Designer Surfaces

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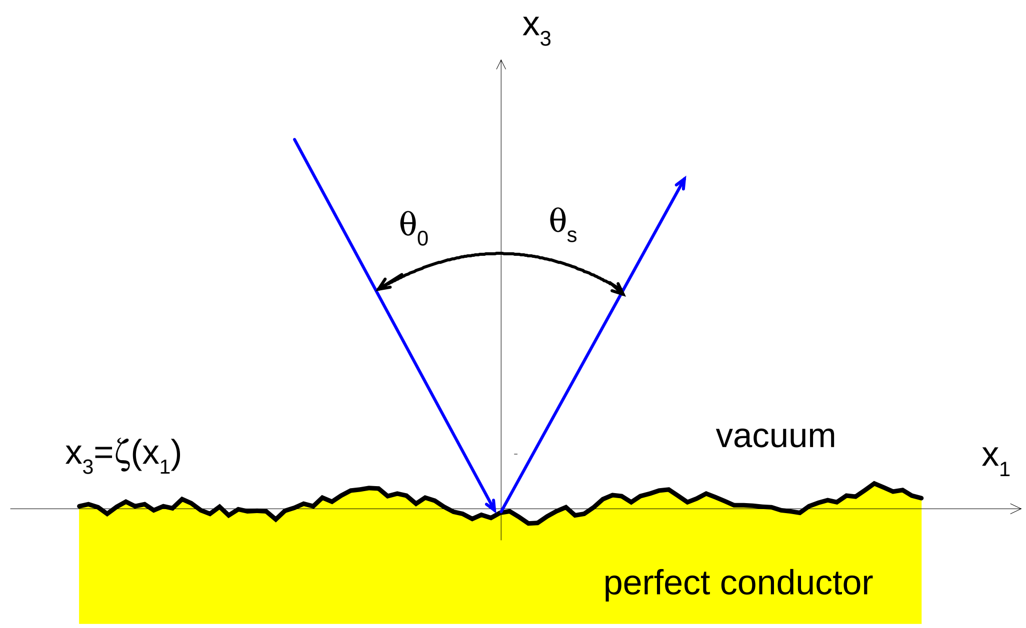
  - Band–Limited Uniform Diffusers
  - Lambertian Diffusers
- Two–Dimensional Surfaces
  - Band–Limited Uniform Diffusers within a Circular Domain
  - Band–Limited Uniform Diffusers within a Rectangular Domain
  - Surfaces that Produce a Specified Distribution of Scattered Intensity Along the x_3- Axis
- Conclusions

Scattering Geometry



The surface profile function $\zeta(x_1)$ is a single-valued function of x_1 that is differentiable and constitutes a random process, but not necessarily a stationary one.

S-Polarization

$$\mathbf{E}(\mathbf{x};t) = (0, E_2(x_1, x_3|\omega), 0)\exp(-i\omega t)$$

$$\mathbf{H}(\mathbf{x};t) = (H_1(x_1, x_3|\omega), 0, H_3(x_1, x_3|\omega))\exp(-i\omega t)$$

$x_3 > \zeta(x_1)_{max}$

$$E_2^{>}(x_1, x_3|\omega) = \exp[ikx_1 - i\alpha_0(k)x_3]$$

$$+ \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k)\exp[iqx_1 + i\alpha_0(q)x_3],$$

where

$$\begin{aligned} \alpha_0(q) &= [(\omega/c)^2 - q^2]^{\frac{1}{2}} \qquad |q| < \omega/c \\ &= i[q^2 - (\omega/c)^2]^{\frac{1}{2}} \qquad |q| > \omega/c. \end{aligned}$$

Mean Differential Reflection Coefficient $\langle \partial R/\partial\theta_s \rangle$:

$$\frac{\partial R}{\partial \theta_s} d\theta_s \quad = \quad \text{fraction of total time-averaged incident flux scattered into} (\theta_s, \theta_s + d\theta_s)$$

$$\frac{\partial R}{\partial \theta_s} \quad = \quad \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2\theta_s}{\cos\theta_0} |R(q|k)|^2,$$

where L_1 is the length of the x_1-axis covered by the random surface, while θ_0 and θ_s are the angles of incidence and scattering, respectively, and are related to the wavenumbers k and q by

$$k = (\omega/c)\sin\theta_0, \qquad q = (\omega/c)\sin\theta_s.$$

Since we are concerned with the scattering of light from a randomly rough surface, it is the mean differential reflection coefficient that we need to calculate. It is given by

$$\boxed{\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2 \theta_s}{\cos \theta_0} \langle |R(q|k)|^2 \rangle,}$$

where the angle brackets denote an average over the ensemble of realizations of $\zeta(x_1)$.

In the Kirchhoff approximation

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{1}{\cos \theta_0} \left[\frac{1 + \cos(\theta_0 + \theta_s)}{\cos \theta_0 + \cos \theta_s} \right]^2$$

$$\times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-i(q-k)(x_1 - x_1')] \langle \exp[-ia(\zeta(x_1) - \zeta(x_1'))] \rangle$$

$$a = \frac{\omega}{c}(\cos \theta_0 + \cos \theta_s).$$

Geometrical Optics Limit of the Kirchhoff Approximation

$$x_1' = x_1 + u$$

$$\zeta(x_1) - \zeta(x_1') = \zeta(x_1) - \zeta(x_1 + u) \cong -u\zeta'(x_1)$$

$$\begin{aligned}\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{1}{\cos\theta_0}\left[\frac{1+\cos(\theta_0+\theta_s)}{\cos\theta_0+\cos\theta_s}\right]^2 \\ &\times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp[i(q-k)u]\langle \exp iau\zeta'(x_1)\rangle.\end{aligned}$$

$$x_n = nb \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$\zeta(x_1) = a_n x_1 + b_n \qquad nb \leq x_1 \leq (n+1)b,$$

where the $\{a_n\}$ are independent identically distributed random deviates, and b is a characteristic length. Therefore, the probability density function (pdf) of a_n,

$$f(\gamma) = \langle \delta(\gamma - a_n) \rangle,$$

is independent of n.
For this surface

$$\zeta'(x_1) = a_n \qquad nb < x_1 < (n+1)b.$$

In order that the surface be continuous at $x_1 = (n+1)b$

$$a_n(n+1)b + b_n = a_{n+1}(n+1)b + b_{n+1}$$

or

$$b_{n+1} = b_n - (n+1)b\,(a_{n+1} - a_n).$$

It is convenient to choose $b_0 = 0$, and we do so.

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp[i(q-k)u]\langle \exp iau\zeta'(x_1)\rangle$$

$$= \int_{-\infty}^{\infty} du \exp[i(q-k)u] \sum_{n=-N}^{N-1} \int_{nb}^{(n+1)b} dx_1 \langle \exp iaua_n \rangle$$

$$= \int_{-\infty}^{\infty} du \exp[i(q-k)u] \sum_{n=-N}^{N-1} \int_{nb}^{(n+1)b} dx_1 \int_{-\infty}^{\infty} d\gamma f(\gamma) \exp(iau\gamma)$$

$$= L_1 \int_{-\infty}^{\infty} du \; \exp[i(q-k)u] \int_{-\infty}^{\infty} d\gamma f(\gamma) \exp(iau\gamma)$$

$$= L_1 \int_{-\infty}^{\infty} d\gamma f(\gamma) 2\pi\delta(q-k+a\gamma)$$

$$= \frac{2\pi L_1}{a} f\left(\frac{k-q}{a}\right) = \frac{2\pi L_1}{(\omega/c)(\cos\theta_0 + \cos\theta_s)} f\left(\frac{\sin\theta_0 - \sin\theta_s}{\cos\theta_0 + \cos\theta_s}\right),$$

where $L_1 = 2Nb$.

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{[1+\cos(\theta_0+\theta_s)]^2}{\cos\theta_0(\cos\theta_0+\cos\theta_s)^3} f\left(\frac{\sin\theta_0-\sin\theta_s}{\cos\theta_0+\cos\theta_s}\right).$$

Set

$$\frac{\sin\theta_0-\sin\theta_s}{\cos\theta_0+\cos\theta_s} = -\gamma,$$

so that

$$\cos\theta_s = \frac{(1-\gamma^2)\cos\theta_0 - 2\gamma\sin\theta_0}{1+\gamma^2},$$
$$\sin\theta_s = \frac{(1-\gamma^2)\sin\theta_0 + 2\gamma\cos\theta_0}{1+\gamma^2}.$$

Then

$$f(\gamma) = \frac{2}{1+\gamma^2}\frac{\cos\theta_0}{\cos\theta_0+\gamma\sin\theta_0}\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle(-\gamma,\theta_0).$$

At normal incidence, $\theta_0 = 0$

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{1+\cos\theta_s} f\left(\frac{-\sin\theta_s}{1+\cos\theta_s} \right).$$

It follows from this expression and the normalization of $f(\gamma)$ that

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\langle \frac{\partial R}{\partial \theta_s} \right\rangle d\theta_s = 1.$$

Set

$$\frac{\sin\theta_s}{1+\cos\theta_s} = \tan\frac{\theta_s}{2} = \gamma.$$

Then

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle (\gamma) = \frac{1}{2}(1+\gamma^2) f(-\gamma),$$

so that

$$f(\gamma) = \frac{2}{1+\gamma^2} \left\langle \frac{\partial R}{\partial \theta_s} \right\rangle (-\gamma),$$

independent of the wavelength of the incident light.

A Band-Limited Uniform Diffuser

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{\theta(\theta_m - |\theta_s|)}{2\theta_m}$$

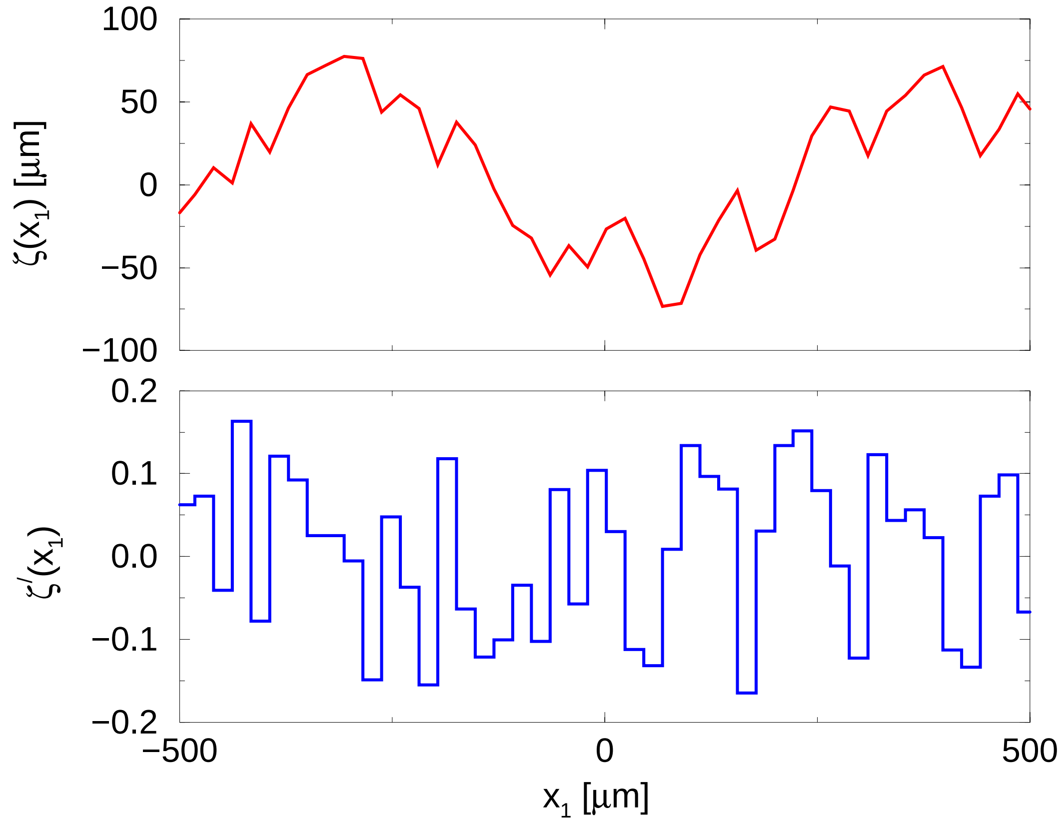
$$= \frac{\theta(\tan(\theta_m/2) - |\tan(\theta_s/2)|)}{2\theta_m}$$

$$= \frac{\theta(\gamma_m - |\gamma|)}{4\tan^{-1}\gamma_m},$$

where $\gamma_m = \tan(\theta_m/2)$. Therefore

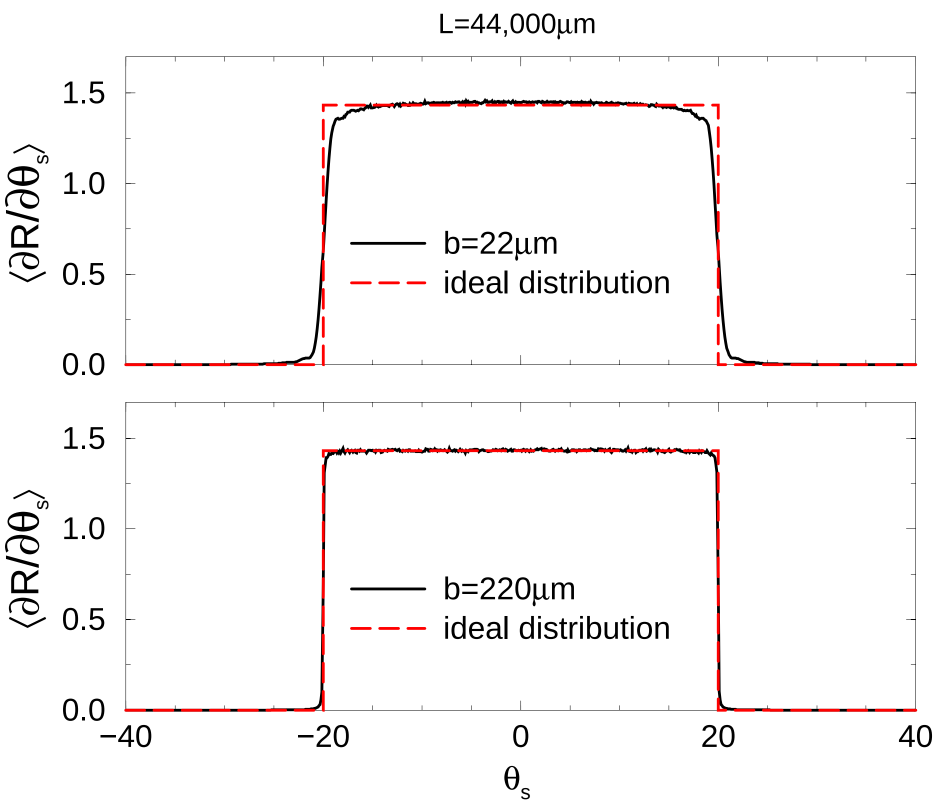
$$f(\gamma) = \frac{1}{2\tan^{-1}\gamma_m}\frac{\theta(\gamma_m - |\gamma|)}{1+\gamma^2}.$$

A Band–Limited Uniform Diffuser



A segment of the surface profile function $\zeta(x_1)$ and the derivative $\zeta'(x_1)$ of this surface profile function. $b = 22\mu\text{m}$.

