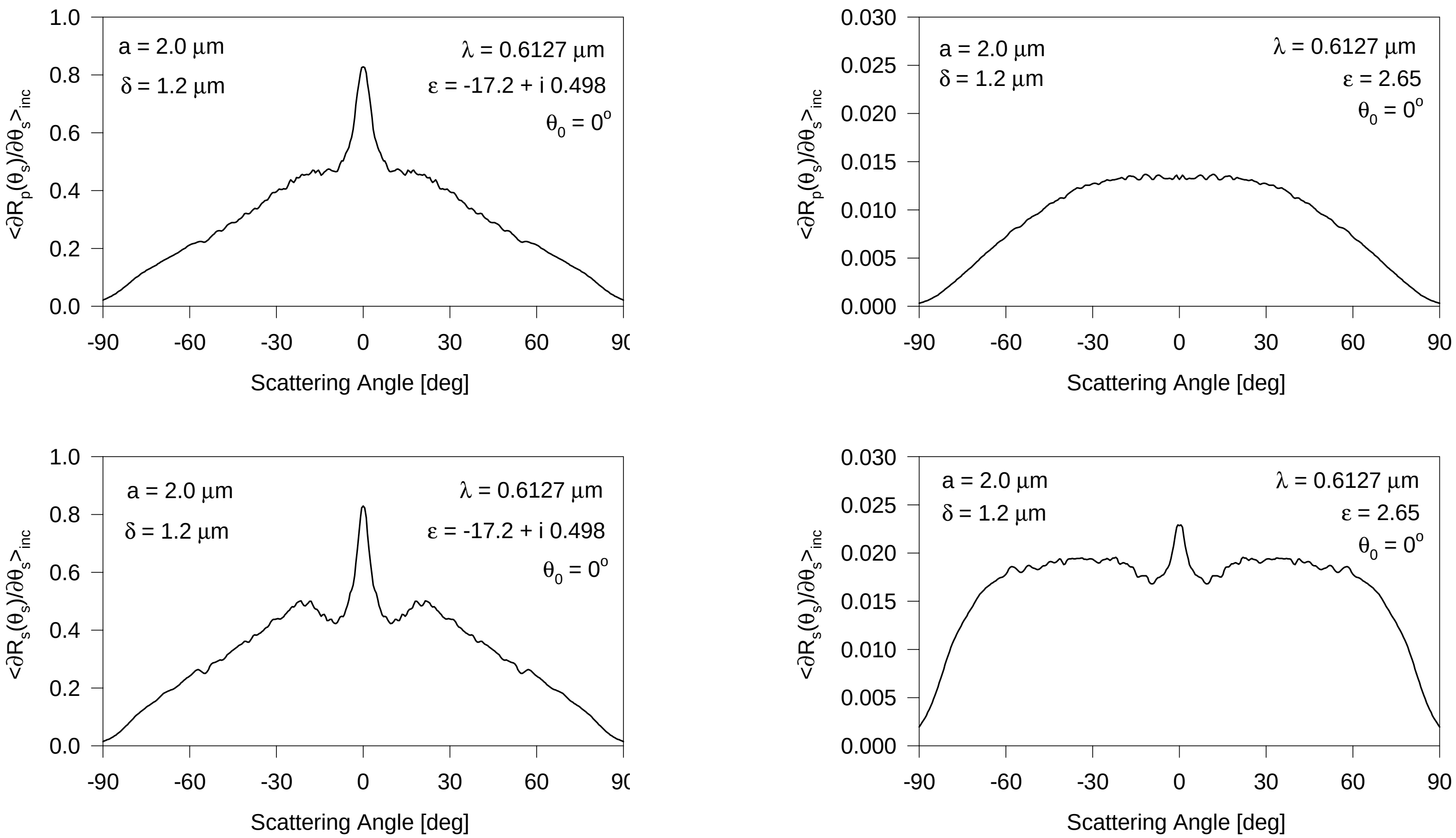
$\delta = 1.5\mu\text{m}$, $a = 4\mu\text{m}$.

Single realization

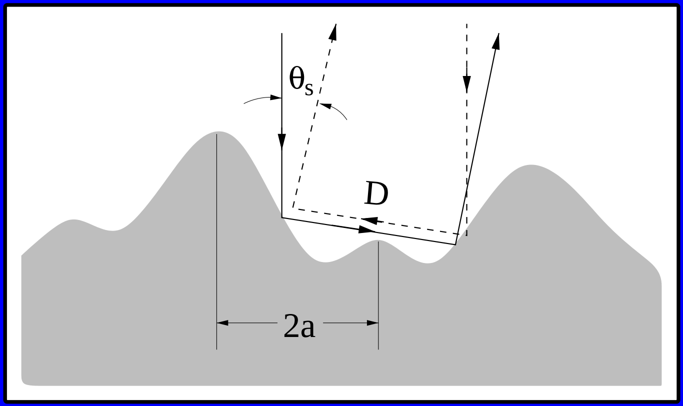
$p-$ polarization, $\theta_0 = 0^\circ$



Silver: $\lambda = 612.7\text{nm}$, $\epsilon(\omega) = -17.2 + i0.498$, $a = 2\mu\text{m}$, $\delta = 1.2\mu\text{m}$



Metal (Silver) Dielectric (Photoresist)



Phase difference between paths ϕ is $\phi \propto \frac{2\pi}{\lambda}\theta_s\langle D\rangle$.

Expect maxima when

$$\langle\phi\rangle = \frac{2\pi}{\lambda}\theta_s\langle D\rangle = n2\pi \quad n = 0, \pm1, \pm2, \ldots,$$

i.e. when

$$\theta_s = \frac{n\lambda}{\langle D\rangle};$$

expect subsidiary minima when

$$\theta_s = \frac{\left(n + \frac{1}{2}\right)\lambda}{\langle D\rangle} \qquad n = 0, \pm1, \pm2, \ldots$$

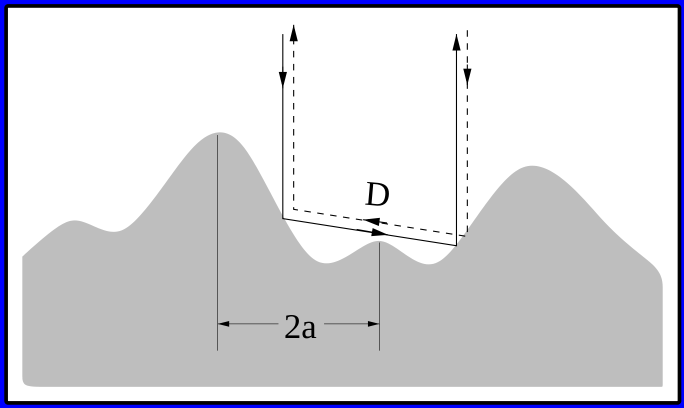
Therefore, the width of the enhanced backscattering peak (*at normal incidence*) is suggested to be

$$\Delta\theta_s = \frac{\lambda}{\langle D\rangle} \approx \frac{\lambda}{\ell} \approx \frac{\lambda}{\langle d\rangle}.$$

The standard deviation of ϕ at $\theta_s = n\lambda/\langle D\rangle$ is

$$\sigma_\phi \mid_{\theta_s = n\lambda/<D>} = 2\pi|n|\frac{\sigma_D}{\langle D\rangle} \approx 2\pi|n|\frac{\sigma_d}{\langle d\rangle}.$$

For $n > 2$, σ_ϕ is large enough to wash out higher order subsidiary maxima.



In the presence of coherence

$$\text{Intensity} = |A + A|^2$$

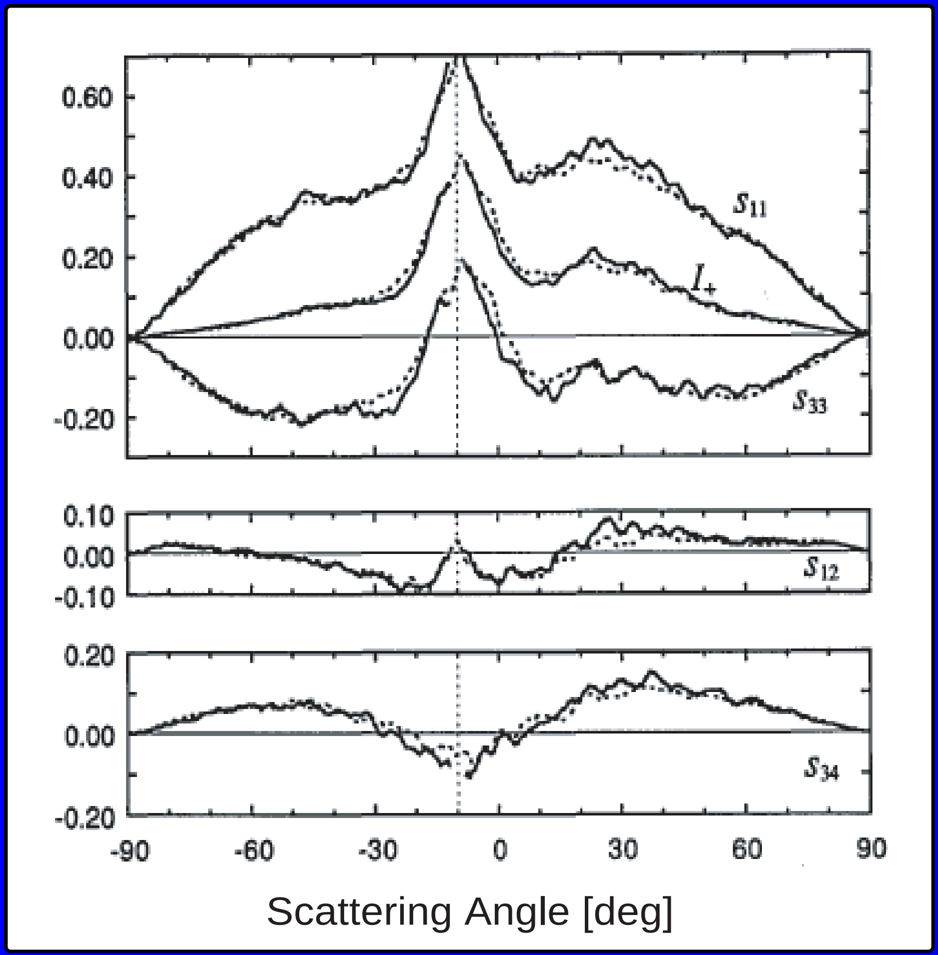
$$= |A|^2 + A^*A + AA^* + |A|^2$$

$$= 4|A|^2 \quad (A = A).$$

When coherence is lost

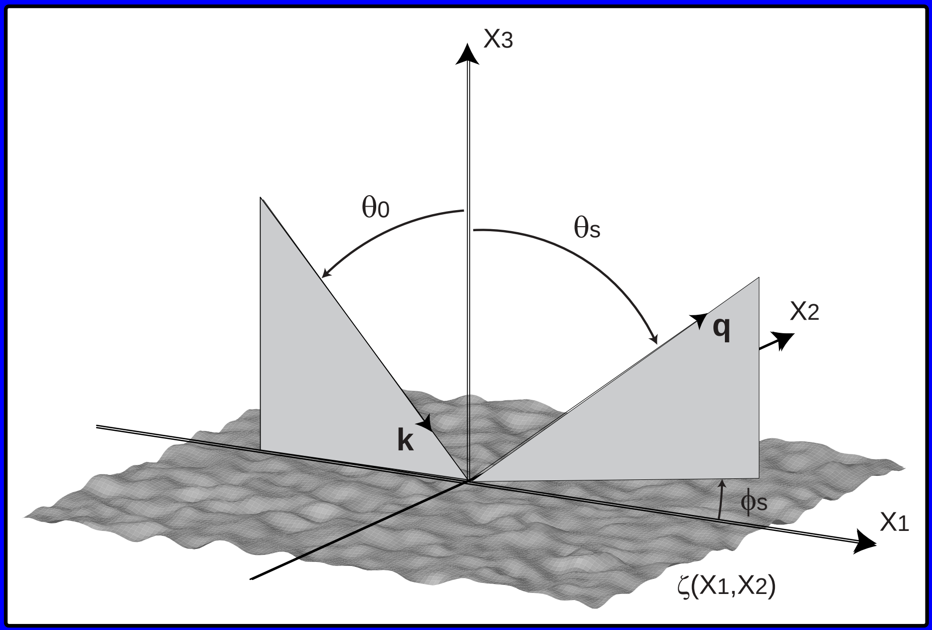
$$\text{Intensity} = |A|^2 + |A|^2 = 2|A|^2 \quad (A \approx A).$$

