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Designer surfaces

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In many practical situations it is desirable to have optical diffusers with specific light scattering properties. For example, a nonabsorbing diffuser that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range, could have applications in projection systems where one wishes to illuminate a screen with uniform intensity but not to waste light by illuminating outside the boundaries of the screen. We will call such an optical element a *band-limited uniform diffuser*. Band-limited uniform diffusers can also be useful in microscope illumination systems, in the fabrication of displays and projection screens, and in Fourier transform holography. A random surface that acts as a band-limited uniform diffuser would consequently be a useful optical element. Lambertian diffusers, which produce a scattered intensity that is proportional to the cosine of the polar scattering angle, are frequently used in the optical industry, e.g. for calibrating scatterometers [1]. Such diffusers have the property that their radiance or luminance is the same in *all* scattering directions. In the visible region of the optical spectrum volume disordered media, e.g. compacted powdered barium sulfate, and freshly smoked magnesium oxide are used as Lambertian diffusers [2]. However, this type of diffuser is inapplicable in the infrared region due to its strong absorption and the presence of a specular component in the scattered light, in this frequency range. The design of a random surface that acts as a Lambertian diffuser, especially in the infrared region of the optical spectrum is therefore a desirable goal, and one that has been regarded as difficult to achieve [3]. Yet another example is provided by the fact that in the scattering of light from a random surface the multiple-scattering processes that give rise to such interesting weak localization effects as enhanced backscattering, enhanced transmission, satellite peaks, and new angular intensity correlation functions, are accompanied by single-scattering processes on which these, often subtle, effects are superimposed. The design of random surfaces that suppress single-scattering in a suitable range of scattering angles could be useful in increasing the visibility of these effects.

The design of band-limited uniform diffusers, some of which employ one- or two-dimensional random surfaces, has been considered by several authors [4–7]. Diffractive optical elements that scatter light uniformly over specified angular regions have become commercially available [8]. These elements, however, are not random and possess the desired characteristics over only a relatively narrow range of wavelengths. Thus, they are not achromatic. Another kind of diffuser, whose design is based on a randomized microlenslet concept, is also available commercially [9]. Although these holographic light shaping diffusers are achromatic, and possess characteristics that approximate the desired ones, the scattering distribution they produce is not uniform, and they do not have a well-defined maximum angle of scattering.

Despite the interest in this subject, until recently there were no clear procedures reported in the literature for designing and fabricating randomly rough surfaces that behave as band-limited uniform diffusers, or scatter light in other specified ways, and it was unclear what kind of surface statistics were required for the production of such optical elements. In this lecture I will present approaches due to my colleagues and myself to the design and fabrication of one- and two-dimensional randomly rough surfaces that possess the scattering properties described above [10–21] These methods are based on the geometrical optics limit of the Kirchhoff approximation, a single-scattering approximation, for the scattering of scalar plane waves from impenetrable surfaces. However, as we will see, the results obtained by these methods have a significantly wider range of applicability. We have chosen to work with random surfaces in designing optical diffusers that scatter light in a prescribed fashion because, as will be shown, the use of such surfaces leads to a precise algorithm for designing them, something that we have been unable to find in dealing with deterministic surfaces.

I begin by considering one-dimensional random surfaces. The physical system that is assumed initially consists of vacuum in the region $x_3 > \zeta(x_1)$ and a perfect conductor in the region $x_3 < \zeta(x_1)$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable and constitutes a random process, but not necessarily a stationary one. The surface $x_3 = \zeta(x_1)$ is illuminated from the vacuum region by an s-polarized plane wave of frequency ω, whose plane of incidence is the x_1x_3-plane. The single nonzero component of the electric field $E_2^>(x_1, x_3; t) = E_2^>(x_1, x_3|\omega)\exp(-i\omega t)$ in the region $x_3 > \zeta(x_1)_{\max}$ is the sum of an incident plane wave and a superposition of outgoing scattered plane waves,

$$E_2^>(x_1, x_3|\omega) = \exp[ikx_1 - i\alpha_0(k)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k) \exp[iqx_1 + i\alpha_0(q)x_3], \quad (1)$$

where $\alpha_0(q) = [(\omega/c)^2 - q^2]^{\frac{1}{2}}$, with $\mathrm{Re}\alpha_0(q) > 0, \mathrm{Im}\alpha_0(q) > 0$.

