Multiple scattering effects in rough surface scattering

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The first theoretical study of the scattering of light from a randomly rough surface was carried out by Mandel'shtam in 1913, in the context of the scattering of light from the surface of a liquid [1]. These calculations were carried out on the basis of a single-scattering approximation, and for more than 70 years after the work of Mandel'shtam single-scattering approaches continued to underlie theoretical investigations of rough surface scattering. These approaches consisted either of small-amplitude perturbation theory [2], in which the amplitude of the scattered or transmitted field was calculated to first order in the surface profile function — the function that gives the departure of a random surface from a planar surface at each point of the latter — or of the Kirchhoff approximation [3], in which the light is assumed to be scattered from and transmitted through the plane tangent to each point of the random surface.

In 1985, however, it was predicted theoretically on the basis of a multiple-scattering calculation of the scattering of p-polarized (transverse magnetic) light from a one-dimensional weakly rough random metal surface that the angular dependence of the intensity of the light scattered incoherently (diffusely) displays a well-defined peak in the retroreflection direction [4]. This is a weak localization effect that is caused by the coherent interaction of the multiply-scattered, p-polarized, surface electromagnetic waves — surface plasmon polaritons — supported by the vacuum metal interface with their reciprocal partners. This effect was observed experimentally two years later [5], in experiments in which, however, metal surfaces considerably rougher than those for which the theory or Ref. [4] is valid were used. These results stimulated the development of computational approaches for calculating the scattering of light from large-amplitude, large slope, random surfaces, and searches for additional phenomena that reveal themselves only when multiple scattering is taken into account in these approaches, activities that continue to this day.

In this lecture some of the techniques that have been used to investigate multiple-scattering phenomena in rough surface scattering are outlined, and effects obtained by their use are described. Where experimental confirmations of the latter exist, they, too, are presented.

The most widely studied weak localization effect in rough surface scattering studied both theoretically and experimentally is the enhanced backscattering effect described above. In the earliest theoretical investigation of it [4] for the scattering of p-polarized light from weakly rough one-dimensional randomly rough metal surfaces, an infinite-order perturbation theory (the solution of a Bethe–Salpeter equation for a two-particle Green's function) that exploited the existence of surface plasmon polaritons at the vacuum-metal interface, was used to calculate the mean differential reflection coefficient. Subsequently, it was shown [6] that if small-amplitude perturbation theory is used for this purpose, it is necessary to work to fourth order in the surface profile function in calculating the mean differential reflection coefficient in order for this effect to reveal itself. Recent perturbative calculations of this effect have included terms of sixth order in the surface profile function [7], and even terms of eighth order [8]. Enhanced backscattering due to the coherent interference of multiply-scattered surface plasmon polaritons with their reciprocal partners in which the surface plasmons scatter from the same points on the surface but in the reverse order, was observed in a clever experiment by West and O'Donnell in 1995 [9] that was analyzed theoretically by Maradudin *et al.* [10].

To deal with scattering from large-amplitude, large-slope one-dimensional randomly rough surfaces, computer simulation approaches were developed [11, 12, 13]. Among other results these methods showed that enhanced backscattering can be observed in the scattering of s-polarized (transverse electric) light from a sufficiently rough metal surface [13]. This is not the case in the scattering of s-polarized light from a weakly rough metal surface, because the latter does not support s-polarized surface plasmon polaritons.

Computer simulation calculations of the scattering of scalar plane waves from two-dimensional randomly rough Dirichlet [14, 15] and Neumann [16] surfaces were carried out, as well as such calculations of the scattering of electromagnetic (vector) waves from two-dimensional perfectly conducting [17] and metallic [18] surfaces.

If one compares the successive terms in the expansion in powers of the surface profile function of the amplitude for the transmission of p-polarized light through a thin, free-standing metal film, whose illuminated surface is a weakly rough one-dimensional random surface, with the corresponding expansion for the scattering amplitude, one finds that the denominators, and hence the poles, in both expansions are the same. These poles give the dispersion relation for the two surface electromagnetic waves supported by the film that are responsible for enhanced backscattering from the film. Consequently, one might expect that an analogous effect should occur in transmission. This expectation was confirmed theoretically in [19], where it was shown that the angular dependence of the intensity of the light transmitted incoherently displays a well-defined peak in the antispecular direction. This peak was subsequently observed experimentally [20].

When the scattering structure with a randomly rough surface supports two or more surface or guided waves, as does the free-standing metal film mentioned in the preceding paragraph, or a dielectric film deposited on a metal substrate, additional features can arise in the angular dependence of the intensity of the light scattered from it or transmitted through it, namely satellite peaks [21]. These are also multiple-scattering effects that arise due to the fact that the scattering structure supports two or more modes, with different wave numbers, at the frequency ω of the incident light. Thus, if the wavenumbers of the $N(\geq 2)$ modes supported by the structure are $k_1(\omega), k_2(\omega), \ldots, k_N(\omega)$, peaks will occur at scattering angles θ_s measured clockwise from the normal to the mean surface, given by $\sin\theta_s = -\sin\theta_0 \pm(\omega/c)[k_m(\omega) - k_n(\omega)]$, where θ_0 is the angle of incidence, measured counterclockwise from the normal to the mean surface. When $m = n$ the peak is the enhanced backscattering peak, when $m \neq n$, the peaks are satellite peaks. These satellite peaks have now been observed experimentally [22]. The satellite peaks that can arise in transmission have yet to be observed.