Comparison of experimental and computer simulations results for the elements of the Stokes matrix



Gold: $\lambda = 3.392\mu\text{m}$, $\delta = 1.73\mu\text{m}$, $a = 3.43\mu\text{m}$, $\theta_0 = 10°$

3. Computer Simulations of Scattering from a Two–Dimensional Random Surface

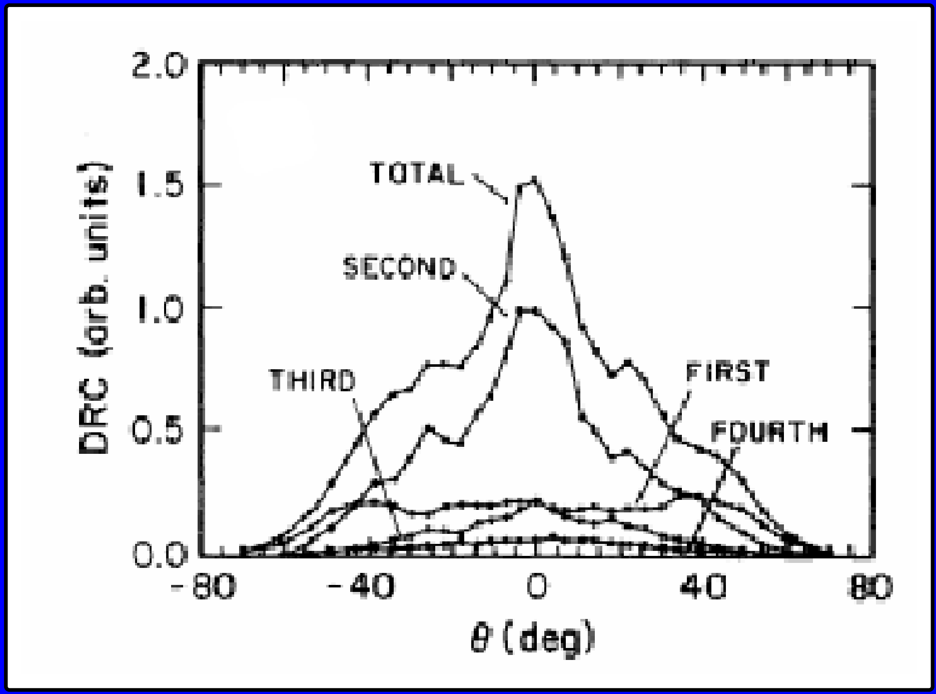


The surface is defined by

$$x_3 = \zeta(\mathbf{x}_\parallel).$$

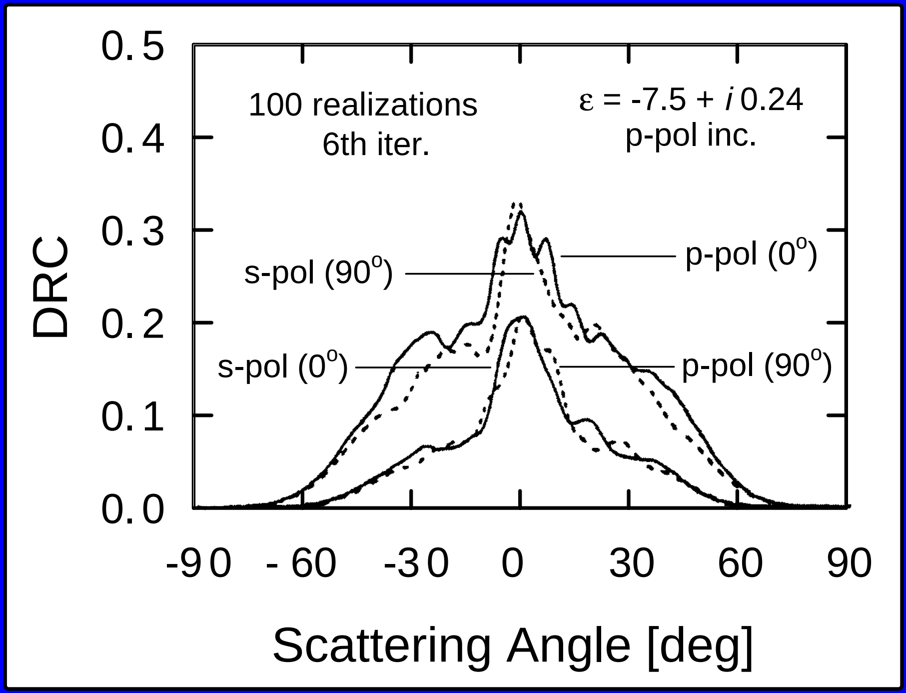
The surface profile function $\zeta(\mathbf{x}_\parallel)$ is a single–valued function of $\mathbf{x}_\parallel$ that is differentiable with respect to x_1 and x_2 and constitutes a zero–mean, stationary, isotropic, Gaussian random process

Perfect Conductor



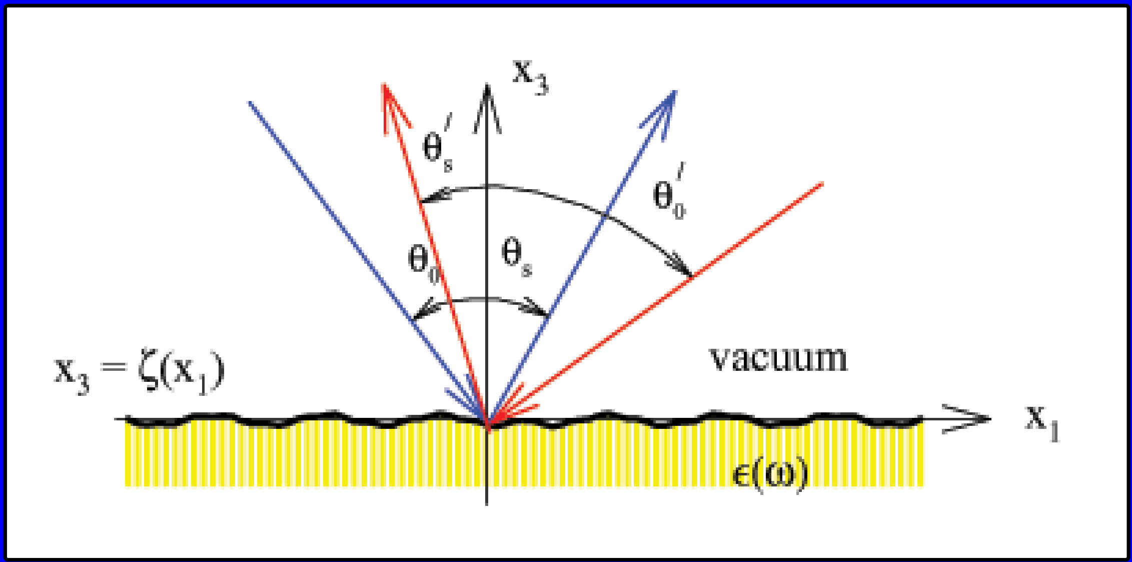
$\lambda = 457.9\text{nm}$, $\delta = \lambda$, $a = 2\lambda$, $W = 3\lambda$, $\theta_0 = 0°$, $\phi_0 = 0°$

Silver



$\lambda = 457.9\text{nm}$, $\delta = \lambda$, $a = 2\lambda$, $W = 3\lambda$, $\theta_0 = 0°$, $\phi_0 = 0°$

4. Angular Intensity Correlation Functions



The angular intensity correlation function we are interested in is

$$\boxed{C(q,k|q',k') = \langle \delta I(q|k)\delta I(q'|k')\rangle.}$$

where

$$\delta I(q|k) = I(q|k) - \langle I(q|k)\rangle.$$

The intensity $I(q|k)$ is given by

$$I(q|k) = \frac{1}{L_1}\left(\frac{\omega}{c}\right)|S(q|k)|^2,$$

$$S(q|k) = \left(\frac{\alpha_0(q,\omega)}{\alpha_0(k,\omega)}\right)^{1/2} R(q|k)$$

is the scattering matrix. In these expressions

$$\boxed{k = \tfrac{\omega}{c}\sin\theta_0 \qquad q = \tfrac{\omega}{c}\sin\theta_s}$$

$$\boxed{k' = \tfrac{\omega}{c}\sin\theta_0' \qquad q' = \tfrac{\omega}{c}\sin\theta_s'}$$

Because of the correlation of an intensity with itself is stronger than the correlation of two different intensities, a peak in $C(q,k|q',k')$ is expected when $q=q'$ and $k=k'$. This is $\boxed{\textit{the memory effect.}}$

Because $S(q|k)$ is reciprocal, $S(q|k)=S(-k|-q)$, a peak in $C(q,k|q',k')$ is also expected when $q=-k'$ and $k=-q'$. This is $\boxed{\textit{the reciprocal memory effect.}}$

In terms of $S(q|k)$

$$\boxed{C(q,k|q',k') = \tfrac{1}{L_1^2}\tfrac{\omega^2}{c^2}\left[\langle S(q|k)S^*(q|k)S(q'|k')S^*(q'|k')\rangle - \langle S(q|k)S^*(q|k)\rangle\langle S(q'|k')S^*(q'|k')\rangle\right].}$$

This expression contains purely specular terms, i.e. terms proportional to $2\pi\delta(q-k)$ and/or $2\pi\delta(q'-k')$. To eliminate these uninteresting terms we introduce

$$\begin{aligned}\delta S(q|k) &= S(q|k) - \langle S(q|k)\rangle \\ &= S(q|k) - 2\pi\delta(q-k)S(k).\end{aligned}$$

Then the contribution to $C(q,k|q',k')$ free from specular contributions can be written

$$\begin{aligned}C(q,k|q',k') &= \frac{1}{L_1^2}\frac{\omega^2}{c^2}\Big[|\langle\delta S(q|k)\delta S^*(q'|k')\rangle|^2 + |\langle\delta S(q|k)\delta S(q'|k'\rangle|^2 \\ &+ \langle\delta S(q|k)\delta S^*(q|k)\delta S(q'|k')\delta S^*(q'|k')\rangle_c\Big],\end{aligned}$$

where $< \cdots >$ denotes the cumulant average.