The differential reflection coefficient, $\partial R/\partial\theta_s$, which is defined in such a way that $(\partial R/\partial\theta_s)d\theta_s$ is the fraction of the total time-averaged incident flux that is scattered into the angular interval $(\theta_s, \theta_s + d\theta_s)$, is given in terms of the scattering amplitude $R(q|k)$ by

$$\frac{\partial R}{\partial \theta_s} = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2\theta_s}{\cos\theta_0} |R(q|k)|^2, \quad (2)$$

where L_1 is the length of the x_1-axis covered by the random surface, while θ_0 and θ_s are the angles of incidence and scattering measured counterclockwise and clockwise from the $+x_3$-axis, respectively, and are related to the wavenumbers k and q by $k = (\omega/c)\sin\theta_0$, $q = (\omega/c)\sin\theta_s$. As we are concerned with the scattering of light from a randomly rough surface, it is the mean differential reflection that we need to calculate. It is defined by

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2\theta_s}{\cos\theta_0} \langle |R(q|k)|^2 \rangle, \quad (3)$$

where the angle brackets here and in all that follows denote an average over the ensemble of realizations of the surface profile function.

In the Kirchhoff approximation, which we adopt here for simplicity, the scattering amplitude $R(q|k)$ is given by[11]

$$R(q|k) = -\frac{(\omega/c)^2 + \alpha_0(q)\alpha_0(k) - qk}{\alpha_0(q)[\alpha_0(q) + \alpha_0(k)]} \times \int_{-\infty}^{\infty} dx_1 \exp[-i(q-k)x_1] \exp\{-i[\alpha_0(q) + \alpha_0(k)]\zeta(x_1)\}. \quad (4)$$

The mean differential reflection coefficient in this approximation thus takes the form

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{1}{\cos\theta_0} \left[\frac{1+\cos(\theta_s+\theta_0)}{\cos\theta_s + \cos\theta_0} \right]^2 \times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-i(q-k)(x_1-x_1')] \langle \exp\{-ia[\zeta(x_1)-\zeta(x_1')]\} \rangle \quad (5)$$

where, to simplify the notation, we have defined $a = \alpha_0(q) + \alpha_0(k) = (\omega/c)(\cos\theta_s + \cos\theta_0)$.

Our goal is to determine the surface profile function $\zeta(x_1)$ that produces a specified form for $\langle \partial R/\partial\theta_s \rangle$ as a function of θ_s and θ_0. As it stands, the expression given by Eq. (5) is too difficult to invert to obtain $\zeta(x_1)$ in terms of $\langle \partial R/\partial\theta_s \rangle$. To simplify it we pass to the geometrical optics limit of the Kirchhoff approximation by making the change of variable $x_1' = x_1 + u$ in Eq. (5), expanding the difference $\zeta(x_1) - \zeta(x_1+u)$ in powers of u, and retaining only the leading nonzero term:

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{1}{\cos\theta_0} \left[\frac{1+\cos(\theta_s+\theta_0)}{\cos\theta_s + \cos\theta_0} \right]^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \, \exp[i(q-k)u] \times \langle \exp[iau\zeta'(x_1)] \rangle. \quad (6)$$

To proceed, we represent the surface profile function $\zeta(x_1)$ in the form

$$\zeta(x_1) = a_n x_1 + b_n, \quad nb \le x_1 \le (n+1)b, \quad n = 0, \pm 1, \pm 2, \ldots, \quad (7)$$

where b is a characteristic length, and the $\{a_n\}$ are independent, identically distributed random deviates. Therefore, the probability density function (pdf) of a_n, $f(\gamma) = \langle \delta(\gamma - a_n) \rangle$, is independent of n. In order that the surface be continuous at $x_1 = (n+1)b$, the relation

$$b_{n+1} = b_n - (n+1)(a_{n+1} - a_n)b \quad (8)$$

must be satisfied. From this recurrence relation the $\{b_n\}$ can be determined from a knowledge of the $\{a_n\}$, provided that an initial value, e.g. that of b_0, is specified. It is convenient to set $b_0 = 0$, and we do so. The double integral in Eq. (6) can now be evaluated, with the result

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \, \exp[i(q-k)u] \langle \exp[iau\zeta'(x_1)] \rangle$$

$$= \frac{2\pi L_1}{(\omega/c)(\cos\theta_0 + \cos\theta_s)} f\left(\frac{\sin\theta_0 - \sin\theta_s}{\cos\theta_0 + \cos\theta_s}\right). \tag{9}$$

The mean differential reflection coefficient thus is given in terms of the pdf of a_n,

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{[1 + \cos(\theta_0 + \cos\theta_s)]^2}{\cos\theta_0(\cos\theta_0 + \cos\theta_s)^3} f\left(\frac{\sin\theta_0 - \sin\theta_s}{\cos\theta_0 + \cos\theta_s}\right). \tag{10}$$