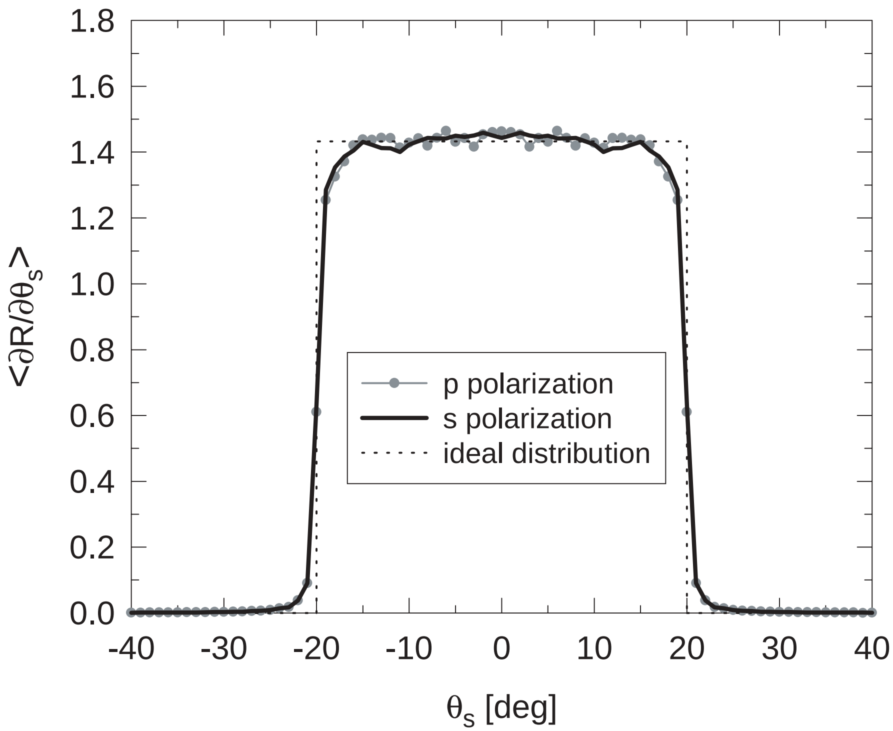
Kirchhoff approximation



The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 120,000$ realizations of perfectly conducting surface profiles: $\theta_m = 20°$, $\lambda = 632.8\text{nm}$.

A rigorous computer simulation calculations

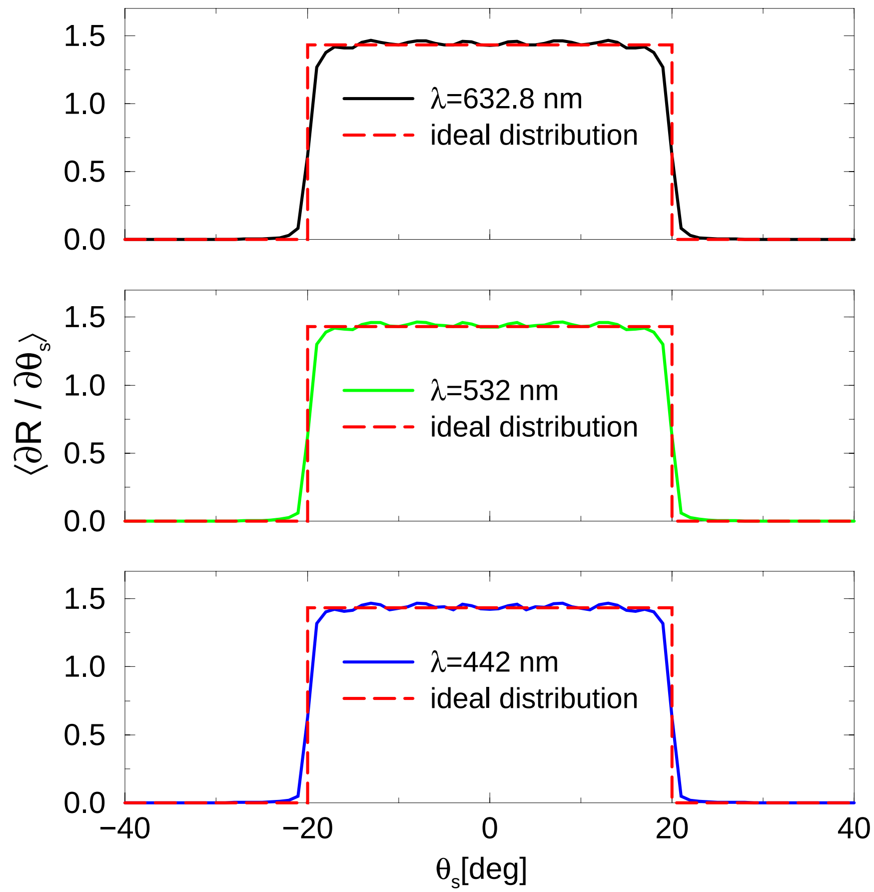
A perfectly conducting random surface



The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 20,000$ realizations of perfectly conducting surface profiles: $\theta_m = 20^\circ$, $b = 22\,\mu$m, $\lambda = 632.8$nm.

A rigorous computer simulation calculations

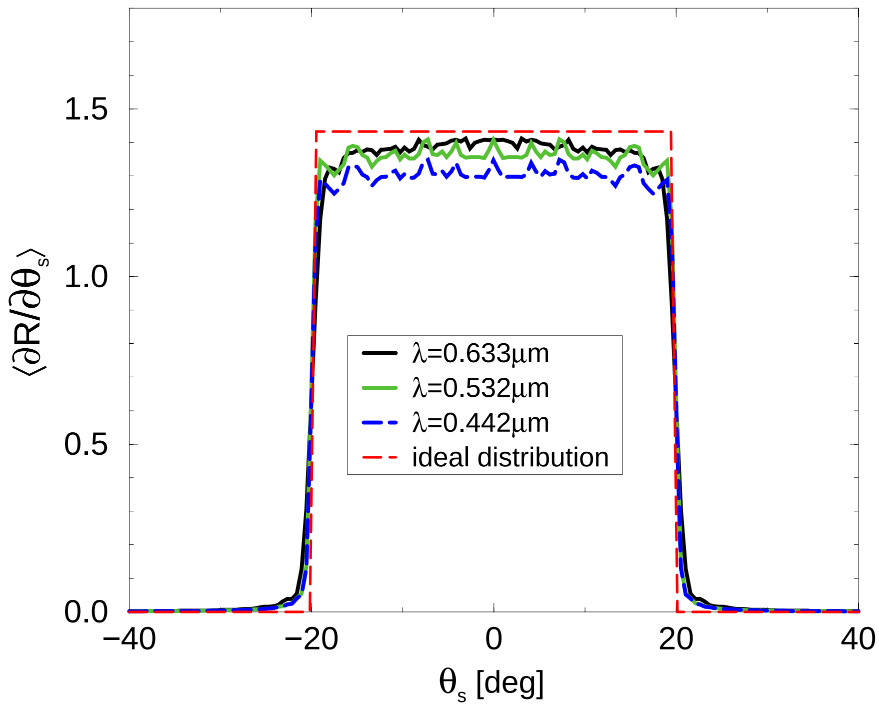
A perfectly conducting random surface



The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 20,000$ realizations of perfectly conducting surface profiles: $\theta_m = 20°$, $b = 22\,\mu$m.

A rigorous computer simulation calculations

A metallic random surface



$\lambda = 0.633\mu\text{m}, \quad \epsilon = -15.91 + i1.07$

$\lambda = 0.532\mu\text{m}, \quad \epsilon = -10.19 + i0.83$

$\lambda = 0.442\mu\text{m}, \quad \epsilon = -5.7 + i0.75$

The mean differential coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 40,000$ realizations of metallic surface profiles in s polarization: $\theta_m = 20^\circ$, $b = 22\,\mu\text{m}$.

A Lambertian Diffuser

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{2}\cos\theta_s \qquad -\frac{\pi}{2} \le \theta_s \le \frac{\pi}{2}$$

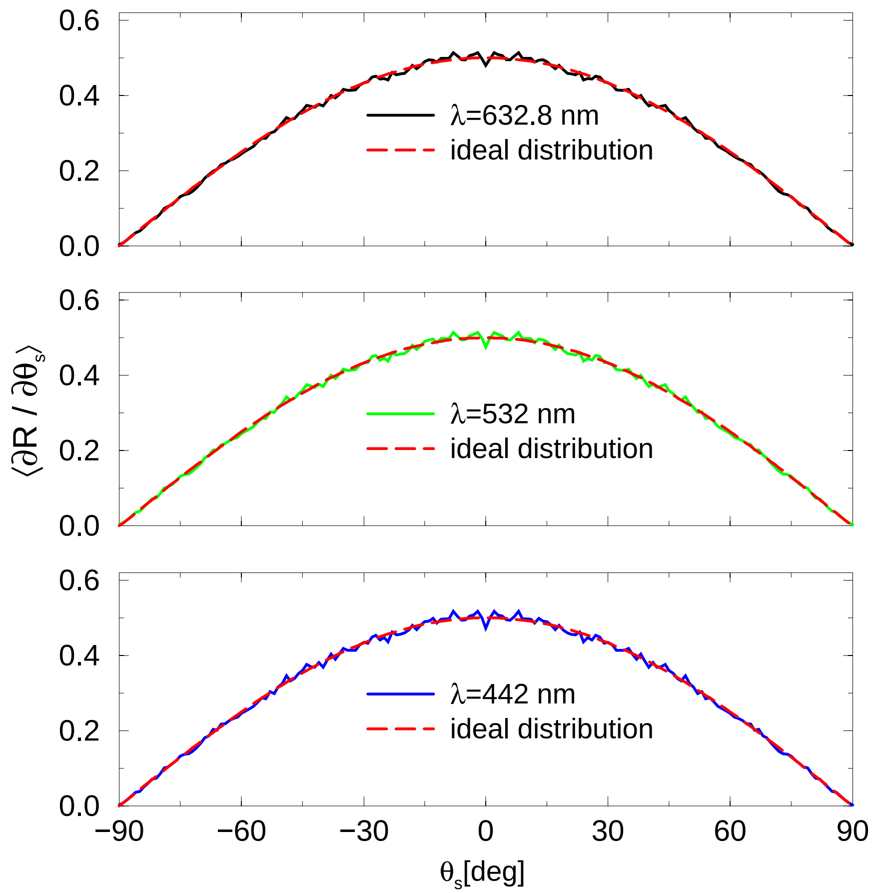
$$= \frac{1}{2}\frac{1-\gamma^2}{1+\gamma^2} \qquad -1 \le \gamma \le 1.$$

Therefore

$$f(\gamma) = \frac{1-\gamma^2}{(1+\gamma^2)^2}\theta(1-|\gamma|).$$

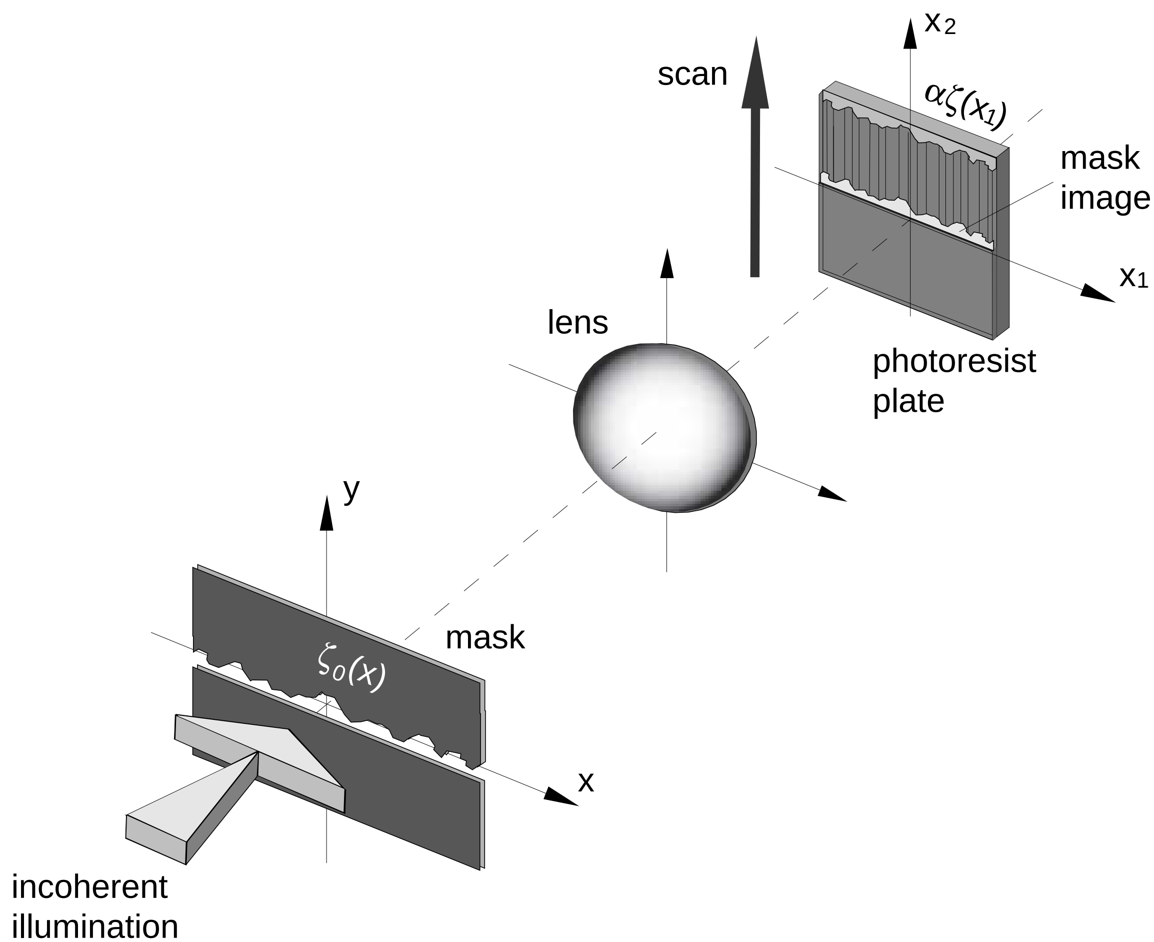
A rigorous computer simulation calculations