Due to the stationarity of $\zeta(x_1)$ $\langle \delta S(q|k)\delta S^*(q'|k')\rangle$ is proportional to $2\pi\delta(q-k-q'+k')$ and gives rise to the $C^{(1)}$ correlation function. The average $\langle \delta S(q|k)\delta S(q'|k')\rangle$ is proportional to $2\pi\delta(q-k+q'-k')$ and gives rise to a correlation function, overlooked in earlier studies of $C(q,k|q',k')$, that is now called the $C^{(10)}$ correlation function. The last term gives rise to the $C^{(2)}$ and $C^{(3)}$ correlation functions, and to a new correlation function called the $C^{(1.5)}$ correlation function.

Since $2\pi\delta(0) = L_1$ in one dimension, when the argument of the delta–functions vanishes, $C^{(1)}$ and $C^{(10)}$ are independent of the length of the surface, because they are proportional to $[2\pi\delta(0)]^2$. At the same time, $\langle \delta S(q|k)\delta S^*(q|k)\delta S(q'|k')\delta S^*(q'|k')\rangle_c$ is proportional to L_1^{-1}, because it is proportional to $2\pi\delta(0)$. Therefore, in the limit of a long surface or a large illumination area, the contributions from the latter term are small, and vanish in the limit of an infinitely long surface. Consequently, they are weak and will not be considered further here.

If it is assumed that $S(q|k)$ obeys complex Gaussian statistics, a consequence of the central limit theorem, the expression for $C(q,k|q',k')$ becomes

$$\begin{aligned} C(q,k|q',k') &= \frac{1}{L_1^2}\frac{\omega^2}{c^2}\left[|\langle\delta S(q|k)\delta S^*(q'|k')\rangle|^2 + |\langle\delta S(q|k)\delta S(q'|k'\rangle|^2\right] \\ &\equiv C^{(1)}(q,k|q',k') + C^{(10)}(q,k|q',k'). \end{aligned}$$

If it is further assumed that $S(q|k)$ follows circular complex Gaussian statistics, the expression for $C(q,k|q',k')$ simplifies to

$$\boxed{C(q,k|q',k') = \frac{1}{L_1^2}\frac{\omega^2}{c^2}|\langle\delta S(q|k)\delta S^*(q'|k')\rangle|^2 \equiv C^{(1)}(q,k|q',k').}$$

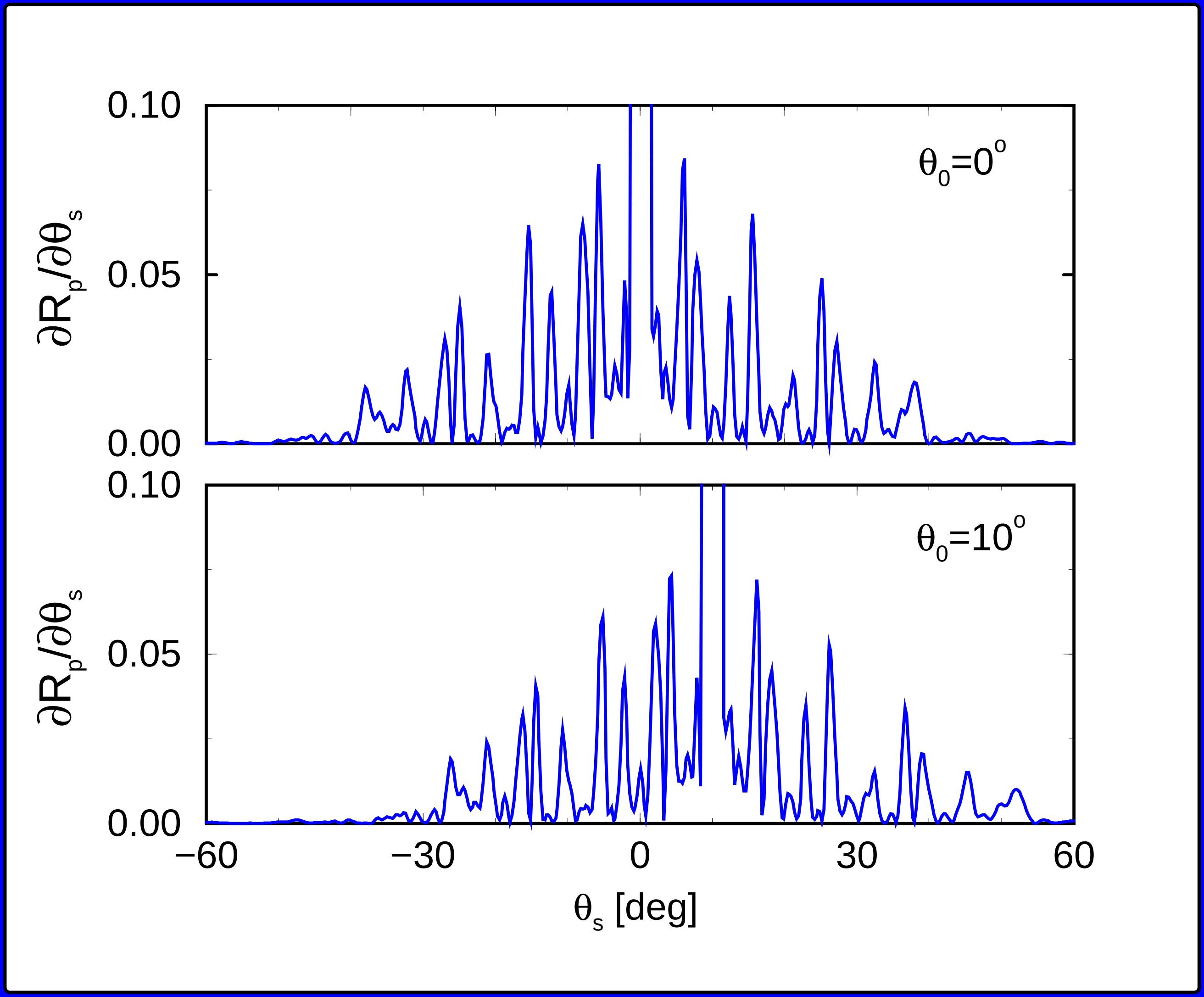
This approximation to $C(q,k|q',k')$ is often called the factorization approximation to it.

Thus, $\langle\delta S(q|k)\delta S^*(q|k)\delta S(q'|k')\delta S^*(q'|k')\rangle_c$ gives the correction to the prediction of the central limit theorem due to the finite length of the random surface.

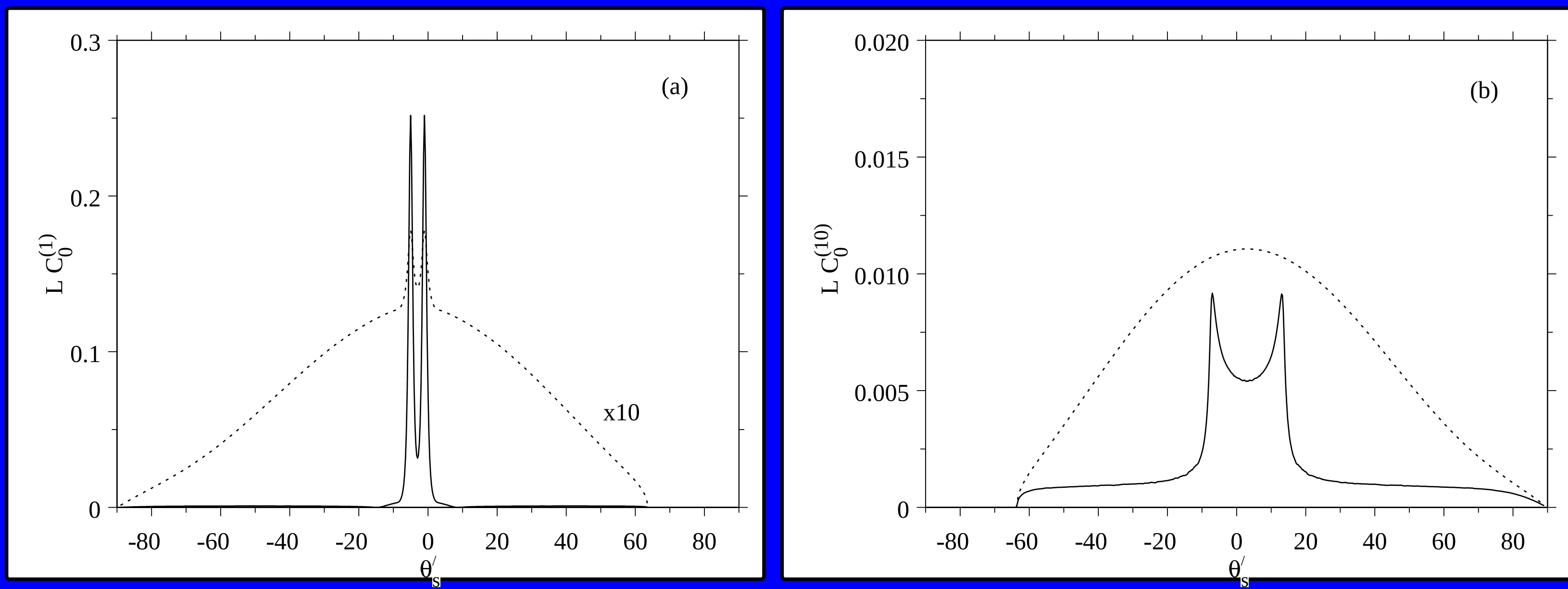
The property of the speckle pattern that is characterized by the presence of the factor $2\pi\delta(q-k-q'+k')$ in $C^{(1)}(q,k|q',k')$ is that if the angle of incidence is changed so that k goes to $k'=k+\Delta k$, the entire speckle pattern shifts in such a way that any feature initially at q moves to $q'=q+\Delta k$.

This is the memory effect.

The property of the speckle pattern that is characterized by the presence of the factor $2\pi\delta(q-k+q'-k')$ in $C^{(10)}(q,k|q',k')$ is that the speckle pattern is symmetric with respect to the specular direction (in wavenumber space).