The change of variable $(\sin\theta_0 - \sin\theta_s)/(\cos\theta_0 + \cos\theta_s) = -\gamma$, allows us to write $f(\gamma)$ in terms of the mean differential reflection coefficient,

$$f(\gamma) = \frac{2}{1+\gamma^2} \frac{\cos\theta_0}{\cos\theta_0 + \gamma\sin\theta_0} \left\langle \frac{\partial R}{\partial \theta_s} \right\rangle (-\gamma, \theta_0), \tag{11}$$

where $\langle \partial R/\partial \theta_s \rangle(\gamma, \theta_0)$ is the expression for $\langle \partial R/\partial \theta_s \rangle$ in which its dependence on θ_s has been replaced by its dependence on γ and θ through the use of the change of variable we have made.

If we seek to design a random surface that gives rise to a mean differential reflection coefficient that is a constant in the angular interval $-\theta_m < \theta_s < \theta_m$, and vanishes outside this interval,i.e. for which

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = A\theta(\theta_s + \theta_m)\theta(\theta_m - \theta_s), \tag{12}$$

where $\theta(z)$ is the Heaviside unit step function, it is found that the corresponding pdf $f(\gamma)$ is given by

$$f(\gamma) = \frac{2A}{1+\gamma^2} \frac{\cos\theta_0}{\cos\theta_0 + \gamma\sin\theta_0} \theta\left(\gamma + \tan\frac{\theta_m - \theta_0}{2}\right) \theta\left(\tan\frac{\theta_m + \theta_0}{2} - \gamma\right). \tag{13}$$

The coefficient A in this expression is obtained by normalizing $f(\gamma)$ to unity. From this result for $f(\gamma)$ a long sequence of $\{a_n\}$ is generated by the rejection method [22], and the corresponding sequence of $\{b_n\}$ is obtained from Eq. (8). The surface profile function $\zeta(x_1)$ is then constructed on the basis of Eq. (7).

To determine whether the random surface generated in this fashion indeed produces the scattering pattern specified by Eq. (12), we proceed as follows. A large number N_p of realizations of the random surface is generated by the method just described, and for each realization the scattering problem is solved by, for example, a rigorous computer-based method [23] to yield the scattering amplitude $R(q|k)$. An arithmetic average of the N_p results for $|R(q|k)|^2$ obtained in this way yields the average $\langle |R(q|k)|^2 \rangle$ entering the expression for the mean differential reflection coefficient given by Eq. (3). The computed result for $\langle \partial R/\partial \theta_s \rangle$ is then compared with the result given by Eq. (12).

Numerical results illustrating this approach to the design of one-dimensional random surfaces with specified scattering properties will be presented in this lecture, as well as a method for fabricating them on photoresist. Experimental results for the mean differential reflection coefficients measured in scattering from surfaces designed in the manner described will be also presented.

Extensions of the method described here for the design of one-dimensional randomly rough surfaces with specified scattering properties to the design of two-dimensional randomly rough surfaces with specified scattering properties will also be described, together with methods for fabricating them on photoresist. Experimental results for scattering from surfaces designed and fabricated by these methods will also be presented.

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Lateral surface superlattices: minibands, transport, and chaos

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The two-dimensional electron gas (2DEG), realized in semiconductor heterostructures and quantum wells, has become a model system for many different investigations. One field of research is based on applying modern semiconductor technologies, to impose a lateral periodic structure onto the 2DEG with periods a (now in the range of 100 nm) which are much smaller than the electron mean-free path $l_{\rm mfp}$ at low temperatures but comparable to the Fermi wavelength λ_F [1–3]. Thus, in a low-temperature transport experiment, the electrons can explore the periodic potential landscape without being scattered by impurities or phonons ($l_{\rm mfp} \ll a$). Considering at the same time that for the prevailing carrier densities $\lambda_F = 2\pi/k_F$ is comparable to a, the electrons in a lateral superlattice represent a model of a solid, but with lattice constants very much larger than the crystalline ones of a few Å. The characteristic energy spectrum of electrons in a periodic potential is a band structure, which due to the scaling of the lattice constant results here in minibands, with widths and gaps in the meV range. The demonstration of miniband formation has been one of the challenges of this system.

Experimental investigations of these systems preferentially employ magneto-transport measurements with the magnetic field applied perpendicular to the plane of the 2DES. With the magnetic length $l_B = \sqrt{\hbar/eB}$ a new scale is introduced