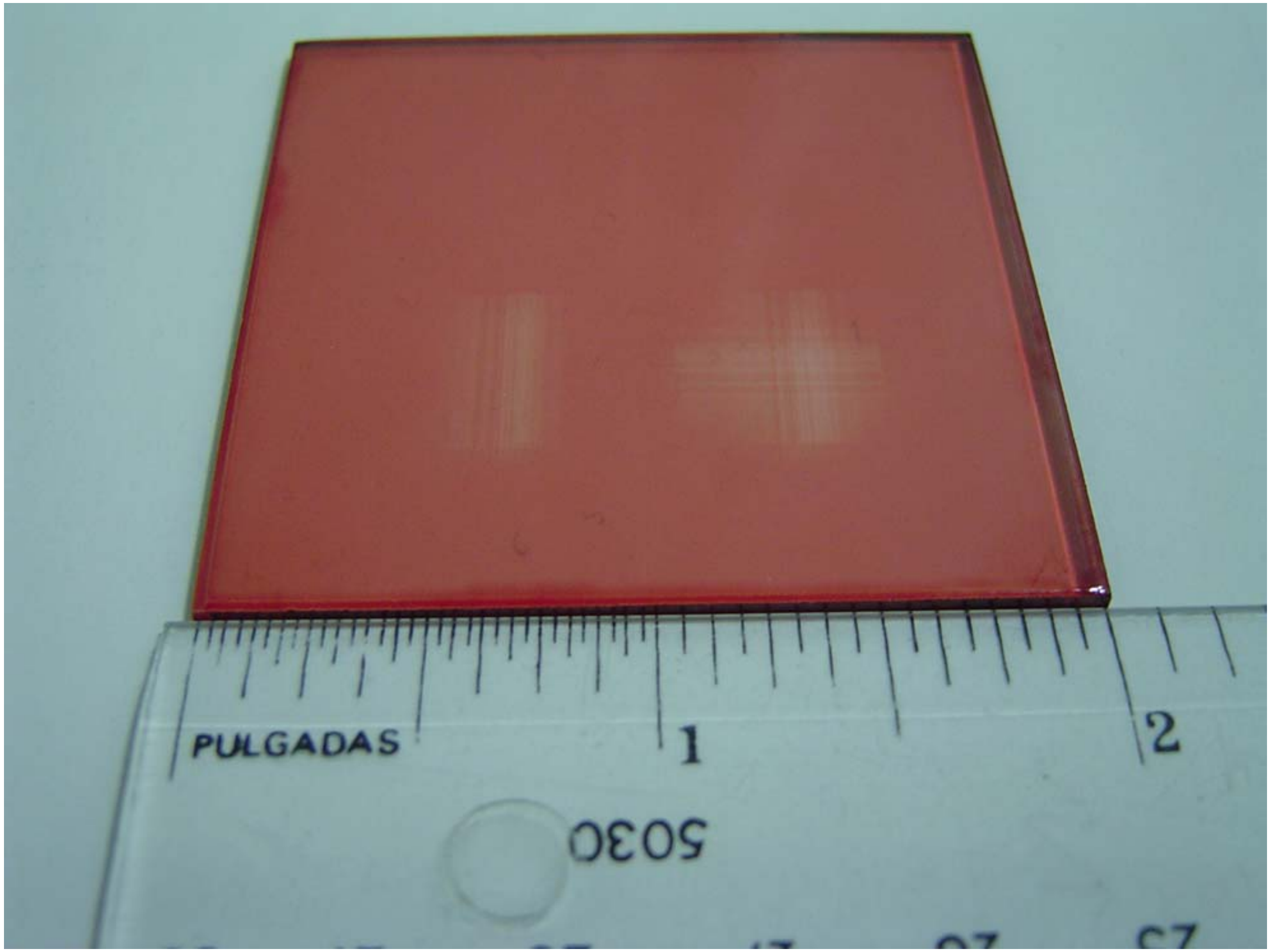
A perfectly conducting random surface



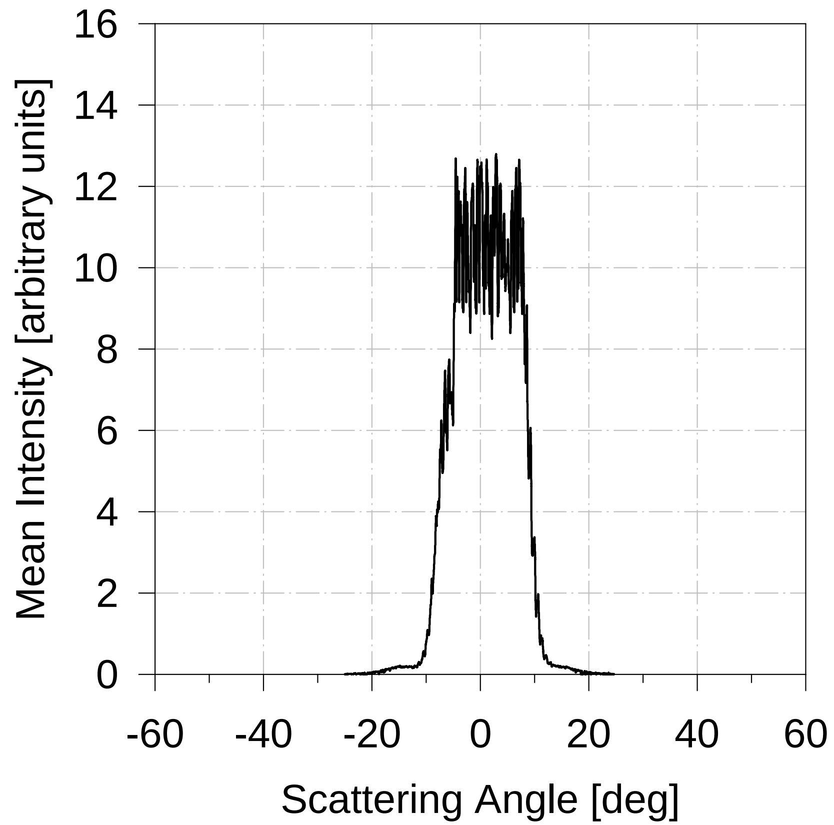
The mean differential reflection coefficient $\langle \partial R/\partial \theta_s \rangle$ estimated from $N_p = 20,000$ realizations of perfectly conducting surface profiles: $b = 22\,\mu$m.

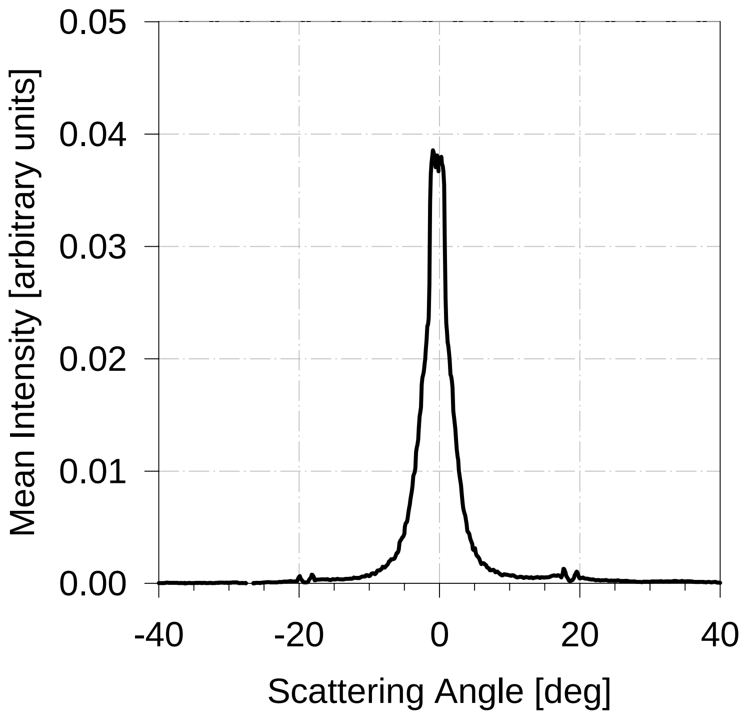
Schematic diagram of the proposed experimental arrangement for the fabrication of surfaces with specified scattering properties




PULGADAS
1
2

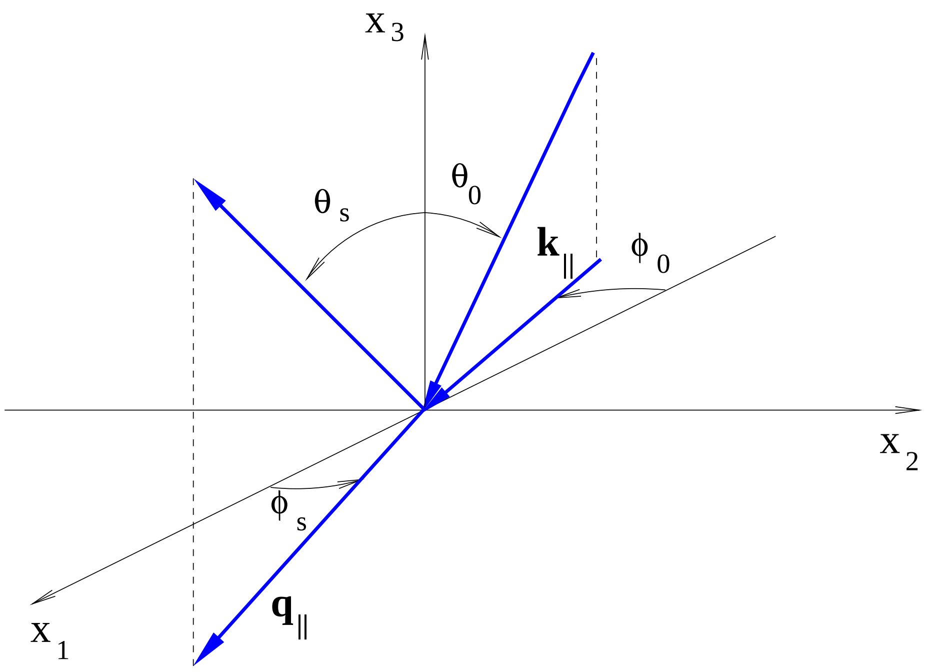
An experimental result for the angular dependence of the mean intensity of $s-$polarized light transmitted through a photoresist film. The angle of incidence is $\theta_0 = 0^\circ$, $\theta_m = 10^\circ$





0.05
0.04
0.03
0.02
0.01
0.00
Mean Intensity [arbitrary units]
-40
-20
0
20
40
Scattering Angle [deg]

Scattering Geometry



The scattering surface is defined by $x_3 = \zeta(\mathbf{x}_\parallel)$, where $\mathbf{x}_\parallel = (x_1, x_2, 0)$. The surface profile function $\zeta(\mathbf{x}_\parallel)$ is a single–valued function of $\mathbf{x}_\parallel$, and is differentiable with respect to x_1 and x_2. It constitutes a random process, but not necessarily a stationary one.

Incident field

$$\Psi(\mathbf{x},t)_{inc} = \hat{\Psi}(\mathbf{x}|\omega)_{inc}\exp(-i\omega t)$$

where

$$\hat{\Psi}(\mathbf{x}|\omega)_{inc} = \exp[i\mathbf{k}_{\parallel}\cdot\mathbf{x}_{\parallel} - i\alpha_0(k_{\parallel})x_3]$$

$$\mathbf{k}_{\parallel} = (k_1, k_2, 0),\ \ \mathbf{x}_{\parallel} = (x_1, x_2, 0)$$

$$\begin{aligned} \alpha_0(k_{\parallel}) &= [(\omega/c)^2 - k_{\parallel}^2]^{\frac{1}{2}} \qquad k_{\parallel} < \omega/c \\ &= i[k_{\parallel}^2 - (\omega/c)^2]^{\frac{1}{2}} \qquad k_{\parallel} > \omega/c. \end{aligned}$$

Scattered field

$$\Psi(\mathbf{x},t)_{sc} = \hat{\Psi}(\mathbf{x}|\omega)_{sc}\exp(-i\omega t)$$

where

$$\hat{\Psi}(\mathbf{x}|\omega)_{sc} = \int \frac{d^2q_{\parallel}}{(2\pi)^2} R(\mathbf{q}_{\parallel}|\mathbf{k}_{\parallel})\exp[i\mathbf{q}_{\parallel}\cdot\mathbf{x}_{\parallel} + i\alpha_0(q_{\parallel})x_3] \qquad x_3 > \zeta(\mathbf{x}_{\parallel})_{max}.$$

Dirichlet boundary condition

$$[\Psi(\mathbf{x},t)_{inc} + \Psi(\mathbf{x},t)_{sc}]|_{x_3=\zeta(\mathbf{x}_{\parallel})} = 0.$$

The differential reflection coefficient $\partial R/\partial\Omega_s$ is defined such that $(\partial R/\partial\Omega_s)d\Omega_s$ is the fraction of the total time-averaged incident flux that is scattered into the element of solid angle $d\Omega_s$ about a given scattering direction. It is given by

$$\frac{\partial R}{\partial\Omega_s} = \frac{1}{S}\left(\frac{\omega}{2\pi c}\right)^2 \frac{\cos^2\theta_s}{\cos\theta_0}|R(\mathbf{q}_{\|}|\mathbf{k}_{\|})|^2,$$

where S is the area of the plane $x_3 = 0$ covered by the random surface, and

$$\begin{aligned} \mathbf{q}_{\|} &= (\omega/c)\sin\theta_s(\cos\phi_s, \sin\phi_s, 0) \\ \mathbf{k}_{\|} &= (\omega/c)\sin\theta_0(\cos\phi_0, \sin\phi_0, 0). \end{aligned}$$