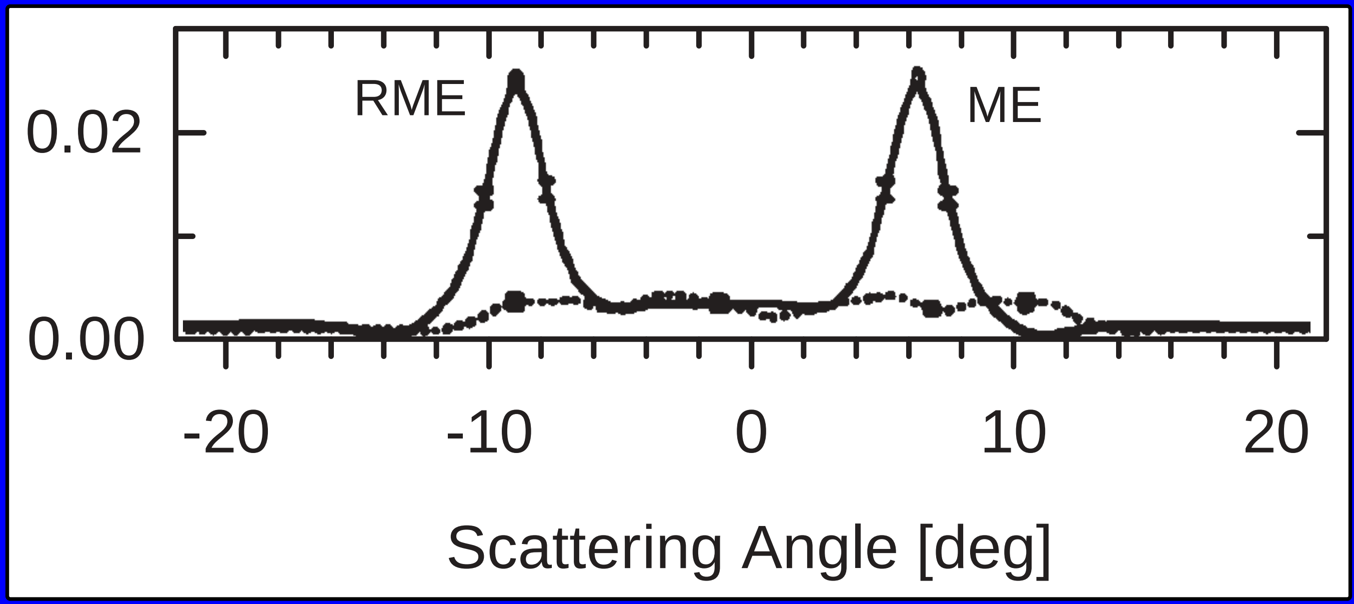
Silver: $\lambda = 612.7\text{nm}$, $\epsilon(\omega) = -17.2 + i0.498$, $\delta = 8\text{nm}$, $a = 400\text{nm}$



______ West–O'Donnell power spectrum $\theta_{max} = 13.5°$

...... Gaussian power spectrum $a = 129.1$nm.

Gold: $\epsilon = -9.0 + i1.29$, $\delta = 10.9$nm, $\theta_0 = 1°$, $\theta_s = -5°$

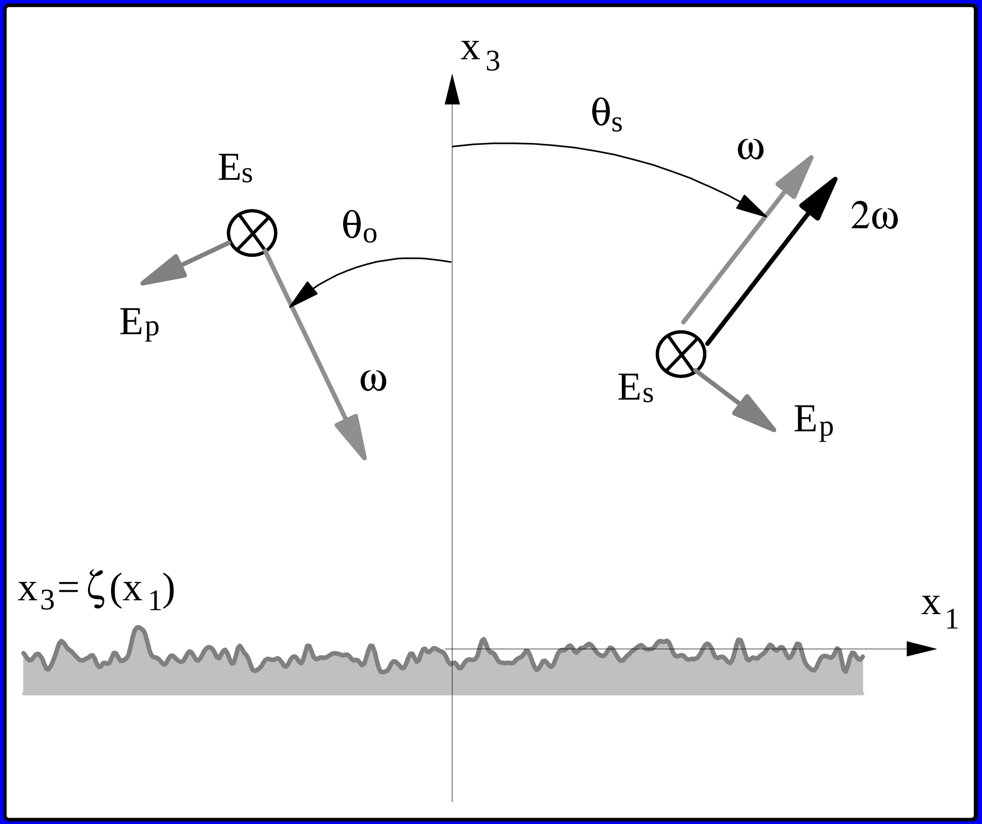


Experimental results for the correlation functions $C^{(1)}$ and $C^{(10)}$

$\theta_0 = 6.3°$, $\theta_s = 8.6°$, $\lambda = 612.7$nm, $\epsilon(\omega) = -9.0 + i1.29$ (Gold)
The surface roughness is characterized by the West–O'Donnell power spectrum with $\delta = 115.5$nm

West and O'Donnell (1999)

5. Second Harmonic Generation in Reflection from a Randomly Rough Metal Surface



The Nonlinear Boundary Conditions

$p-$**polarized incident light**

$$\boxed{H(x_1|2\omega) - H^<(x_1|2\omega) = \tfrac{2ic}{\omega}\mu_3\phi^{-2}(x_1)L(x_1|\omega)\tfrac{d}{dx_1}H(x_1|\omega),}$$

$$\boxed{L(x_1|2\omega) - \tfrac{1}{\epsilon(2\omega)}L^<(x_1|2\omega) = \tfrac{2ic}{\omega}\tfrac{d}{dx_1}\left\{\phi^{-2}(x_1)\left[\mu_1\left(\tfrac{d}{dx_1}H(x_1|\omega)\right)^2 + \mu_2 L(x_1|\omega)^2\right]\right\}}$$

where $\phi(x_1) = \sqrt{1+(\zeta'(x_1))^2}$,

$$H(x_1|\Omega) = H_2^>(x_1,\zeta(x_1)|\Omega),$$

$$H^<(x_1|\Omega) = H_2^<(x_1,\zeta(x_1)|\Omega),$$

$$L(x_1|\Omega) = \frac{\partial}{\partial N}H_2^>(x_1,x_3|\Omega)\bigg|_{x_3=\zeta(x_1)},$$

$$L^<(x_1|\Omega) = \frac{\partial}{\partial N}H_2^<(x_1,x_3|\Omega)\bigg|_{x_3=\zeta(x_1)}.$$

s–polarized incident light

$$\boxed{H(x_1|2\omega) - H^<(x_1|2\omega) = 0,}$$

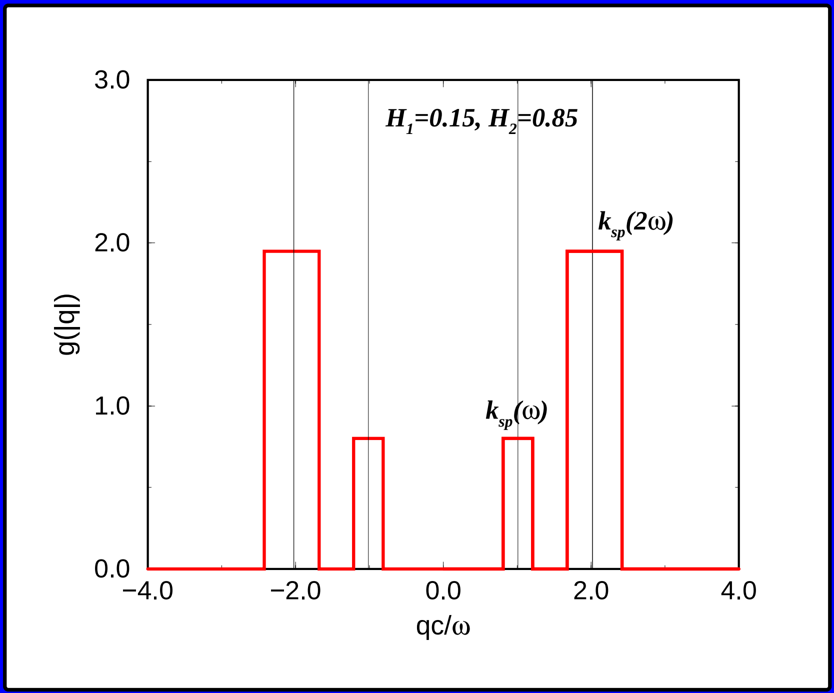
$$\boxed{L(x_1|2\omega) - L^<(x_1|2\omega) = -\frac{2i\omega}{c}\mu_2\frac{d}{dx_1}E^2(x_1|\omega),}$$

where

$$E(x_1|\omega) = E_2^>(x_1, \zeta(x_1)|\omega).$$

$$
\begin{aligned}
g(|q|) &= \frac{\pi H_1}{k_{max}^{(1)} - k_{min}^{(1)}} \left[\Theta(q - k_{min}^{(1)})\Theta(k_{max}^{(1)} - q) + \Theta(-q - k_{min}^{(1)})\Theta(k_{max}^{(1)} + q) \right] \\
&+ \frac{\pi H_2}{k_{max}^{(2)} - k_{min}^{(2)}} \left[\Theta(q - k_{min}^{(2)})\Theta(k_{max}^{(2)} - q) + \Theta(-q - k_{min}^{(2)})\Theta(k_{max}^{(2)} + q) \right],
\end{aligned}
$$

where $\Theta(z)$ is the Heaviside unit step function.