In a series of papers published in 1987 [23] E. Wolf considered radiation from a three-dimensional quasihomogeneous source and showed that if the spectral coherence of the source i.e. the correlation in the fluctuations of the source, is appropriately chosen, the spectrum of the emitted radiation can be redshifted or blueshifted with respect to that of the source, even when the source is at rest with respect to the observer, and the radiation propagates in free space. There is an analogy between scattering and radiation that has its origin in the following circumstances. In the scattering of polychromatic light from a static random medium the different frequency components of the incident light, which are scattered in any particular direction, will be scattered with different strengths. Consequently, the spectrum of the scattered light will differ from that of the incident light, even though the different frequency components are uncorrelated. The possibility of generating a spectral redistribution by scattering is analogous to the possibility of generating a spectral redistribution in light emitted from a source caused by correlations in the fluctuations of the source. The only difference is that in scattering one is dealing with secondary sources, namely with the polarization induced in the scattering medium by the incident field. The induced polarization, in general, is correlated over finite distances of the scattering medium, and thus imitates correlations in primary sources. This analogy between scattering and radiation prompted theoretical [24, 25] and experimental [25, 26] investigations of spectral redistributions of light scattered from randomly rough surfaces that showed that the spectrum of the scattered light indeed differs from that of the incident light.

Up to now I have considered only the intensity of the light scattered from, or transmitted through, a randomly rough surface, namely a second moment of the scattered field. Recently, angular intensity correlation functions of light scattered from randomly rough surfaces have begun to be studied both theoretically and experimentally. These are correlation functions of the type $\langle \delta I(q|k)\delta I(q'|k')\rangle$, where $\delta I(q|k) = I(q|k) - \langle I(q|k)\rangle$, and the angle brackets denote an average over the ensemble of realizations of the surface profile. They represent a fourth moment of the scattered field. $I(q|k)$ is the intensity with which an incident plane wave whose wave vector has a component k parallel to the mean scattering surface is scattered into a plane wave whose wave vector has a component q parallel to the mean scattering surface. These wavenumbers are related to angles of incident θ_0 and scattering θ_s by $k = (\omega/c)\sin\theta_0$, $q = (\omega/c)\sin\theta_s$, $k' = (\omega/c)\sin\theta_0'$, $q' = (\omega/c)\sin\theta_s'$. The theoretical calculations, which take multiple-scattering processes into account, have predicted a memory effect and a reciprocal memory effect, as well as a new correlation function that exists when the scattered field obeys complex Gaussian statistics and vanishes when it obeys circular complex Gaussian statistics [27, 28]. These angular intensity correlation functions have been observed in recent experiments [29].

When highly intense light is incident on a metal surface second harmonic light is generated in reflection. When the metal surface is randomly rough, weak localization effects are present in the angular dependence of the intensity of the second harmonic light. When the surface is weakly rough and is characterized by a power spectrum that enhances the excitation of surface plasmon polaritons of frequency 2ω, by incident light of frequency ω, computer simulation and small-amplitude perturbation theory calculations of the second harmonic intensity predict a dip in the retroreflection direction for all angles of incidence [30, 31]. This prediction is in agreement with experimental results presented in [32]. When the surface roughness is characterized by a power spectrum that enhances the excitation of surface plasmon polaritons of frequency ω by incident light of frequency ω, peaks are predicted at scattering angles defined by $q = k \pm k_{sp}(\omega)$ [31], where $k = (\omega/c)\sin\theta_0$, $q = (w/c)\sin\theta_s$, and $k_{sp}(\omega)$ is the wavenumber of surface plasmon polaritons of frequency ω. They are associated with the resonant nonlinear interaction of the excited surface plasmon polariton with the incident light. These peaks are observed in the experimental results of [33]. In the case of second harmonic generation from strongly rough metal surfaces, computer simulation calculations[34] show dips in the retroreflection direction in the angular dependence of the scattered light at the harmonic frequency, in agreement with the experimental results presented in [35].

The preceding discussion does not cover all the weak localization effects predicted theoretically and observed experimentally when multiple-scattering effects are taken into account in studies of rough surface scattering. Yet they illustrate the richness of the phenomena multiple scattering produces, and raise the expectation that there are still more to be discovered.

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Designer surfaces

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In many practical situations it is desirable to have optical diffusers with specific light scattering properties. For example, a nonabsorbing diffuser that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range, could have applications in projection systems where one wishes to illuminate a screen with uniform intensity but not to waste light by illuminating outside the boundaries of the screen. We will call such an optical element a *band-limited uniform diffuser*. Band-limited uniform diffusers can also be useful in microscope illumination systems, in the fabrication of displays and projection screens, and in Fourier transform holography. A random surface that acts as a band-limited uniform diffuser would consequently be a useful optical element. Lambertian diffusers, which produce a scattered intensity that is proportional to the cosine of the polar scattering angle, are frequently used in the optical industry, e.g. for calibrating scatterometers [1]. Such diffusers have the property that their radiance or luminance is the same in *all* scattering directions. In the visible region of the optical spectrum volume disordered media, e.g. compacted powdered barium sulfate, and freshly smoked magnesium oxide are used as Lambertian diffusers [2]. However, this type of diffuser is inapplicable in the infrared