In scattering from a random surface it is the mean differential reflection coefficient that is of interest. It is given by

$$\left\langle\frac{\partial R}{\partial\Omega_s}\right\rangle = \frac{1}{S}\left(\frac{\omega}{2\pi c}\right)^2 \frac{\cos^2\theta_s}{\cos\theta_0}\langle|R(\mathbf{q}_{\|}|\mathbf{k}_{\|})|^2\rangle.$$

In the Kirchhoff approximation

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle = \frac{1}{S}\left(\frac{\omega}{2\pi c}\right)^2 \frac{[1+\cos\theta_0\cos\theta_s - \sin\theta_0\sin\theta_s\cos(\phi_s-\phi_0)]^2}{\cos\theta_0(\cos\theta_0+\cos\theta_s)^2}$$

$$\times \int d^2x_\parallel \int d^2x'_\parallel \exp[-i(\mathbf{q}_\parallel - \mathbf{k}_\parallel)\cdot(\mathbf{x}_\parallel - \mathbf{x}'_\parallel)]$$

$$\times\langle \exp[-i(\omega/c)(\cos\theta_0+\cos\theta_s)(\zeta(\mathbf{x}_\parallel)-\zeta(\mathbf{x}'_\parallel))]\rangle.$$

In the case of normal incidence, $\theta_0 = 0$, $\mathbf{k}_\parallel = 0$,

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle = \frac{1}{S}\left(\frac{\omega}{2\pi c}\right)^2 \int d^2x_\parallel \int d^2x'_\parallel \exp[-i\mathbf{q}_\parallel\cdot(\mathbf{x}_\parallel - \mathbf{x}'_\parallel)]$$

$$\times\langle \exp[-ia(\zeta(\mathbf{x}_\parallel)-\zeta(\mathbf{x}'_\parallel))]\rangle,$$

where

$$a = (\omega/c)(1+\cos\theta_s).$$

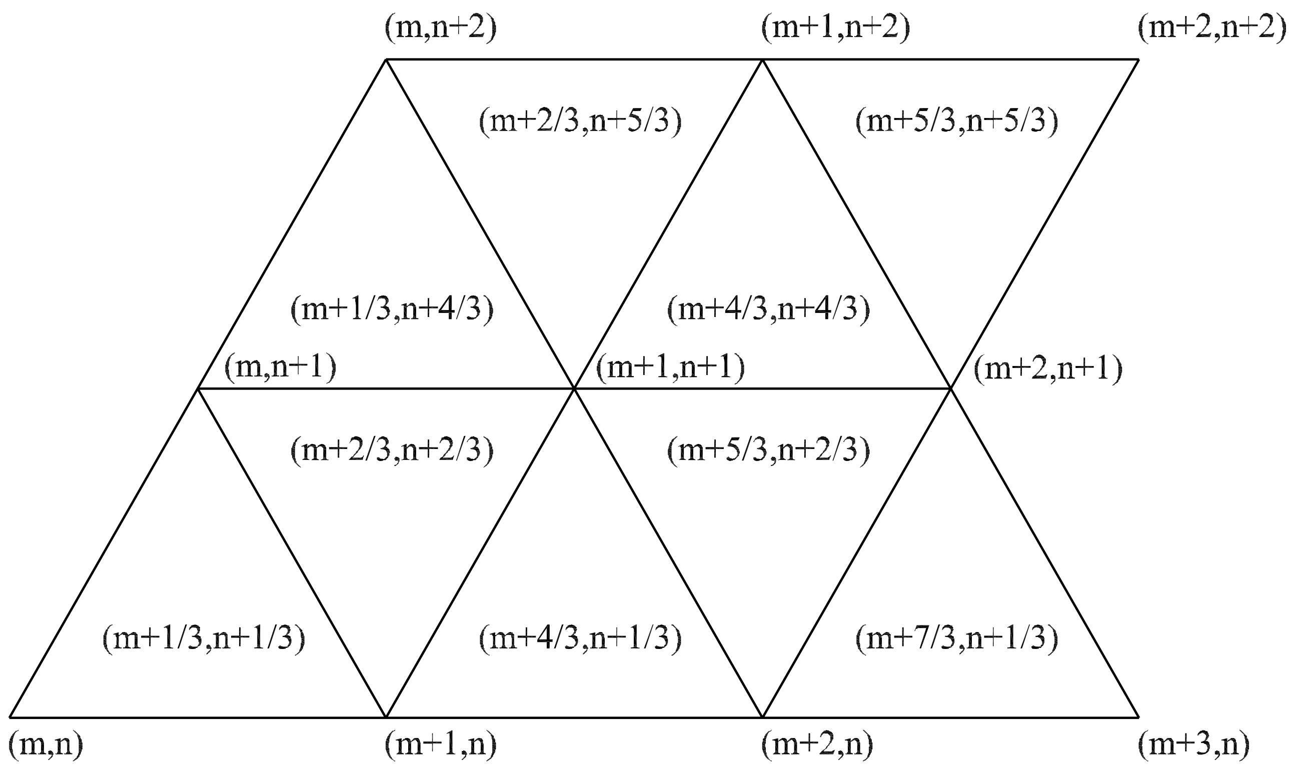
Geometrical optics limit of the Kirchhoff approximation:

$$
\begin{aligned}
\mathbf{x}'_{\|} &= \mathbf{x}_{\|} - \mathbf{u}_{\|} \\
\zeta(\mathbf{x}_{\|}) - \zeta(\mathbf{x}'_{\|}) &\rightarrow \zeta(\mathbf{x}_{\|}) - \zeta(\mathbf{x}_{\|} - \mathbf{u}_{\|}) \\
&\cong \mathbf{u}_{\|} \cdot \nabla\zeta(\mathbf{x}_{\|}).
\end{aligned}
$$

Therefore

$$
\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle = \frac{1}{S}\left(\frac{\omega}{2\pi c}\right)^2 \int d^2u \exp(-i\mathbf{q}_{\|} \cdot \mathbf{u}_{\|}) \int d^2x_{\|} \langle \exp[-ia\mathbf{u}_{\|} \cdot \nabla\zeta(\mathbf{x}_{\|})] \rangle.
$$

To evaluate the double integral we begin by covering the x_1x_2-plane by equilateral triangles of edge b:



The vertices of these triangles are given by the vectors $\mathbf{x}_\parallel(m,n) = m\mathbf{a}_1 + n\mathbf{a}_2$, where $\mathbf{a}_1 = (b,0)$, $\mathbf{a}_2 = \left(\frac{b}{2}, \frac{\sqrt{3}b}{2}\right)$. Each triangle is labeled by the coordinates of its center of gravity.

For (x_1, x_2) contained in the triangle $(m + \frac{1}{3}, n + \frac{1}{3})$

$$\zeta(\mathbf{x}_\parallel) = b^{(0)}_{m+\frac{1}{3},n+\frac{1}{3}} + a^{(1)}_{m+\frac{1}{3},n+\frac{1}{3}} x_1 + a^{(2)}_{m+\frac{1}{3},n+\frac{1}{3}} x_2.$$

For (x_1, x_2) contained in the triangle $(m + \frac{2}{3}, n + \frac{2}{3})$

$$\zeta(\mathbf{x}_\parallel) = b^{(0)}_{m+\frac{2}{3},n+\frac{2}{3}} + a^{(1)}_{m+\frac{2}{3},n+\frac{2}{3}} x_1 + a^{(2)}_{m+\frac{2}{3},n+\frac{2}{3}} x_2.$$

The coefficients $a^{(1,2)}_{m+\frac{1}{3},n+\frac{1}{3}}$ and $a^{(1,2)}_{m+\frac{2}{3},n+\frac{2}{3}}$ are assumed to be independent indentically distributed random deviates. Therefore the joint probability density function

$$\begin{aligned}
&\langle \delta(\gamma_1 - a^{(1)}_{m+\frac{1}{3},n+\frac{1}{3}}) \delta(\gamma_2 - a^{(2)}_{m+\frac{1}{3},n+\frac{1}{3}}) \rangle \\
&\quad = \langle \delta(\gamma_1 - a^{(1)}_{m+\frac{2}{3},n+\frac{2}{3}}) \delta(\gamma_2 - a^{(2)}_{m+\frac{2}{3},n+\frac{2}{3}}) \rangle \\
&\quad = f(\gamma_1, \gamma_2)
\end{aligned}$$

is independent of the subscripts to these coefficients.

The double integral becomes

$$\int d^2u_\| e^{-i(q_1u_1+q_2u_2)} \sum_{m,n} \left\{ \int\limits_{m+\frac{1}{3},n+\frac{1}{3}} d^2x_\| \left\langle e^{-iau_1 a^{(1)}_{m+\frac{1}{3},n+\frac{1}{3}} - iau_2 a^{(2)}_{m+\frac{1}{3},n+\frac{1}{3}}} \right\rangle \right.$$

$$\left. + \int\limits_{m+\frac{2}{3},n+\frac{2}{3}} d^2x_\| \left\langle e^{-iau_1 a^{(1)}_{m+\frac{2}{3},n+\frac{2}{3}} - iau_2 a^{(2)}_{m+\frac{2}{3},n+\frac{2}{3}}} \right\rangle \right\}$$

$$= \int d^2u_\| e^{-i(q_1u_1+q_2u_2)} \sum_{m,n} \left\{ \int\limits_{m+\frac{1}{3},n+\frac{1}{3}} d^2x_\| \int d^2\gamma_\| f(\gamma_1,\gamma_2) e^{-iau_1\gamma_1 - iau_2\gamma_2} \right.$$

$$\left. + \int\limits_{m+\frac{2}{3},n+\frac{2}{3}} d^2x_\| \int d^2\gamma_\| f(\gamma_1,\gamma_2) e^{-iau_1\gamma_1 - iau_2\gamma_2} \right\}$$

$$= S \int d^2u_\| \int d^2\gamma_\| f(\gamma_1,\gamma_2) e^{-i(q_1u_1+q_2u_2)} e^{-iau_1\gamma_1 - iau_2\gamma_2}$$

$$= S \int d^2\gamma_\| f(\gamma_1,\gamma_2) 2\pi\delta(q_1 + a\gamma_1) 2\pi\delta(q_2 + a\gamma_2)$$

$$= S \left(\frac{2\pi}{a}\right)^2 f\left(-\frac{q_1}{a}, -\frac{q_2}{a}\right).$$

The mean differential reflection coefficient

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle = \left(\frac{\omega}{ac} \right)^2 f\left(-\frac{q_1}{a}, -\frac{q_2}{a} \right).$$

We can invert this equation to obtain

$$f\left(\frac{q_1}{a}, \frac{q_2}{a} \right) = \left(1 + \cos\theta_s \right)^2 \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle (-q_1, -q_2),$$

where $\langle \partial R/\partial \Omega_s \rangle (q_1, q_2)$ is the mean DRC expressed in terms of the components of the wave vector $\mathbf{q}_{\|}$.

Make the changes of variables

$$\frac{q_1}{a} = s_1 = \frac{\sin\theta_s \cos\phi_s}{1+\cos\theta_s} = \tan\frac{\theta_s}{2}\cos\phi_s$$
$$\frac{q_2}{a} = s_2 = \frac{\sin\theta_s \sin\phi_s}{1+\cos\theta_s} = \tan\frac{\theta_s}{2}\sin\phi_s .$$

It follows that

$$\sin\theta_s = \frac{2s_\parallel}{1+s_\parallel^2}, \cos\theta_s = \frac{1-s_\parallel^2}{1+s_\parallel^2},$$

where

$$s_\parallel = \left(s_1^2 + s_2^2\right)^{\frac{1}{2}} .$$

Then

$$f(s_1, s_2) = \frac{4}{(1+s_\parallel^2)^2}\left\langle \frac{\partial R}{\partial \Omega_s}\right\rangle(-s_1, -s_2),$$

where $\langle \partial R/\partial\Omega_s\rangle(s_1, s_2)$ is the form that $\langle \partial R/\partial\Omega_s\rangle(q_1, q_2)$ takes when q_1 and q_2 are replaced by s_1 and s_2.

A Band-Limited Uniform Diffuser within a Circular Domain

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle (q_1, q_2) = A\theta \left(q_m \frac{\omega}{c} - \sqrt{q_1^2 + q_2^2} \right)$$

$$\begin{aligned} f(s_1, s_2) &= \frac{4A}{(1+s_\parallel^2)^2} \theta \left(q_m - \frac{2s_\parallel}{1+s_\parallel^2} \right) \\ &= \frac{4A}{(1+s_\parallel^2)^2} \theta \left(q^* - s_\parallel \right), \end{aligned}$$

where

$$q^* = \frac{1 - \sqrt{1 - q_m^2}}{q_m}.$$

A is determined by the normalization of $f(s_1, s_2)$:

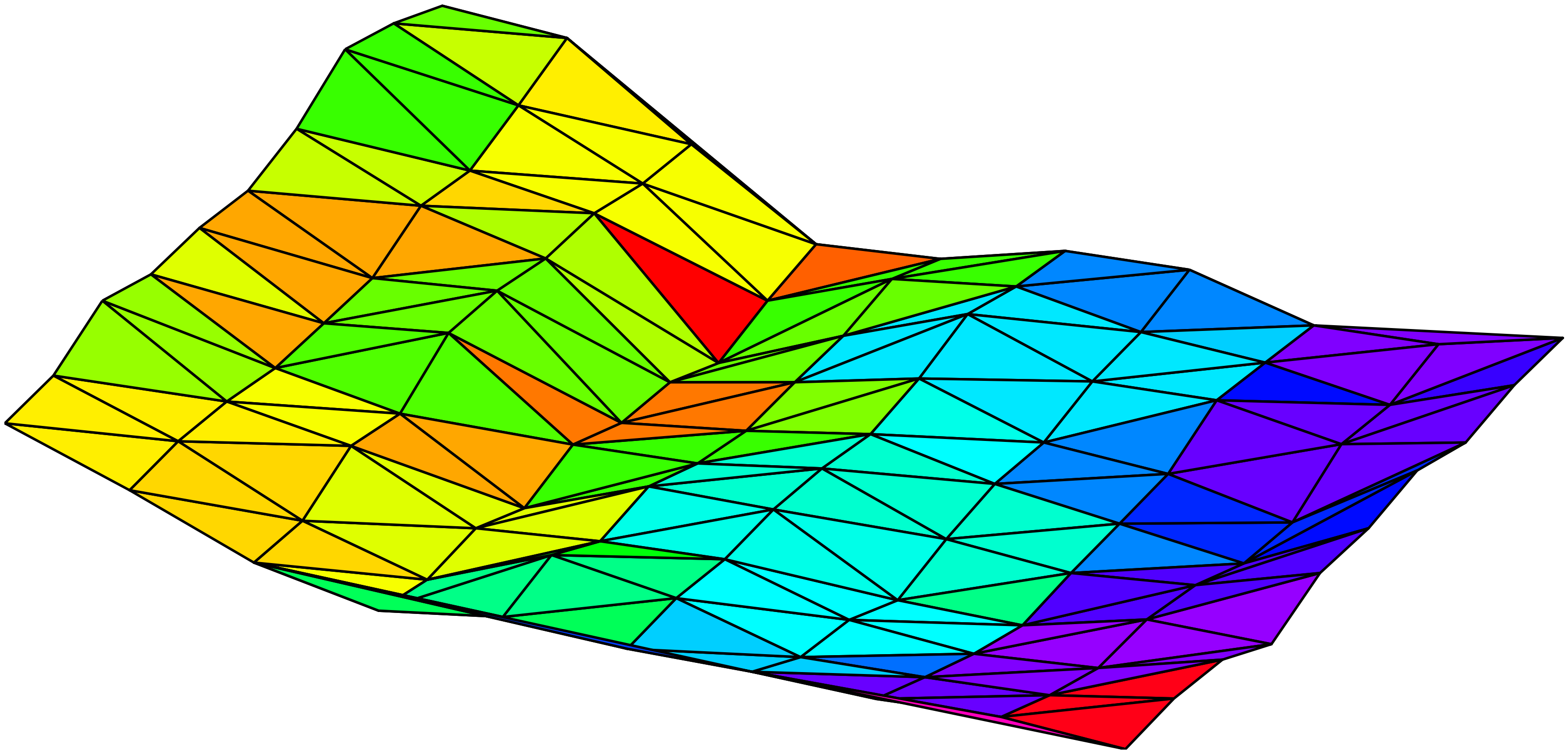
$$A = \frac{1}{4\pi} \frac{1 + q^{*2}}{q^{*2}}.$$

The marginal pdf $f(s_1)$ is

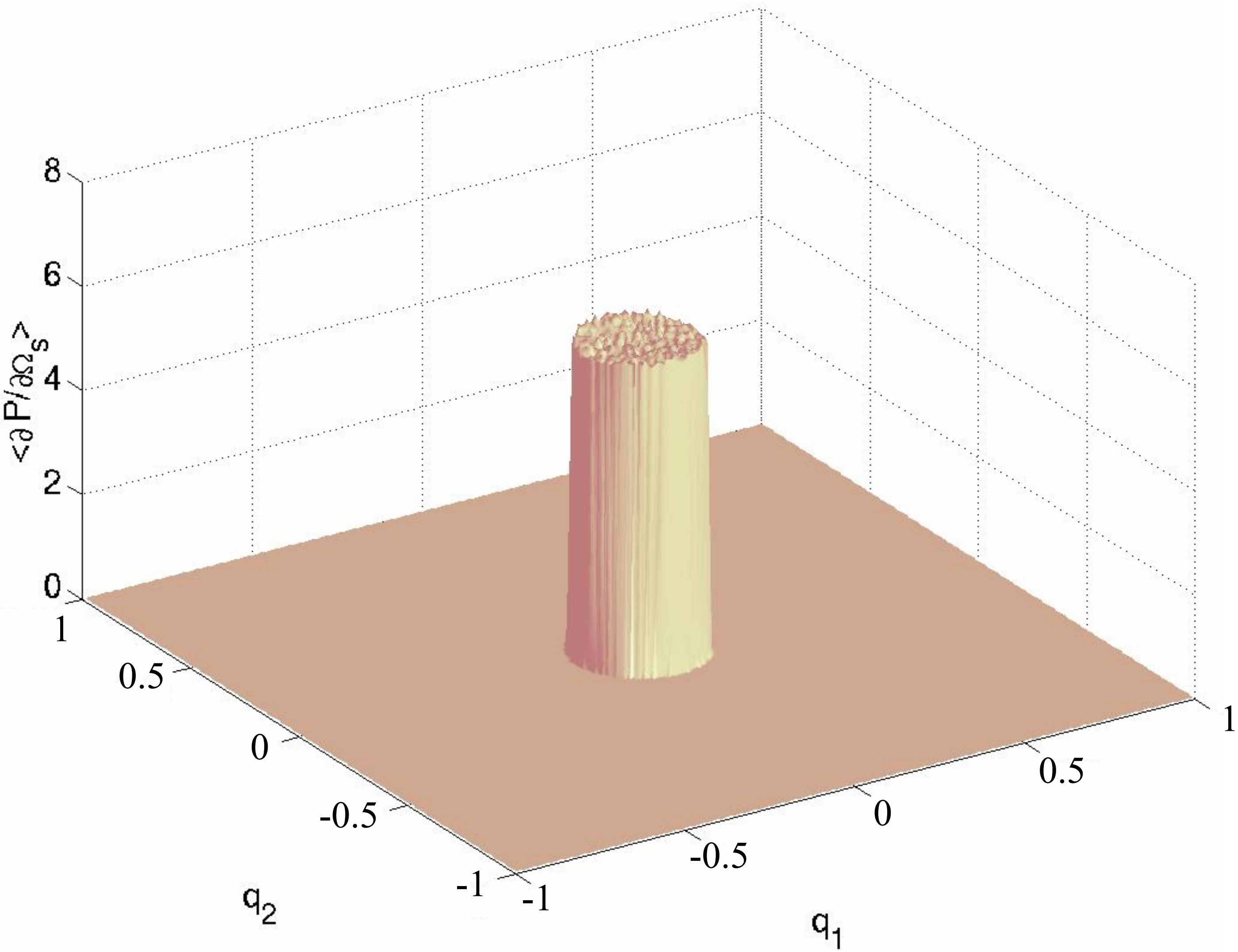
$$
\begin{aligned}
f(s_1) &= \int_{-\infty}^{\infty} ds_2 f(s_1, s_2) = \langle \delta(s_1 - a^{(1)}_{m+\frac{1}{3},n+\frac{1}{3}}) \rangle \\
&\qquad\qquad\qquad\qquad\quad = \langle \delta(s_1 - a^{(1)}_{m+\frac{2}{3},n+\frac{2}{3}}) \rangle \\
&= \frac{4A}{(1+s_1^2)^{3/2}} \left[\tan^{-1} \left(\frac{q^{*2} - s_1^2}{1+s_1^2} \right)^{1/2} + \frac{(q^{*2} - s_1^2)^{1/2}(1+s_1^2)^{1/2}}{1+q^{*2}} \right] \theta(q^* - |s_1|).
\end{aligned}
$$

The conditional pdf of $a^{(2)}_{m+\frac{1}{3},n+\frac{1}{3}}$ $\left(a^{(2)}_{m+\frac{2}{3},n+\frac{2}{3}}\right)$ given $a^{(1)}_{m+\frac{1}{3},n+\frac{1}{3}}$ $\left(a^{(1)}_{m+\frac{2}{3},n+\frac{2}{3}}\right)$ is

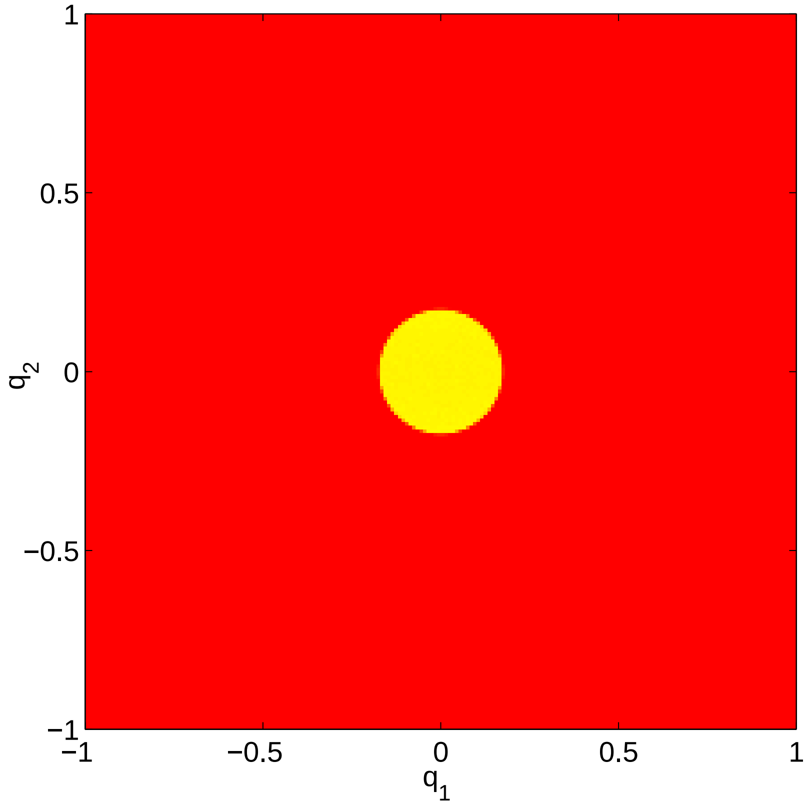
$$
f(s_2|s_1) = \frac{f(s_1, s_2)}{f(s_1)}.
$$

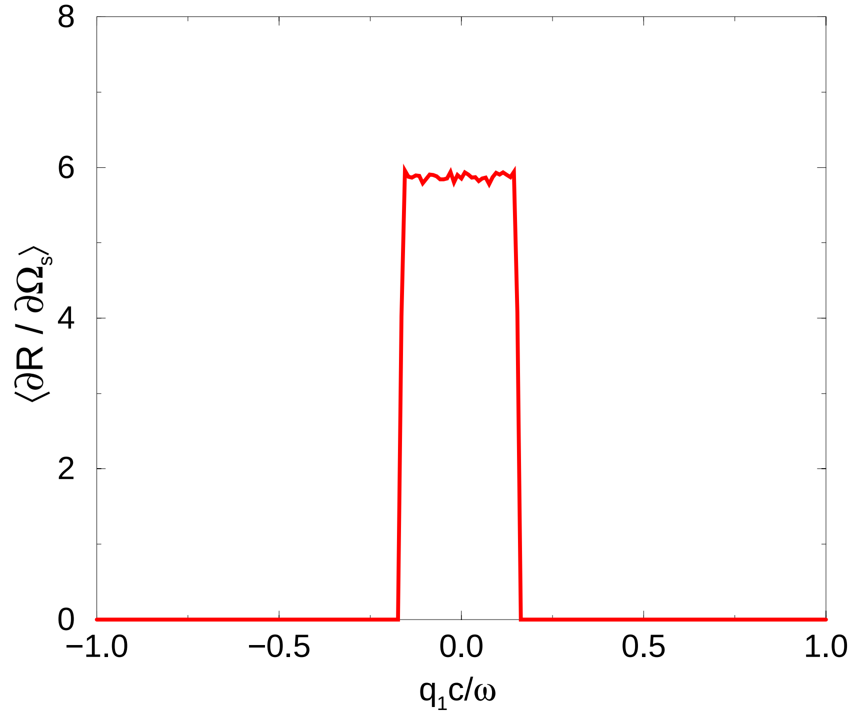


A segment of the surface profile function $\zeta(\mathbf{x}_{\|})$.



The mean differential reflection coefficient estimated from N_p = 10,000 realizations of surface profiles : θ_m =20^o


1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
q1
q2


8
6
4
2
0
−1.0
−0.5
0.0
0.5
1.0
q1c/ω
⟨∂R / ∂Ωs⟩

A Band-Limited Uniform Diffuser within a Rectangular Domain

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle (q_1, q_2) = A\theta \left(q_1^{(m)} - |q_1| \right) \theta \left(q_2^{(m)} - |q_2| \right).$$

When $s_\parallel$ is small enough that $1 + s_\parallel^2 \cong 1$,

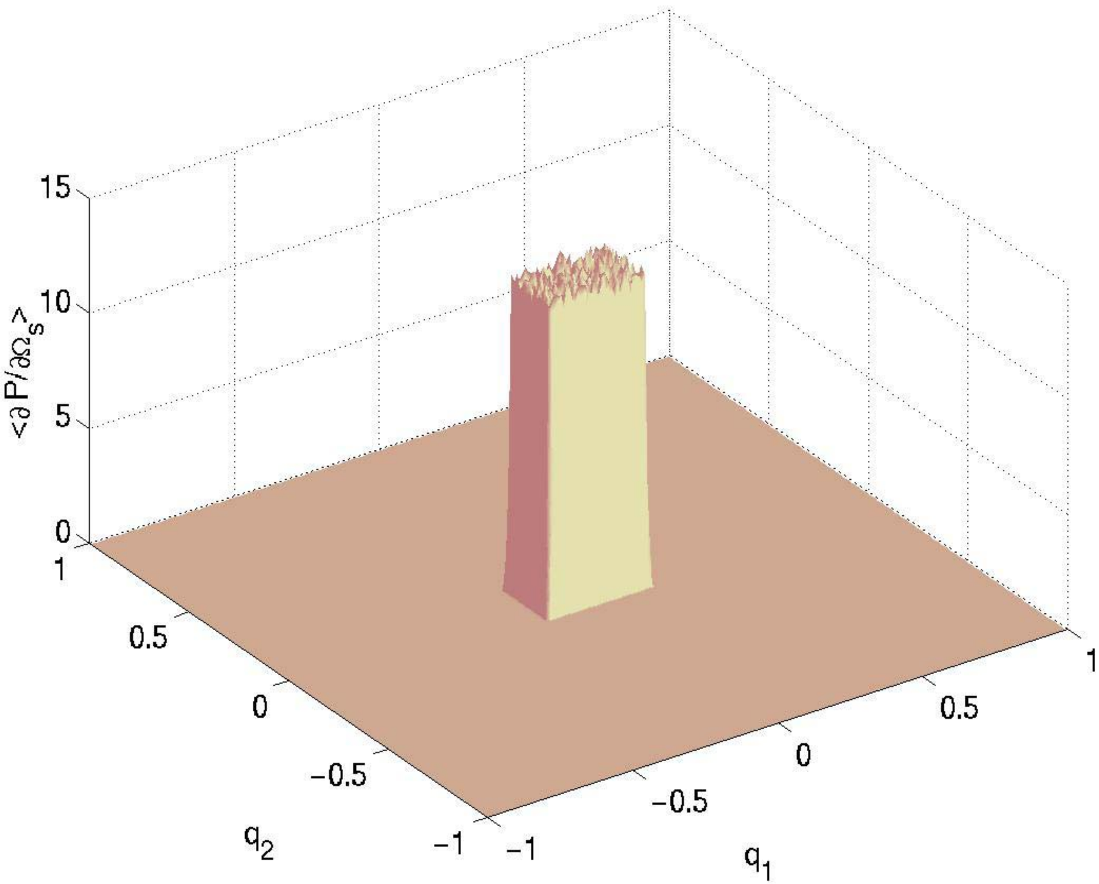
$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle (s_1, s_2) = A\theta \left(s_1^{(m)} - |s_1| \right) \theta \left(s_2^{(m)} - |s_2| \right),$$