$$H_1 + H_2 = 1$$

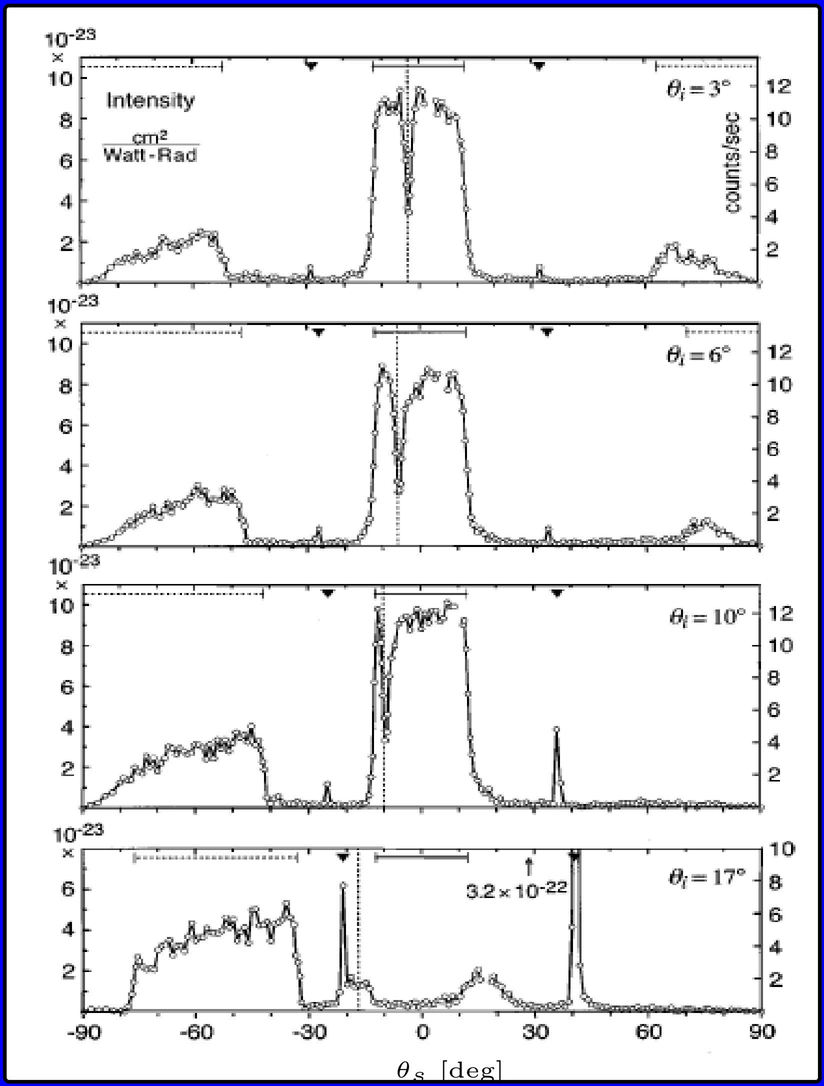
$$k_{min}^{(1)} < k_{sp}(\omega) < k_{max}^{(1)},$$

$$k_{min}^{(2)} < k_{sp}(2\omega) < k_{max}^{(2)},$$

$$k_{sp}(\Omega) = Re\left[\frac{\Omega}{c}\sqrt{\frac{\epsilon(\Omega}{\epsilon(\Omega)+1}}\right].$$

Rectangular power spectrum centered at $k_{sp}(2\omega)$

$H_1 = 0$



$$\delta = 11.1\text{nm}$$

$$\theta_{max} = 12.2°$$

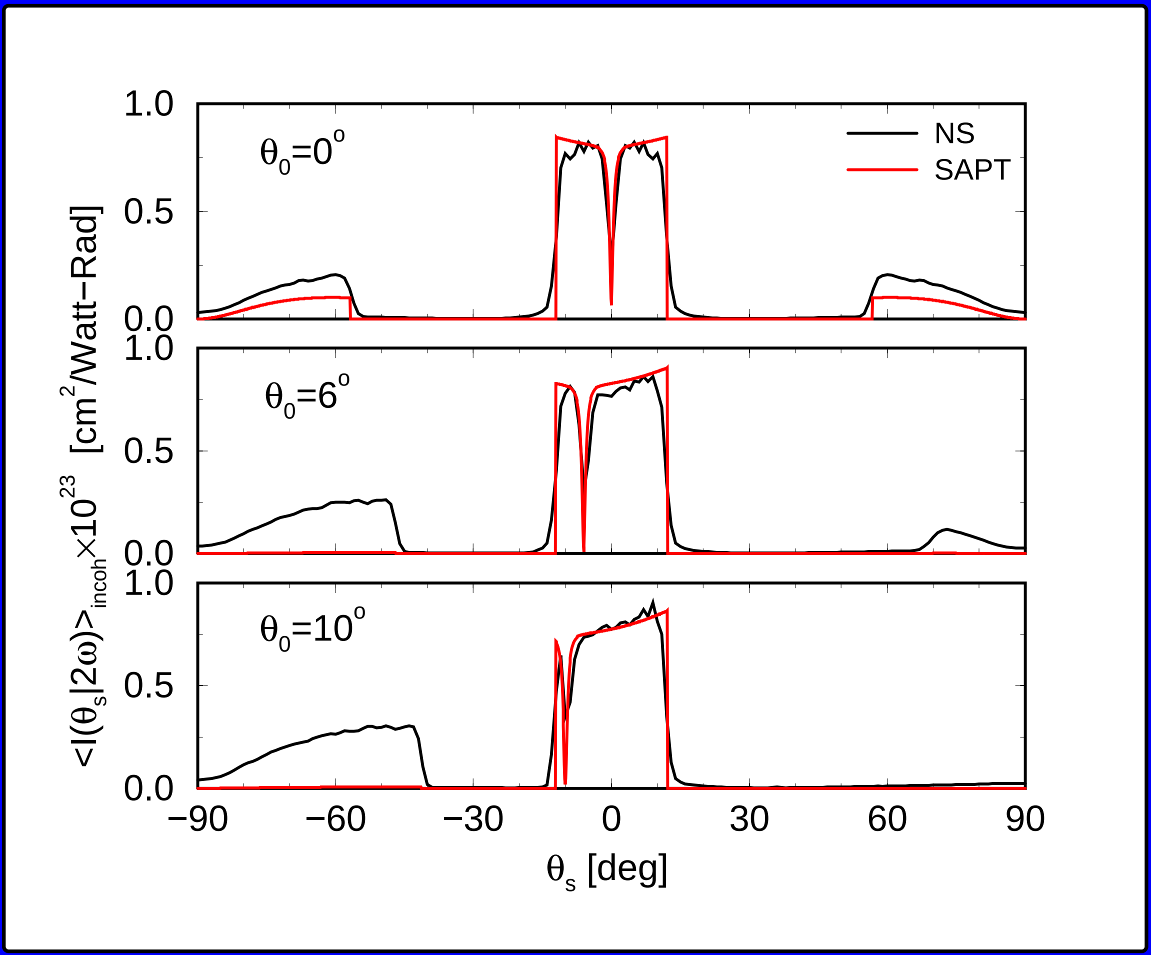
$$\epsilon(\omega) = -56.25 + i0.60$$

$$\epsilon(2\omega) = -11.56 + i0.37$$

$$\lambda = 1064\text{nm}$$

Rectangular power spectrum centered at $k_{sp}(2\omega)$

$H_1 = 0$



$\delta = 11.1\text{nm}$

$\theta_{max} = 12.2^\circ$

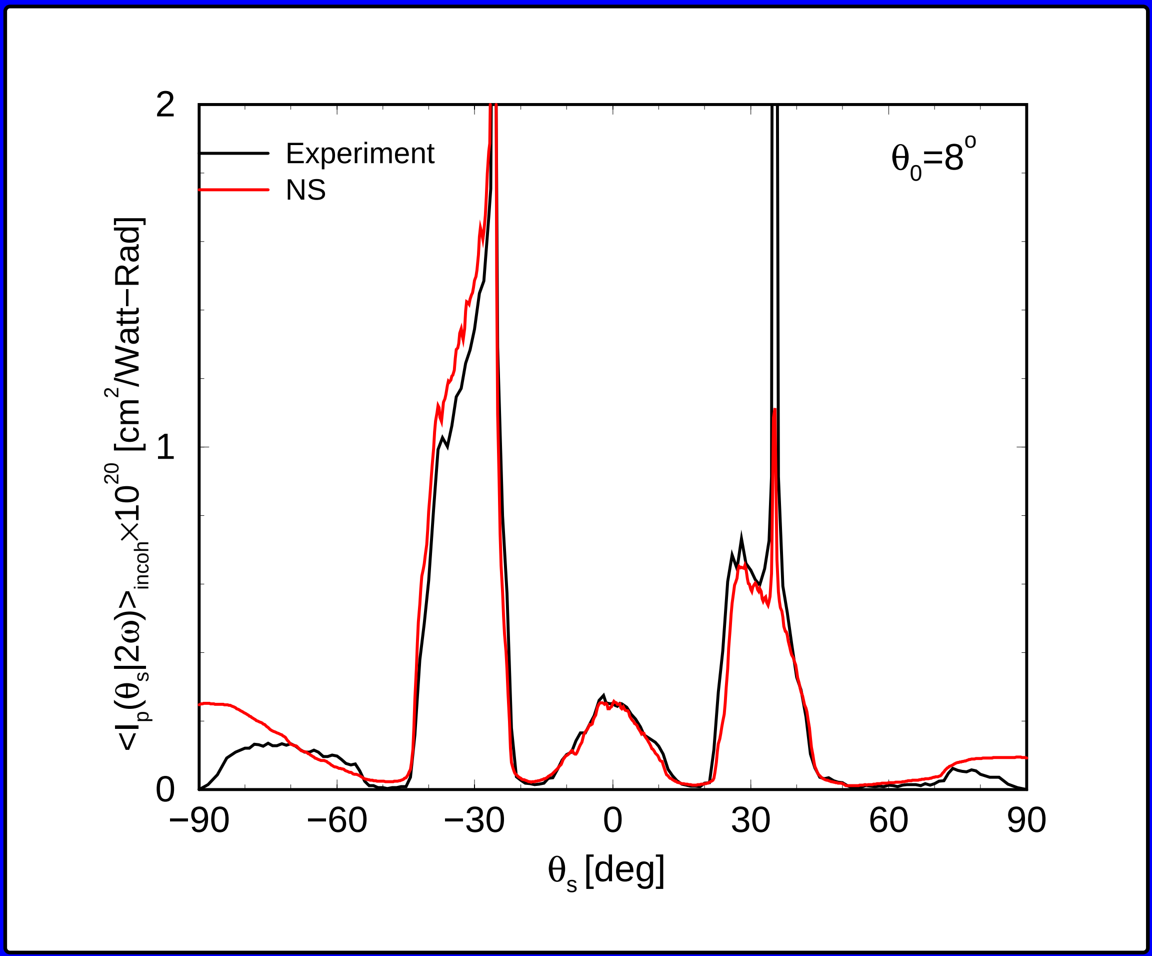
$\epsilon(\omega) = -56.25 + i0.60$

$\epsilon(2\omega) = -11.56 + i0.37$

$\lambda = 1064\text{nm}$

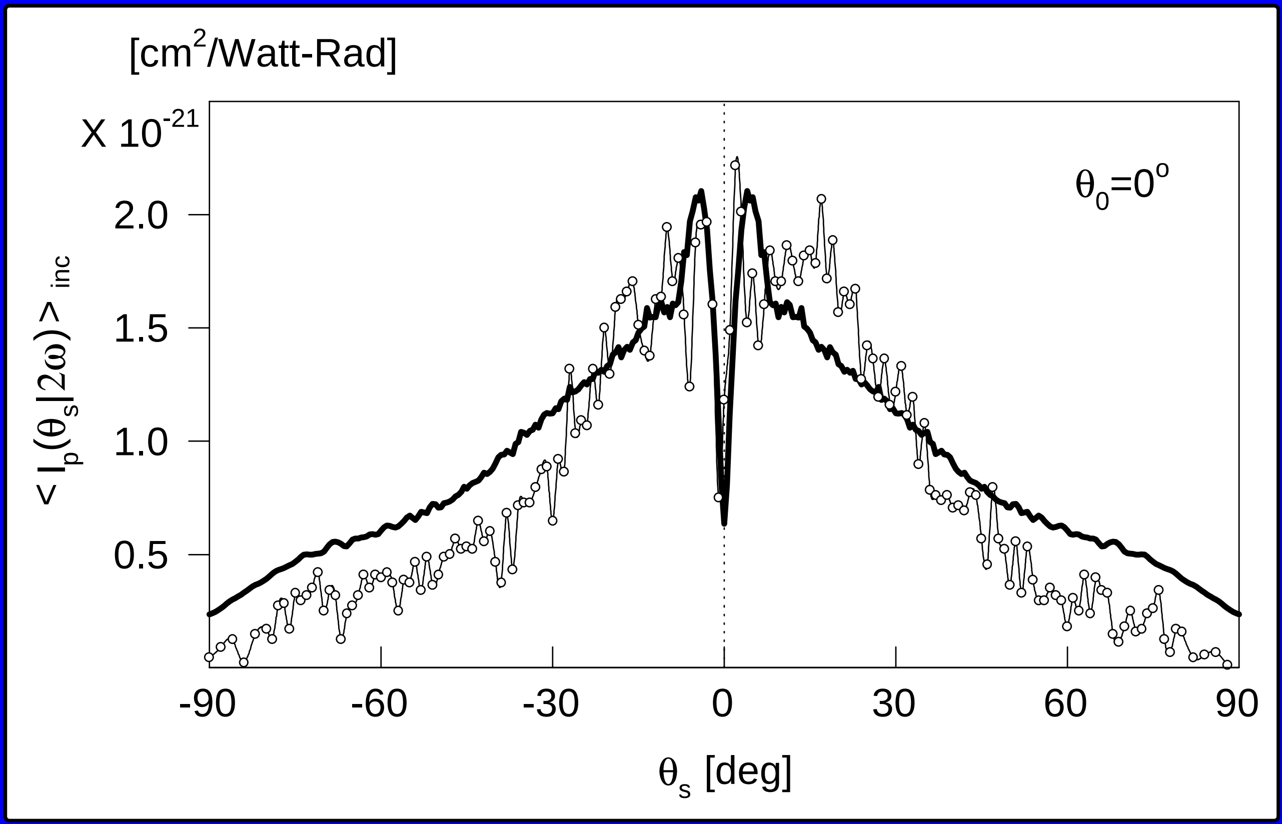
Rectangular power spectrum centered at $k_{sp}(\omega)$

$H_2 = 0$, $\delta = 28.3\text{nm}$, $\theta_{max} = 15°$



Gaussian power spectrum

$\delta = 1.81\mu\text{m}$, $a = 3.4\mu\text{m}$

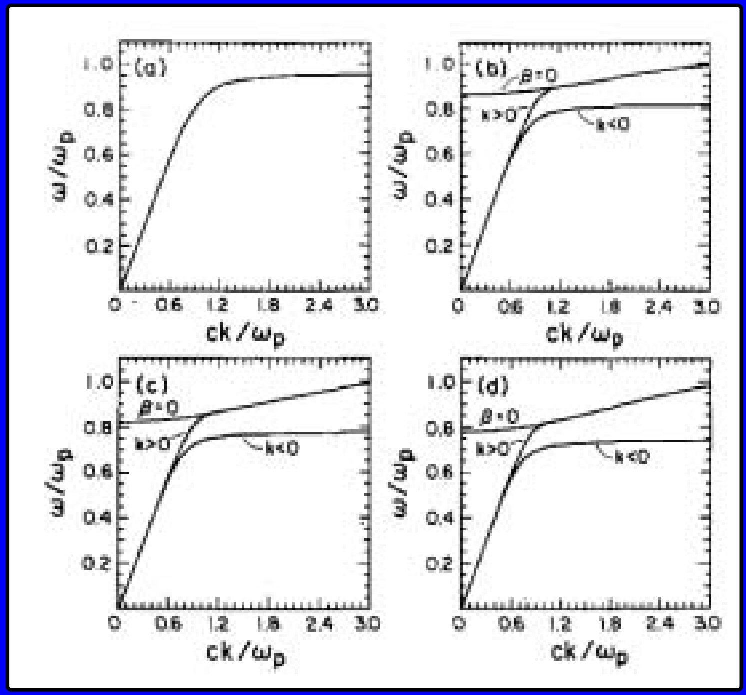


6. Some Directions for Future Research

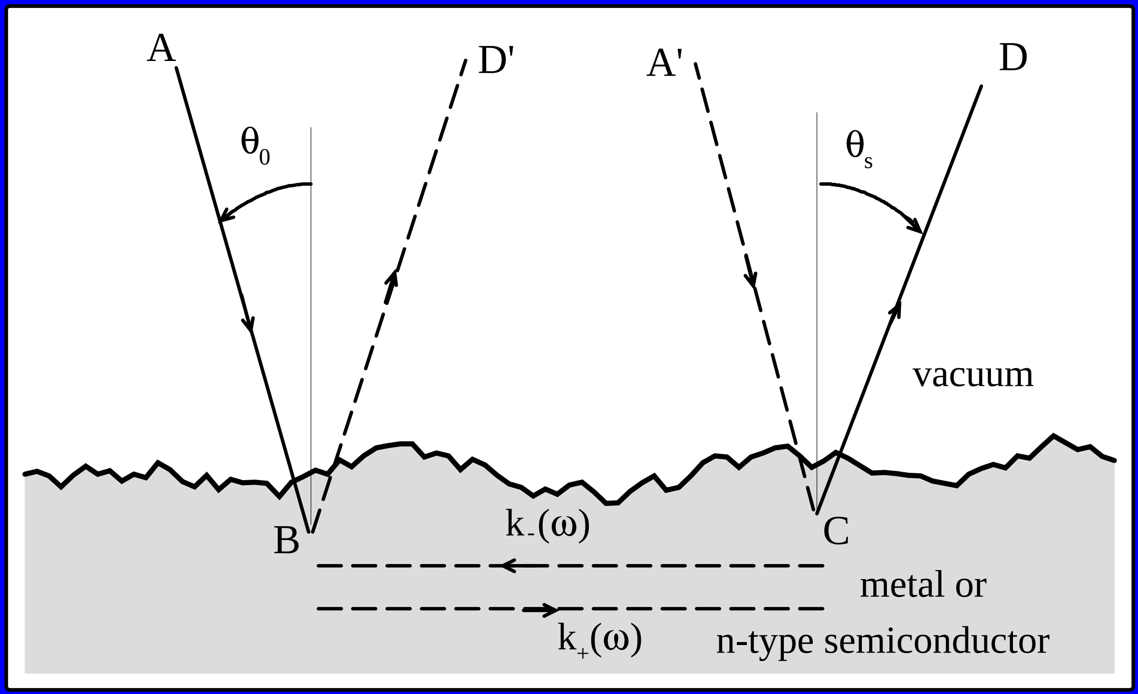
- *Accurate and fast numerical algorithms and accurate approximate analytic approaches to the scattering of light from two–dimensional randomly rough surfaces*
- *Grazing angle scattering*
- *Scattering from the randomly rough surface of an inhomogeneous medium*
- *The inverse problem*

Coherent Interference of Nonreciprocal Scattering Sequences

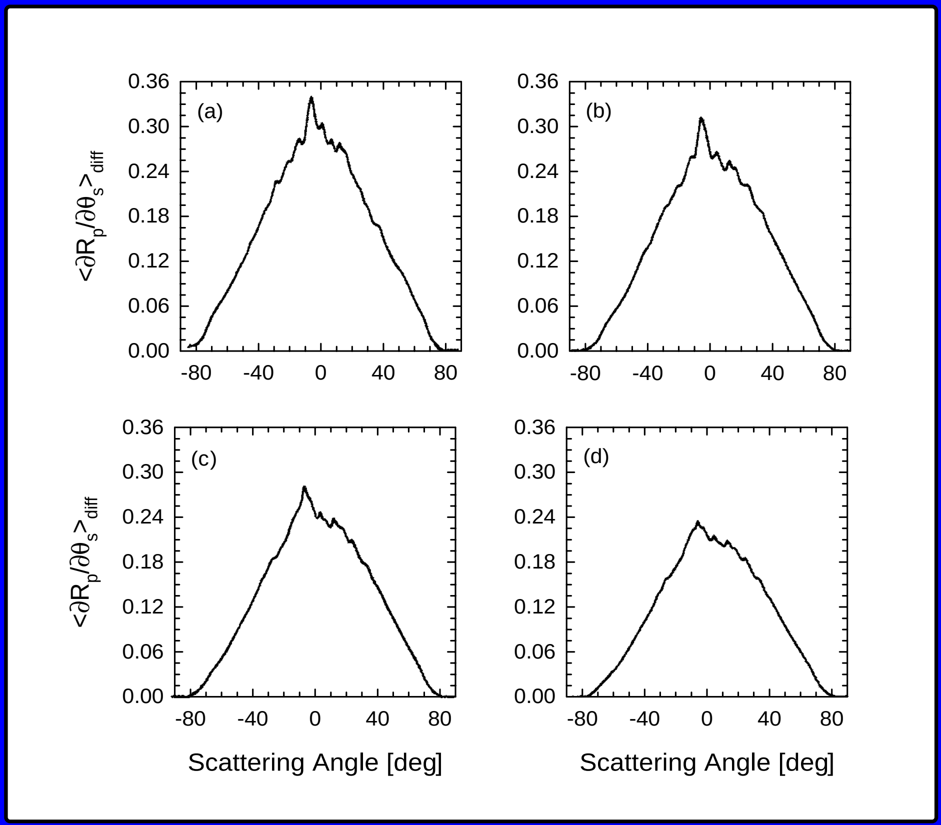
N–type semiconductor in a magnetic field



The dispersion curves for surface polaritons at a planar $n-$GaAs/vacuum interface. The external magnetic field is parallel to the interface and perpendicular to the direction of propagation of surface polaritons.
(a) $H = 0$; (b) $H = 2.46 \cdot 10^4 G$; (c) $H = 3.28 \cdot 10^4 G$; (d) $H = 4.1 \cdot 10^4 G$.