where

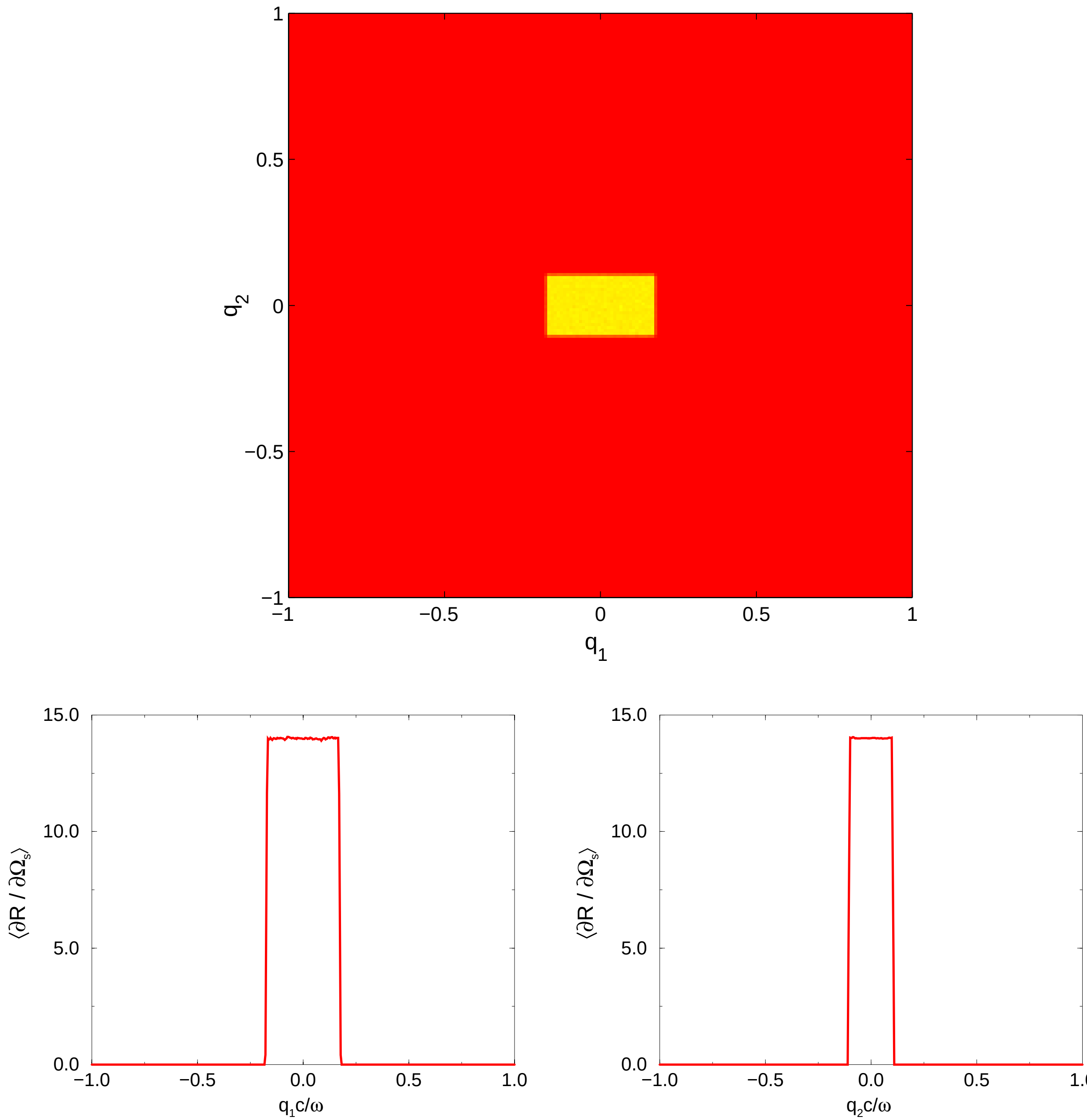
$$A = \frac{1}{16 s_1^{(m)} s_2^{(m)}}, \qquad s_j^{(m)} = \frac{c q_j^{(m)}}{2\omega} \qquad j = 1, 2.$$

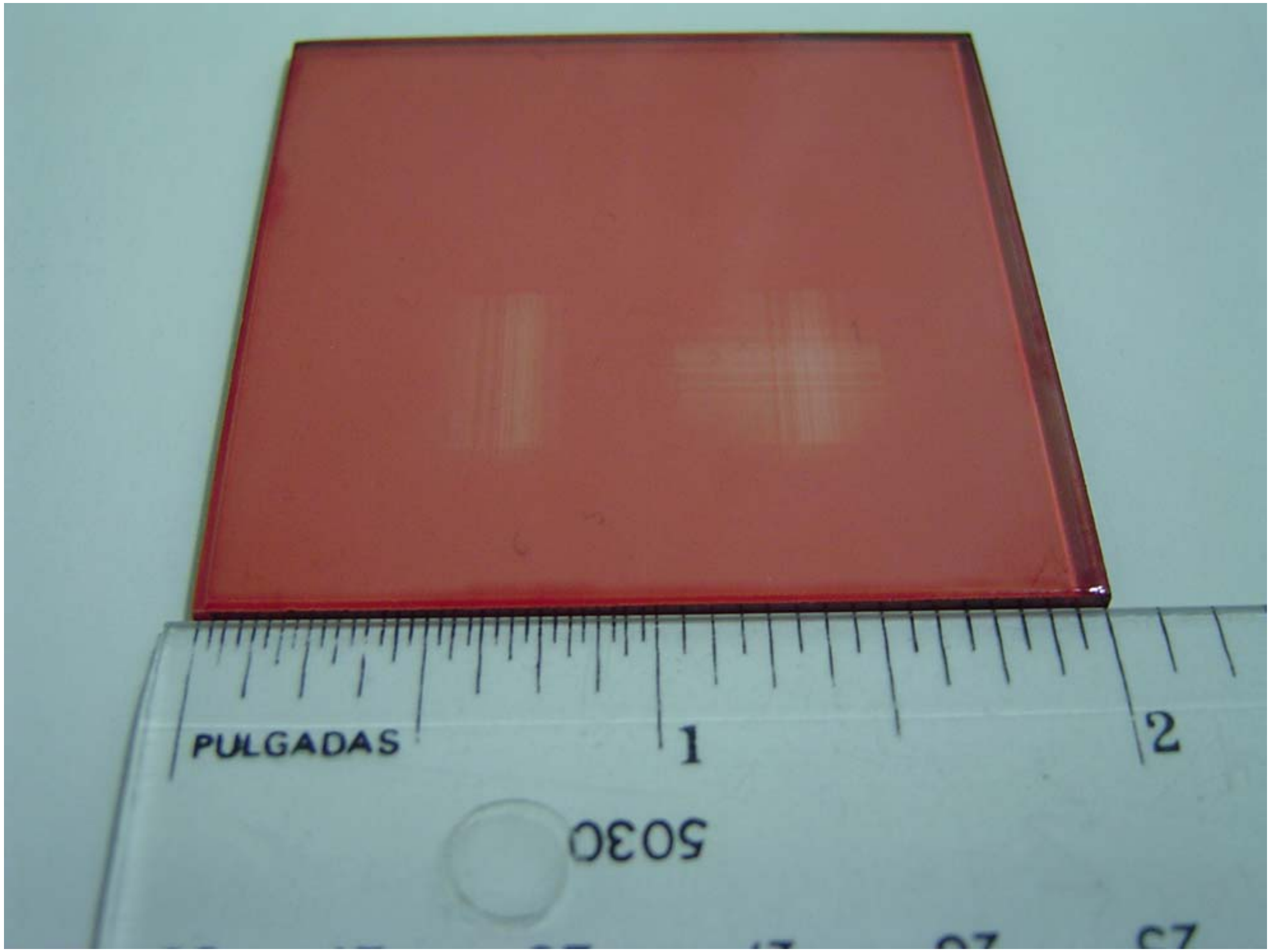
$$f(s_1, s_2) = 4A \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle (s_1, s_2) = A\theta \left(s_1^{(m)} - |s_1| \right) \theta \left(s_2^{(m)} - |s_2| \right),$$

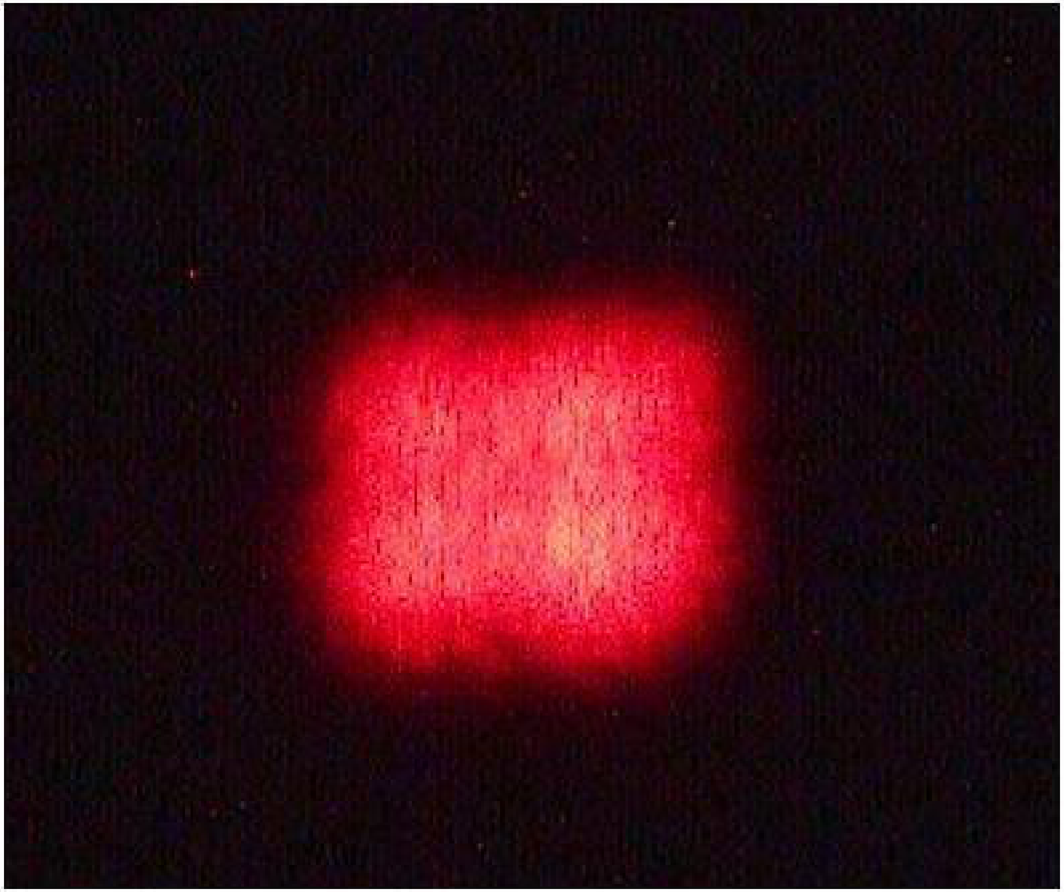
$$f(s_1) = \frac{\theta \left(s_1^{(m)} - |s_1| \right)}{2 s_1^{(m)}}, \qquad f(s_2|s_1) = \frac{\theta \left(s_2^{(m)} - |s_2| \right)}{2 s_2^{(m)}}.$$



The mean differential reflection coefficient estimated from Np = 10,000 realizations of surface profiles : θ_1 =10°, θ_2 =6°


1
0.5
0
−0.5
−1
q_2
−1
−0.5
0
0.5
1
q_1
15.0
10.0
5.0
0.0
$\langle \partial R / \partial \Omega_s \rangle$
−1.0
−0.5
0.0
0.5
1.0
$q_1 c/\omega$
15.0
10.0
5.0
0.0
$\langle \partial R / \partial \Omega_s \rangle$
−1.0
−0.5
0.0
0.5
1.0
$q_2 c/\omega$


PULGADAS
1
2



The surface $x_3 = \zeta(\mathbf{x}_\parallel)$ is illuminated from the vacuum side by a scalar plane wave of frequency ω,

$$\psi(\mathbf{x}|\omega)_{inc} = \exp[i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel - i\alpha_0(k_\parallel)x_3],$$

where $\alpha_0(k_\parallel) = \sqrt{(\omega/c)^2 - k_\parallel^2}$.
By the use of Green's second integral identity the scattered field in this region can be written in the form

$$\psi(\mathbf{x}|\omega)_{sc} = -\frac{1}{4\pi}\int d^2x'_\parallel [g_0(\mathbf{x}|\mathbf{x}')]_{x'_3=\zeta(\mathbf{x}'_\parallel)} L(\mathbf{x}'_\parallel|\omega),$$

where $g_0(\mathbf{x}|\mathbf{x}')$ is the scalar free-space Green's function

$$g_0(\mathbf{x}|\mathbf{x}') = \frac{e^{i\frac{\omega}{c}|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}.$$

In the Kirchhoff approximation the source function $L(\mathbf{x}_\parallel|\omega)$ is given by

$$\begin{aligned} L(\mathbf{x}_\parallel|\omega) &= 2\left(-\frac{\partial\zeta(\mathbf{x}_\parallel)}{\partial x_1}\frac{\partial}{\partial x_1} - \frac{\partial\zeta(\mathbf{x}_\parallel)}{\partial x_2}\frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3}\right)\psi(\mathbf{x}|\omega)_{inc}\Big|_{x_3=\zeta(\mathbf{x}_\parallel)} \\ &= -2i\left(\frac{\partial\zeta(\mathbf{x}_\parallel)}{\partial x_1}k_1 + \frac{\partial\zeta(\mathbf{x}_\parallel)}{\partial x_2}k_2 + \alpha_0(k_\parallel)\right)e^{i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel - i\alpha_0(k_\parallel)\zeta(\mathbf{x}_\parallel)}. \end{aligned}$$