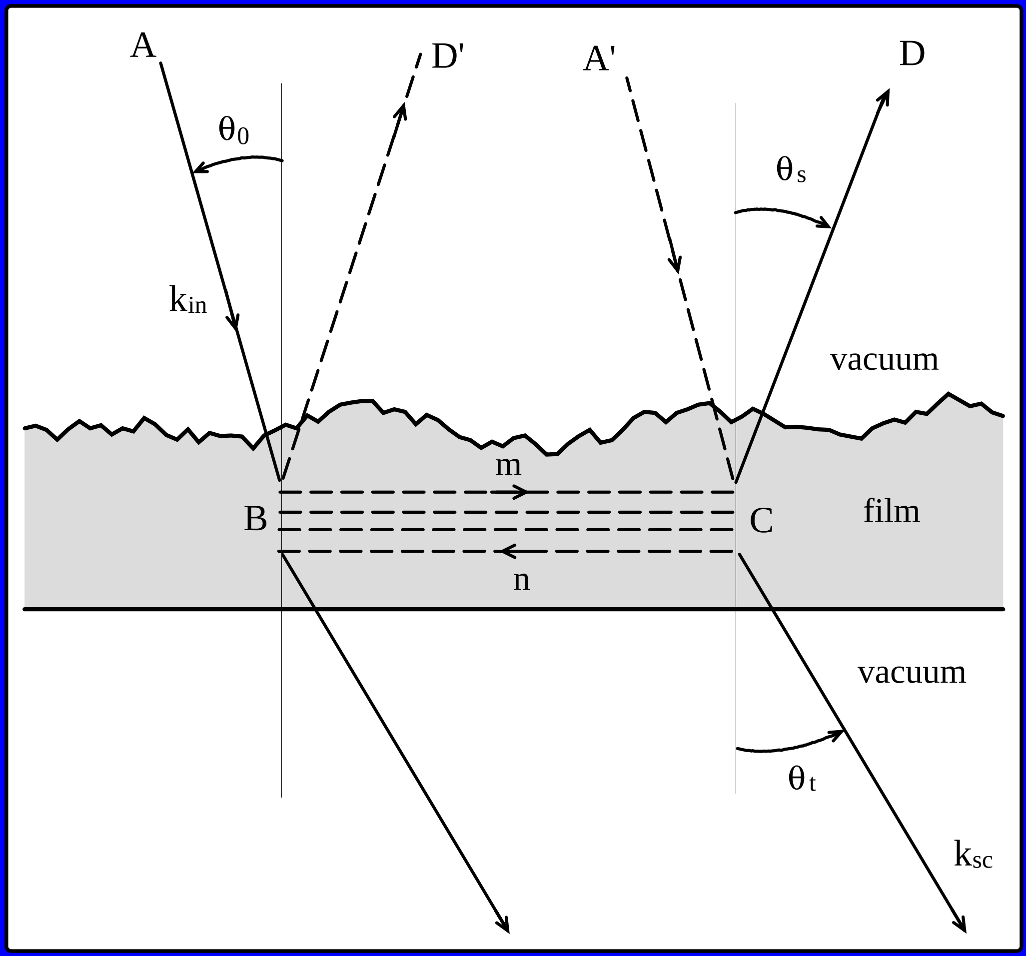


$$\sin\theta_s = -\sin\theta_0 + \frac{c}{\omega}\left[k_+(\omega) - k_-(\omega)\right]$$



$n-$GaAs: $\delta = 3.14\mu$m, $a = 15.14\mu$m, $\lambda = 43.2\mu$m, and $\theta_0 = 5°$.
(a) $H = 0$; (b) $H = 2.46 \cdot 10^4 G$; (c) $H = 3.28 \cdot 10^4 G$; (d) $H = 4.1 \cdot 10^4 G$.

Scattering of Light from, and its Transmission through, Guided Wave–Supporting Structures Possessing Random Surfaces



In a structure that support N guided waves with wavenumbers $k_1(\omega), k_2(\omega), \ldots k_N(\omega)$, the phase difference between a scattering path $(ABCD)_m$ and its reciprocal partner $(A'BCD')_n$ is

$$\Delta\phi_{nm} = \mathbf{r}_{BC} \cdot (\mathbf{k}_{in} + \mathbf{k}_{sc}) + |\mathbf{r}_{BC}|(k_n - k_m)$$

Constructive interference ($\Delta\phi = 0$) now occurs if

$$\begin{aligned} k_n &= k_m \qquad \mathbf{k}_{sc} = -\mathbf{k}_{in} \qquad \text{(enhanced backscattering)} \\ k_n &\neq k_m \qquad \mathbf{k}_{sc} \neq -\mathbf{k}_{in} \qquad \text{(satellite peaks).} \end{aligned}$$

In an $N-$ mode structure, $k_1(\omega), k_2(\omega), \ldots k_N(\omega)$, satellite peaks occur at scattering angles θ_s given by

$$\sin\theta_s^{n,m} = -\sin\theta_0 \pm \frac{c}{\omega}\left[k_n(\omega) - k_m(\omega)\right],$$

where θ_0 is the angle of incidence.

In transmission the phase difference between a scattering path $(ABCE)_m$ and its reciprocal partner $(A'BCE')_n$ is

$$\begin{aligned} \Delta\phi_{nm} &= \mathbf{r}_{BC} \cdot (\mathbf{k}_{in} + \mathbf{k}^*_{sc}) + |\mathbf{r}_{BC}|(k_n - k_m) \\ \mathbf{k}^*_{sc} &= k_{sc,\|}\hat{x}_1 + k_{sc,\perp}(-\hat{x}_3) \end{aligned}$$

Constructive interference ($\Delta\phi = 0$) now occurs if

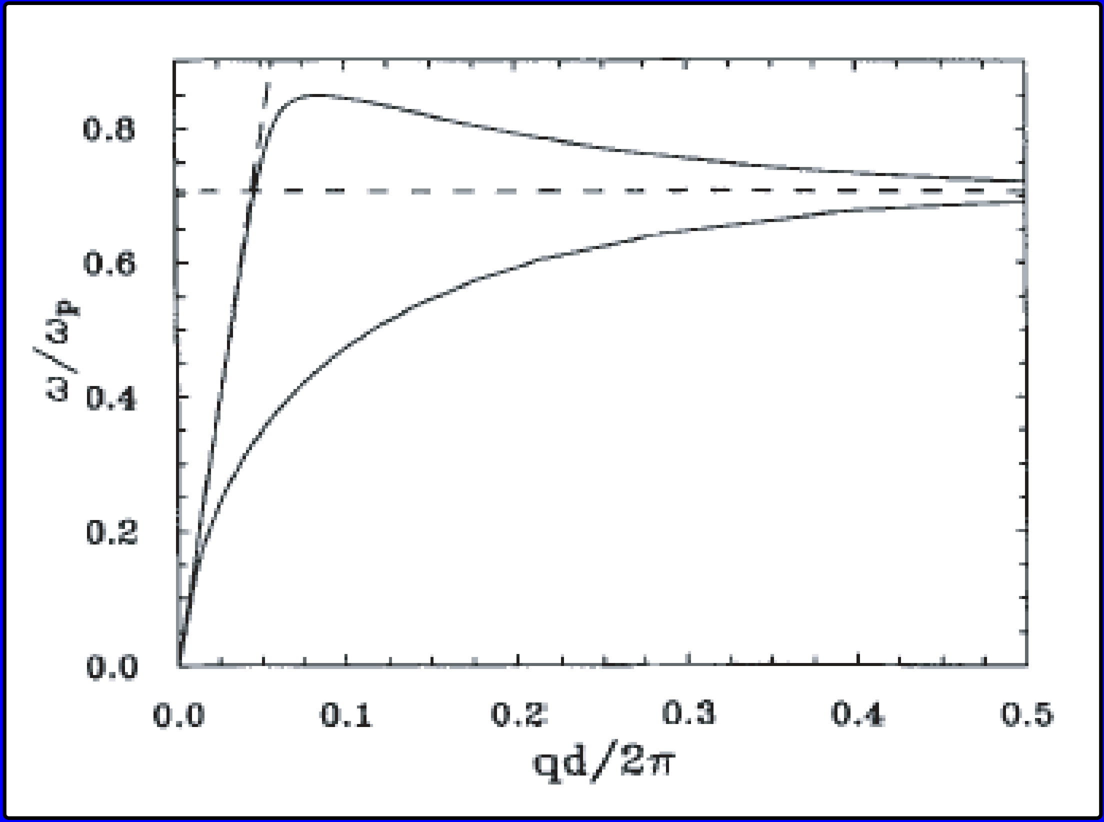
$$\begin{aligned} k_n &= k_m \qquad \mathbf{k}^*_{sc} = -\mathbf{k}_{in} \qquad \text{(enhanced transmission)} \\ k_n &\neq k_m \qquad \mathbf{k}^*_{sc} \neq -\mathbf{k}_{in} \qquad \text{(satellite peaks).} \end{aligned}$$

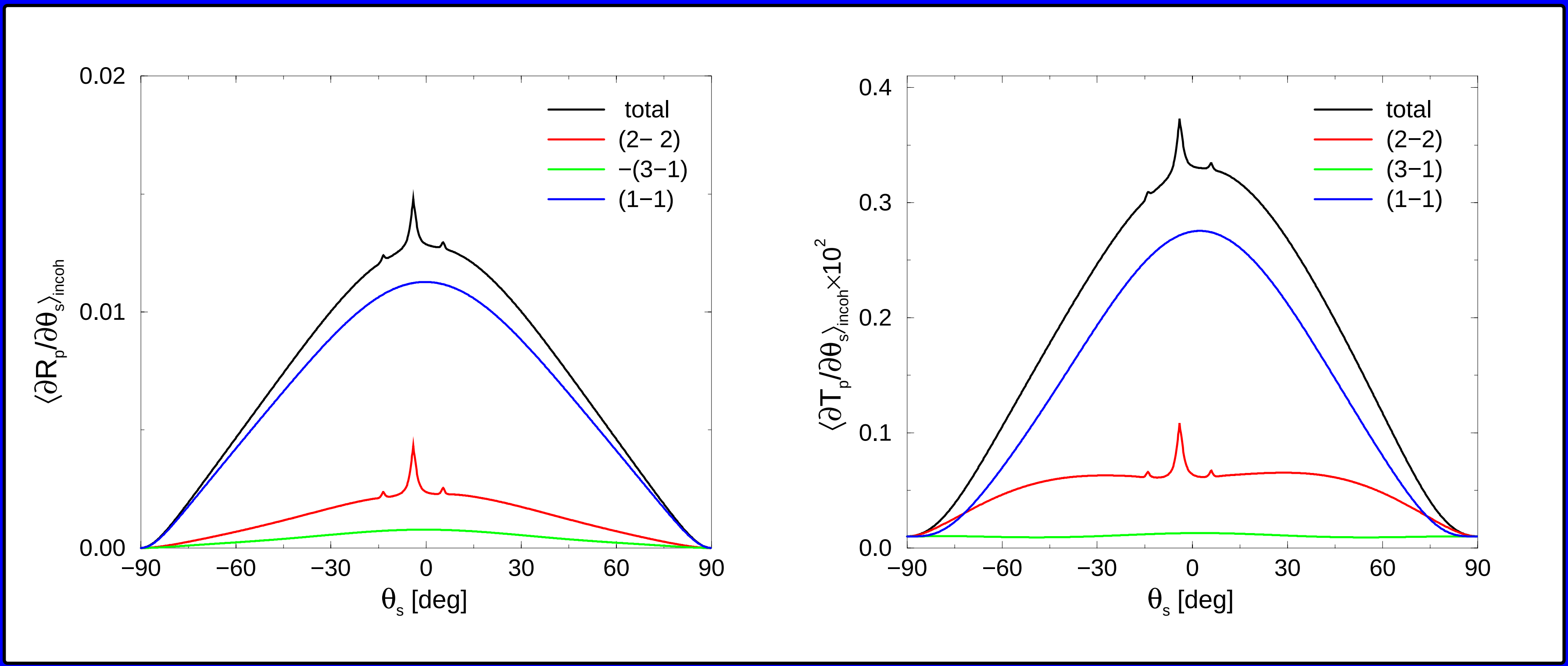
In an $N-$ mode structure, $k_1(\omega), k_2(\omega), \ldots k_N(\omega)$, satellite peaks occur at transmission angles θ_t given by

$$\sin\theta_t^{n,m} = -\sin\theta_0 \pm \frac{c}{\omega}\left[k_n(\omega) - k_m(\omega)\right],$$