In the case of normal incidence, $\mathbf{k}_\parallel = 0$, the scattered field in the Fresnel limit $x_3 \gg x_1, x_2$ and $x_3 \gg x_1', x_2'$ has the form

$$\psi(\mathbf{x}|\omega)_{sc} \cong \frac{i}{2\pi}\frac{\omega}{c}\frac{e^{i\frac{\omega}{c}x_3}}{x_3}\int d^2x_\parallel' \exp\left\{-2i\frac{\omega}{c}\zeta(\mathbf{x}_\parallel') + i\frac{\omega}{c}\frac{(\mathbf{x}_\parallel - \mathbf{x}_\parallel')^2}{2x_3}\right\}.$$

We assume that the surface profile function $\zeta(\mathbf{x}_\parallel)$ is a function of $\mathbf{x}_\parallel$ only through its magnitude, $|\mathbf{x}_\parallel| = r$, and write

$$\zeta(\mathbf{x}_\parallel) = H(r).$$

Then, the expression for the scattered field takes the form

$$\psi(\mathbf{x}|\omega)_{sc} \quad \cong \quad i\left(\frac{\omega}{c}\right)\frac{e^{i\frac{\omega}{c}(x_3+\frac{r^2}{2x_3})}}{x_3}\int_0^\infty dr'r'J_0\left(\frac{\omega r}{cx_3}r'\right)e^{i\frac{\omega}{2cx_3}r'^2 - i\frac{2\omega}{c}H(r')}.$$

Our goal is to find the function $H(r)$ that produces a specified form for the mean intensity of the scattered field along the x_3-axis, $\langle I(x_3)\rangle = \langle|\psi(0,0,x_3|\omega)_{sc}|^2\rangle$, where

$$\psi(0,0,x_3|\omega)_{sc} = i\left(\frac{\omega}{c}\right)\frac{e^{i\frac{\omega}{c}x_3}}{x_3}\int_0^\infty dr\, r\, e^{-2i\frac{\omega}{c}H(r)+i\frac{\omega}{c}\frac{r^2}{2x_3}}.$$

The solution is given by a surface defined by

$$H(r) = \frac{a_n}{b}r^2 + b_n, \quad \sqrt{n}b \le r \le \sqrt{n+1}b, \qquad n = 0,1,2,\ldots,N-1,$$

where $\{a_n\}$ are independent identically distributed random deviates. Consequently, the probability density function (pdf) of a_n, $f(\gamma) = \langle\delta(\gamma - a_n)\rangle$, is independent of n. The $\{b_n\}$ are determined from the condition that the surface profile function $H(r)$ be a continuous function of r, and are given by

$$b_n = b_0 + (a_0 + a_1 + \cdots + a_{n-1} - na_n)b \qquad n \ge 1.$$

We find that

$$f(\gamma) = \frac{1}{4\pi}\frac{c}{\omega}\frac{1}{Nb}\frac{\langle I(\frac{b}{4\gamma})\rangle}{\gamma^2}.$$

For example, we seek to design a surface that produces a constant scattered intensity within the interval $z_1 < x_3 < z_2$ of the x_3-axis, and zero scattered intensity along the rest of the x_3-axis,

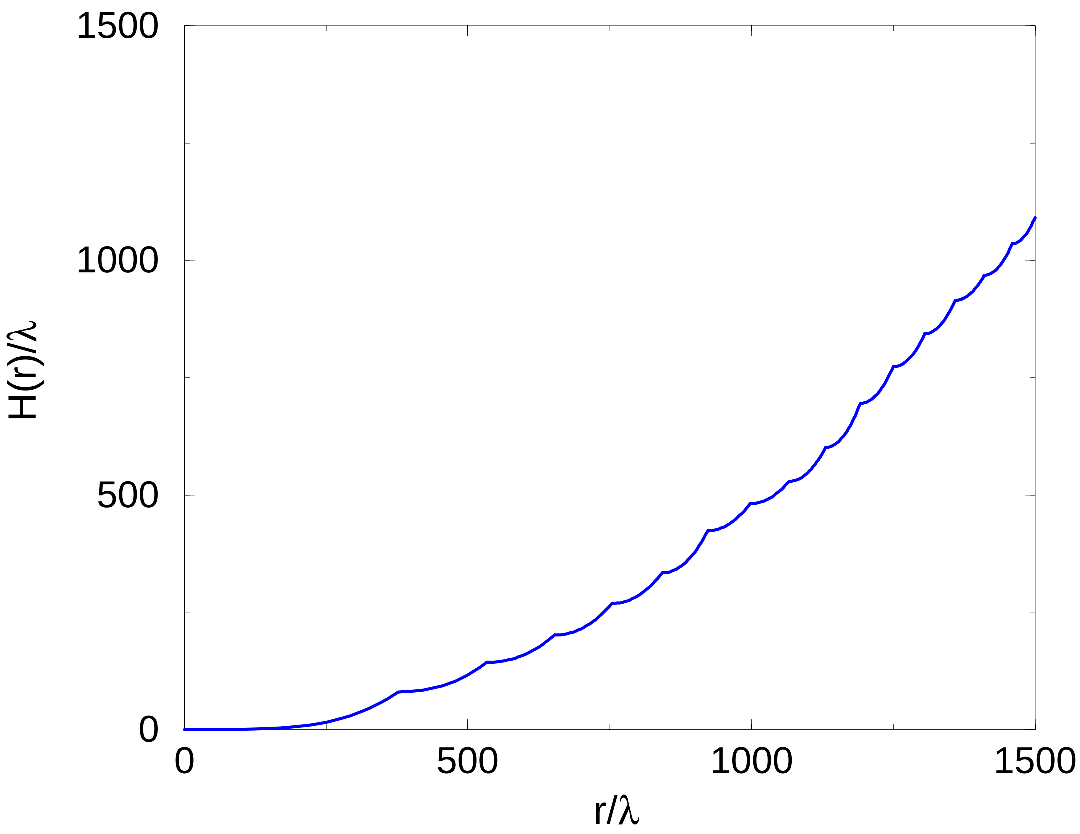
$$\langle I(x_3)\rangle = \pi\frac{\omega}{c}\frac{Nb^2}{z_2 - z_1}\theta(x_3 - z_1)\theta(z_2 - x_3) \qquad z_2 > z_1,$$

where $\theta(z)$ is the Heaviside unit step function.

The probability density function (pdf) of a_n then has the form

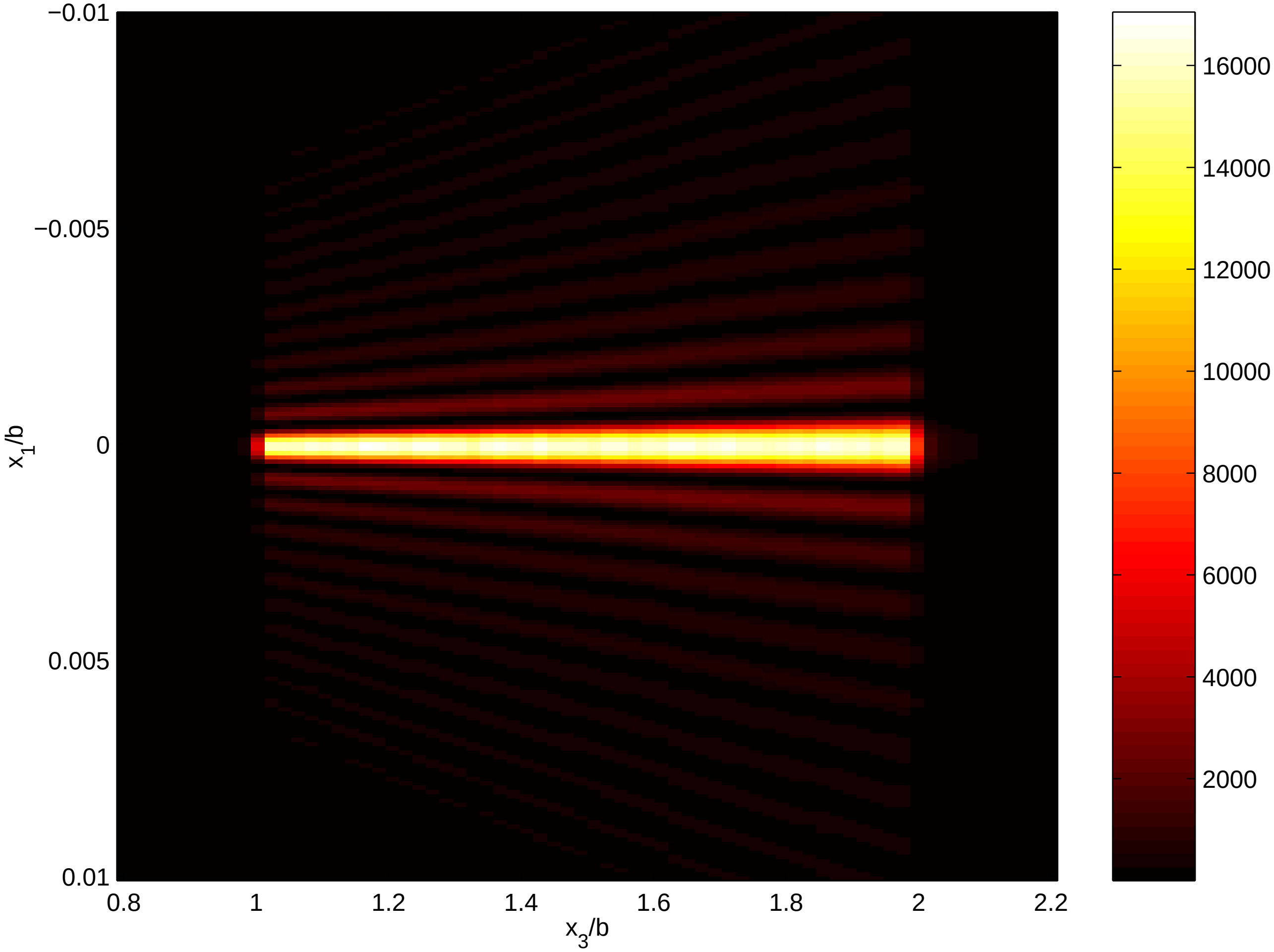
$$f(\gamma) = \frac{b}{4(z_2 - z_1)}\frac{1}{\gamma^2}\theta\left(\gamma - \frac{b}{4z_2}\right)\theta\left(\frac{b}{4z_1} - \gamma\right).$$

A segment of the surface profile function $H(r)$

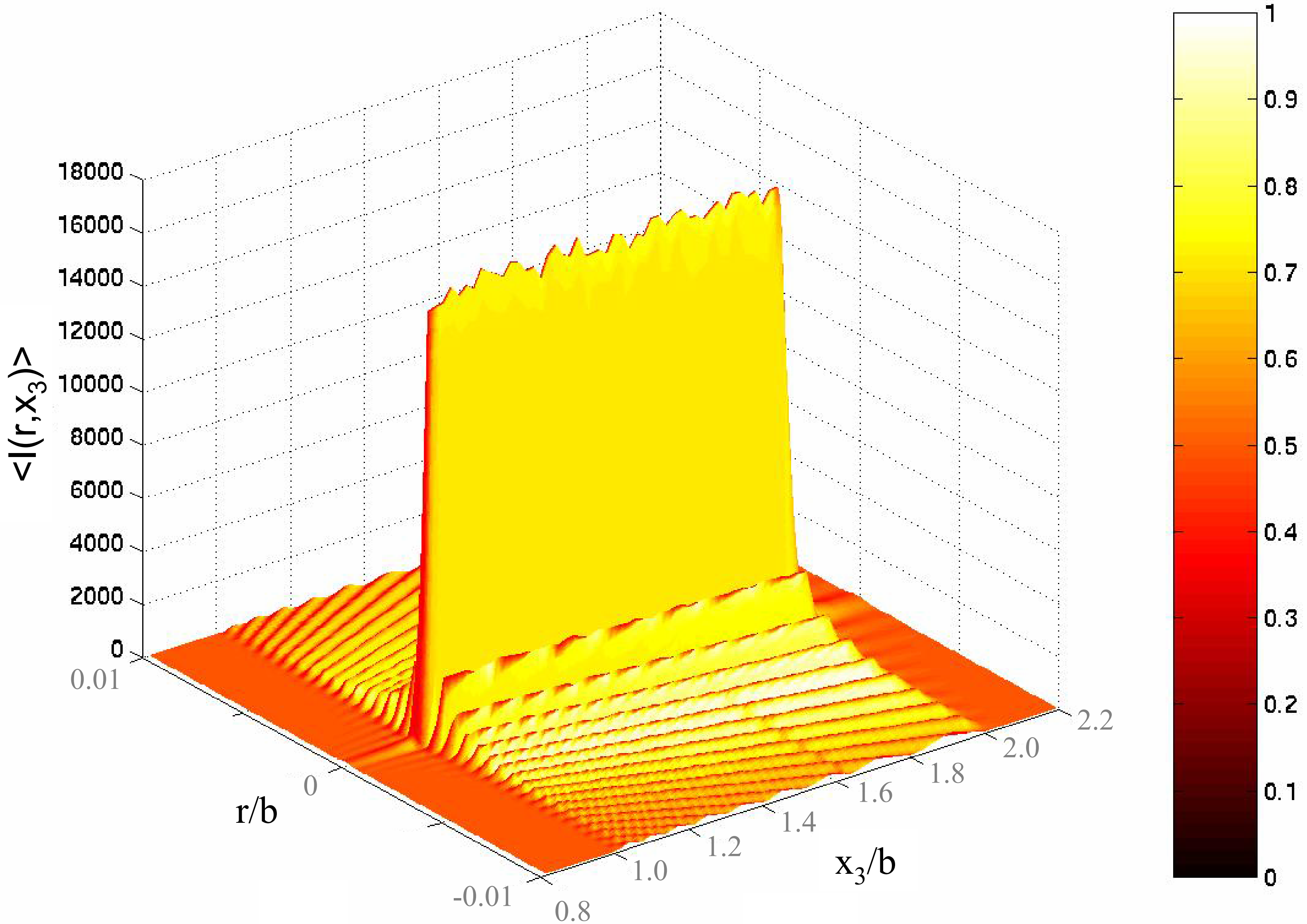


The parameters employed in generating this segment were $z_1 = b$, $z_2 = 2b$, and $b = 400\lambda$.

$z_1 = b, z_2 = 2b$

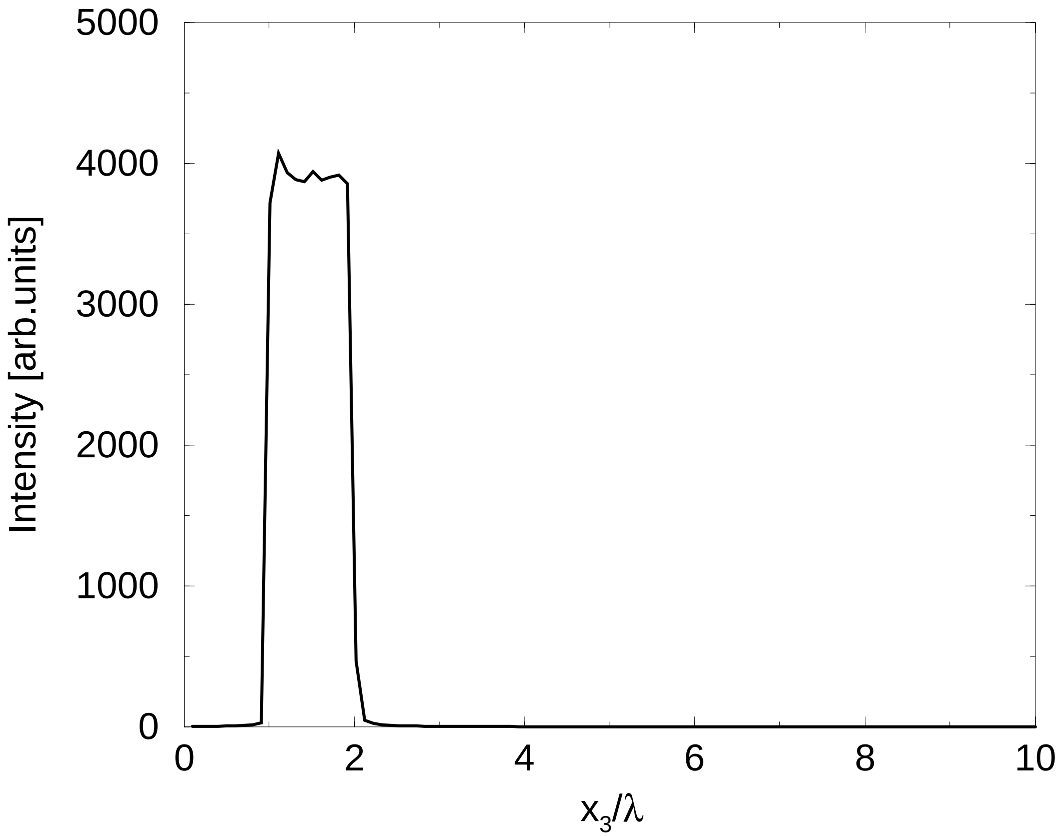


A color–level plot of $\langle I(x_1, x_3)\rangle$ calculated in the Kirchhoff approximation. The parameters employed are $z_1 = b$, $z_2 = 2b$, $b = 200\lambda$, $N = 200$, and $N_p = 80,000$.



<I(r,x3)>
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0.01
0
-0.01
r/b
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
x3/b
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

$z_1 = b, z_2 = 2b$



A plot of $\langle I(x_3)\rangle$ calculated by a rigorous computer simulations method. estimated from $N_p = 30,000$ realizations of the surface profile function. The parameters employed are $z_1 = b$, $z_2 = 2b$, $b = 200\lambda$, $N = 4$, and $N_p = 30,000$.

The nondiffracting beam introduced by Durnin is a solution of the free-space wave equation of the form

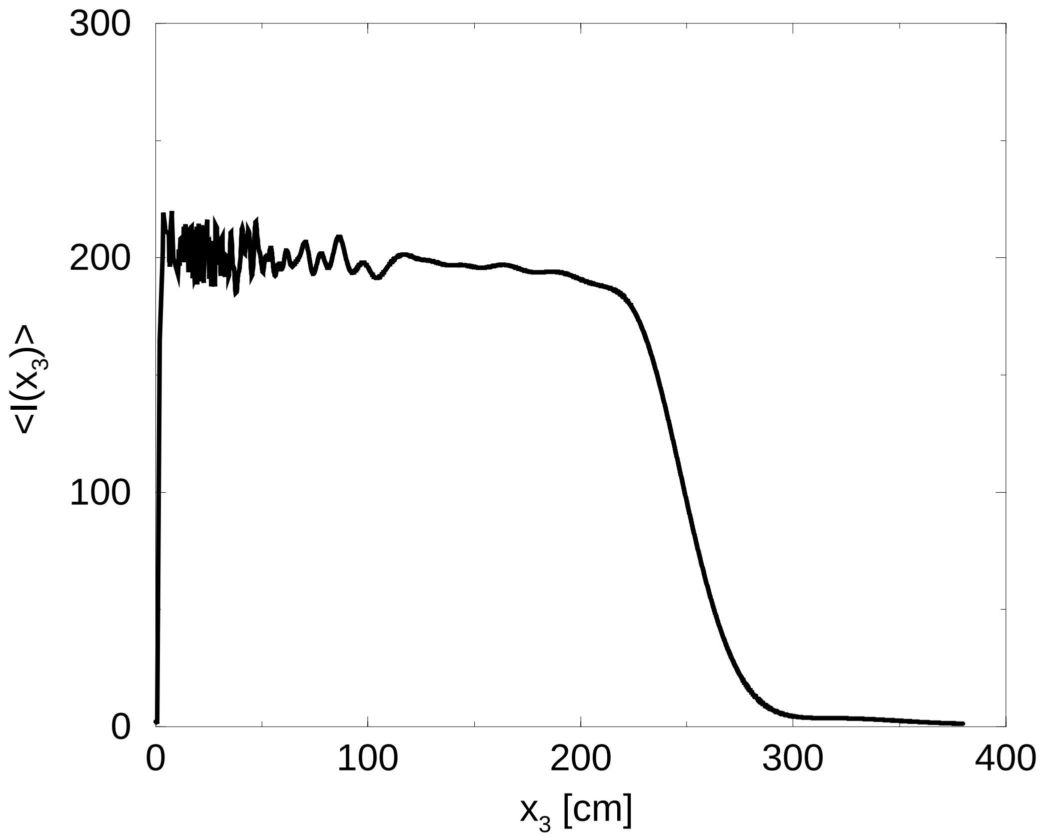
$$E(\rho, z) = J_0(\alpha\rho)\exp(i\beta z),$$

in which

$$\alpha^2 + \beta^2 = k^2,$$

where k is the wave number, $J_0(x)$ is the zero-order Bessel function, and (ρ, θ, z) are the cylindrical coordinates.
This beam has an infinite extent in the transverse plane, and is capable of propagating to infinity in the z-direction without spreading. Such an ideal nondiffracting beam contains an infinite amount of energy, and is impossible to realize in practice.

Consequently, most recent studies of nondiffracting beams have focused on pseudo-nondiffracting beams, which have a finite beam aperture. Such beams have a finite propagation range, have variation in transverse beam profiles, and intensity peaks in the direction of propagation. Nevertheless, the propagation length can extend to several tens of centimeters, long enough for many applications.



A plot of $\langle I(x_3)\rangle$ calculated in the Kirchhoff approximation. The parameters employed are $z_1 = 1.26$cm, $z_2 = 253$cm, $\lambda = 0.6328\mu$m, $b = 12$mm, and $N_p = 80,000$.

Conclusions

- The results presented show that it is possible to design, and to fabricate, one- and two–dimensional randomly rough surfaces that scatter light in a prescribed fashion.

We now design a random surface that gives rise to a mean differential reflection coefficient that is a constant in the angular interval $|\theta_s| < \theta_m < \pi/2$, and vanishes for $|\theta_s| > \theta_m$, while the angle of incidence is θ_0.

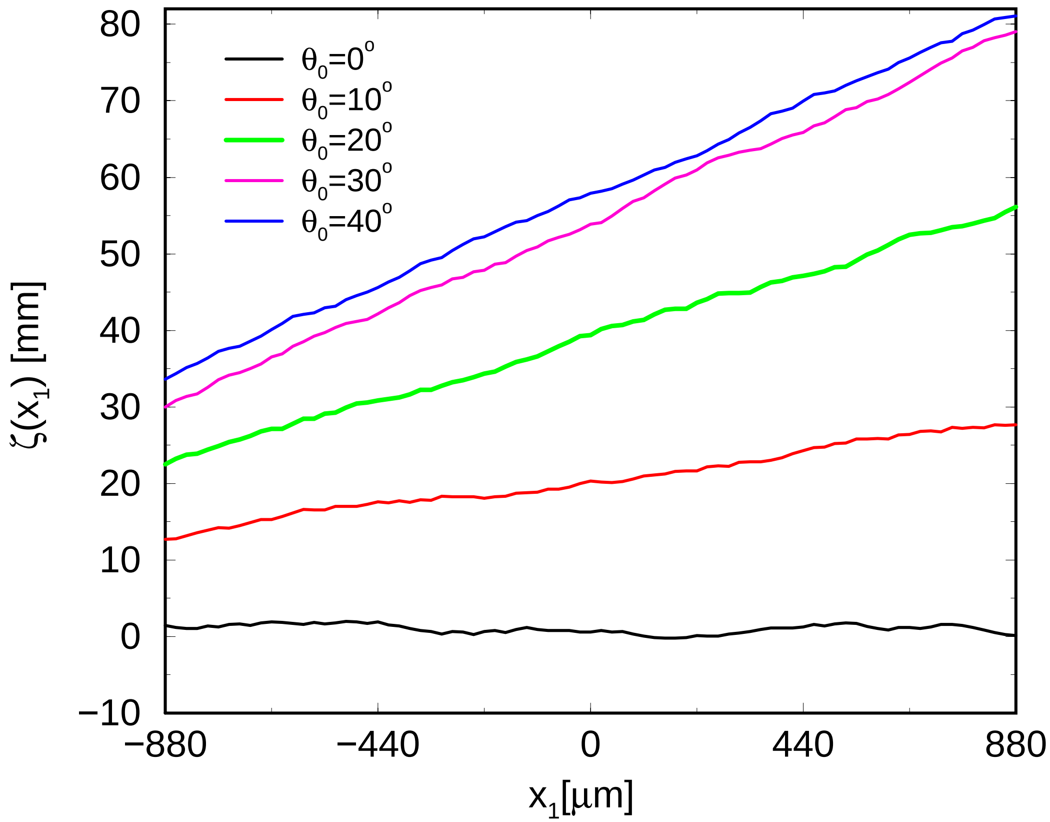
$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = A\theta(\sin\theta_s + \sin\theta_m)\theta(\sin\theta_m - \sin\theta_s)$$

In this case the pdf of a_n has the form

$$\begin{aligned} f(\gamma) &= \frac{2A}{1+\gamma^2} \frac{\cos\theta_0}{\cos\theta_0 + \gamma\sin\theta_0} \\ &\times \theta\left(\gamma + \tan\frac{\theta_m - \theta_0}{2}\right)\theta\left(\tan\frac{\theta_m + \theta_0}{2} - \gamma\right). \end{aligned}$$

where the coefficient A is obtained from the normalization condition for $f(\gamma)$

$$A = \frac{1}{2\cos\theta_0}\left[\theta_m\cos\theta_0 + \sin\theta_0 \ell n \frac{\cos\left(\frac{\theta_m - \theta_0}{2}\right)}{\cos\left(\frac{\theta_m + \theta_0}{2}\right)}\right]^{-1}.$$

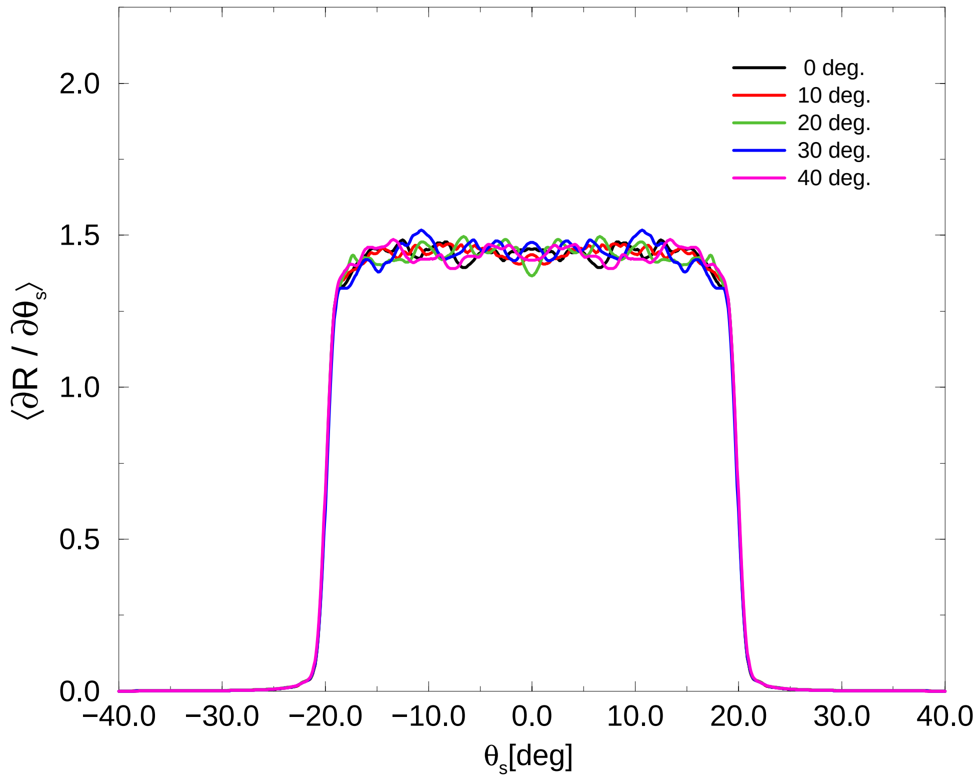


Segments of the surface profile functions $\zeta(x_1)$ for different angles of incidence. $b = 22\mu$m.

A Band–Limited Uniform Diffuser

A rigorous computer simulation calculations

A perfectly conducting random surface



The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 20,000$ realizations of perfectly conducting surface profiles: $\theta_m = 20°$, $\lambda = 632.8$nm, $L = 100\mu$m, $b = 22\mu$m.