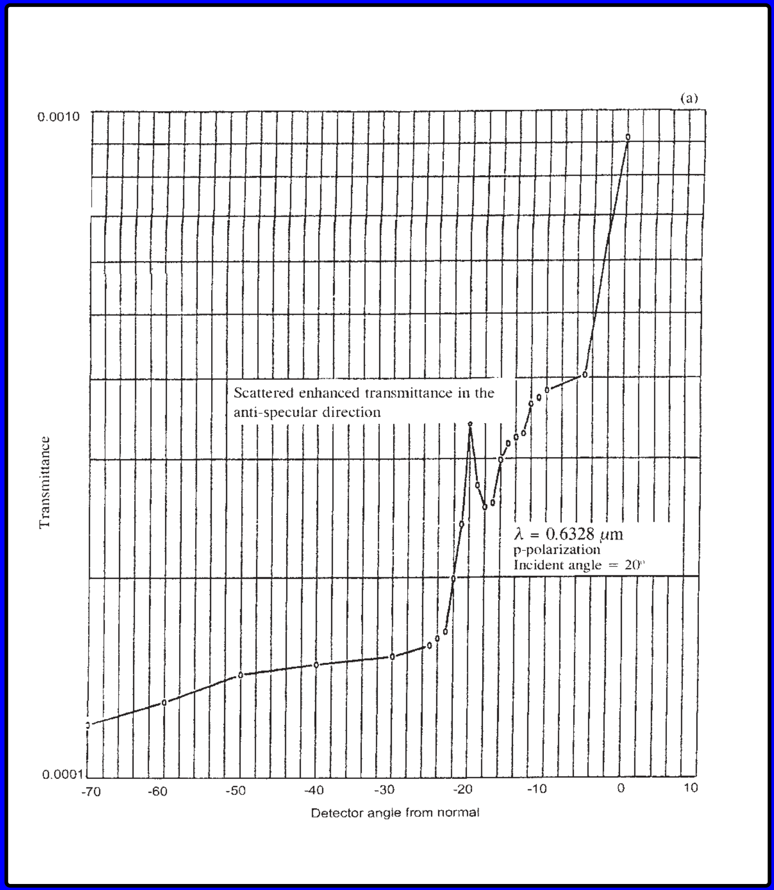
where θ_0 is the angle of incidence.

A Free–Standing Metal Film



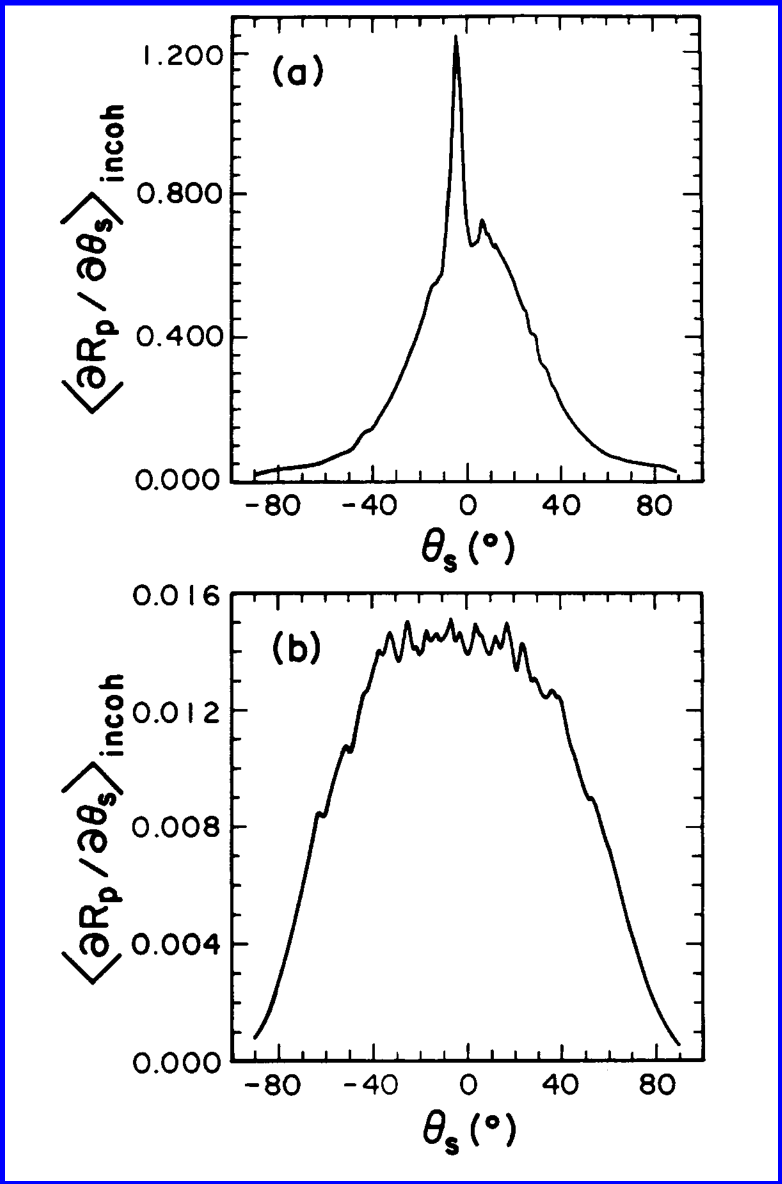


Silver: $\lambda = 457.9$nm, $\epsilon(\omega) = -7.5 + i0.24$, $\delta = 5$nm, $a = 125$nm, $d = 35$nm, $\theta_0 = 4^\circ$



Silver/Glass: $\lambda = 632.8\text{nm}$, $\delta = 11.8\text{nm}$, $a = 120\text{nm}$, $d = 85\text{nm}$, $D = 5\text{mm}$, $\theta_0 = 20^\circ$

A Thick Dielectric Film on a Perfect Conductor



$BaSO_4$

$n = 1.628 + i0.003$

$\delta = 1.2\mu\text{m}$

$a = 2\mu\text{m}$

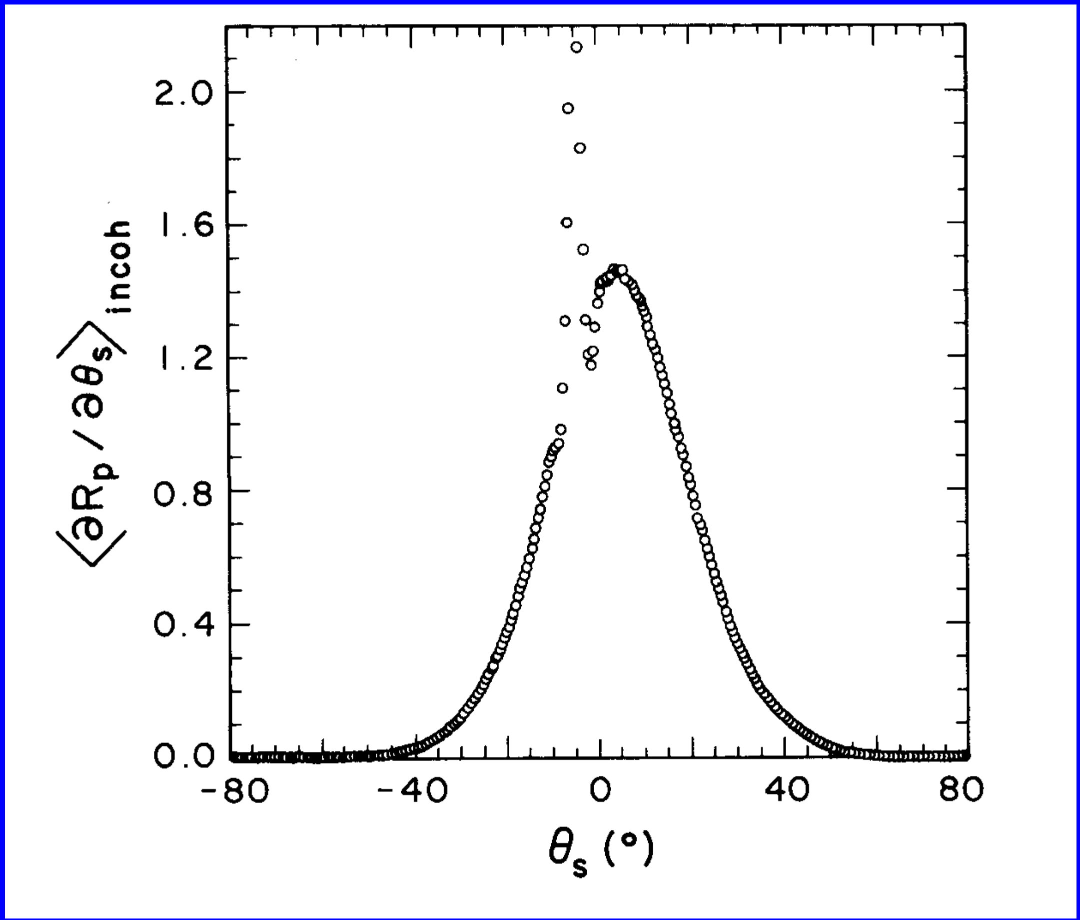
$\lambda = 632.8\text{nm}$

$d = 4.8\mu\text{m}$

$\theta_0 = 5°$

(b) Semi-infinite $BaSO_4$

A Dielectric Film on a Planar Gold Substrate



$n = 1.41$
$\delta = 1.08\mu\text{m}$
$a = 3.06\mu\text{m}$
$\lambda = 632.8\text{nm}$
$d = 8.5\mu\text{m}$
$\theta_0 = 5°$