различия между магнитными полупроводниками p- и n-типа. Кроме того, она может быть обобщена на случаи пониженной размерности (гетероструктуры, квантовые ямы), существенные для многочисленных практических приложений.

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Temperature scaling in the integer quantum hall effect regime: experiments

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A concise review of temperature scaling experiments in the plateau-to-plateau transitions on the integral quantum Hall effect is presented. Some new results on temperature scaling in the plateau-to-plateau transitions as well as on the $i = 1$ plateau-to-insulating phase transitions in two-dimensional electron gas in $In_{0.53}Ga_{0.47}As/InP$ heterostructures are also presented.

1. Introduction

The transport properties of two-dimensional electron gas (2DEG) continue to attract much interest both experimentally and theoretically. Recently there has been an upsurge of interest in the scaling behaviour of transport coefficients in the quantum Hall effect in connection with the question of universality of the scaling exponent.

The steps connecting the quantum Hall plateaus in ρ_{xy} as well as the peaks of the longitudinal resistivity ρ_{xx} become sharper with decreasing temperature. The scaling theory of the integer quantum Hall effect (IQHE) predicts [1] that a universal critical exponent $\varkappa$ describes the temperature dependence of the linewidth of the ρ_{xx} peaks and the maximum slope of ρ_{xy} between the plateaus

$$\Delta B \sim T^{\varkappa} \text{ and } (d\rho_{xy}/dB)_{\max} \sim T^{-\varkappa}.$$

These relationships follow from the scaling theory result that ρ_{xx} and ρ_{xy} both depend on the temperature and magnetic field only through the single variable $(B-B_c)T^{-\varkappa}$ [2, 3]. The exponent $\varkappa$ is given by the ratio $\varkappa = p/2\alpha$, where p is the inelastic scattering length exponent and α is the localization length exponent. The localization length ξ of the levels near the Fermi energy diverges like a universal power law

$$\xi = \xi_0 \mid B - B_c \mid^{-\alpha}$$

here B_c is the critical magnetic field corresponding to the singular point in the free electron spectrum of the Landau level at $T = 0$ [1]. At finite but low temperature the characterisitc length is the unelastic scattering length given by

$$L_{in}(T) \sim T^{-p/2}.$$

In a number of experiments, [4–12] (for reviews see [13–15]) it was found that the width of the peaks in ρ_{xx} shrinks as a power law $T^{\varkappa}$, as stipulated by the relevant theory. Early experiments on InGaAs/InP heterostructures [4, 6], and later on other material systems resulted in $\varkappa = 0.42 \pm 0.04$, subsequently considered as a universal value, while other experiments [5, 7–9] mainly on GaAlAs/GaAs heterostructures show that the scaling exponent $\varkappa$ depends on both density and type of doping and is also different for transitions between different Landau levels, yielding $0.2 \leq \varkappa \leq 1.0$. The main difference between these two material systems is in the type and character of the dominant electron scattering mechanism. In the GaAs based system longer range potential fluctuations prevail, while in the InGaAs/InP heterojunction the electron scattering is dominated by alloy disorder, which approximately can be described by uncorrelated δ-function potentials also used in deriving the predictions of the scaling theory. Further on there are also some experiments which even claim the absence of scaling in the transitions between the IQHE plateaus [16]. Thus the apparent power law with a possible non-universal exponent $\varkappa$ is still a controversial issue.

In the past years the experiments extended the reach of scaling studies to the transition from the quantum Hall (QH) state to the insulating phase (IP) below the $i = 1$ quantum Hall plateau in the range of fractional filling of the lowest Landau level, the IP being supposed to be a Hall insulator [17–24]. Striking similarities in the scaling behaviour in the transitions between the different QH states were observed [18, 19], hinting the same universality class for both types of transitions [19–23]. However, also for the $i = 1$ quantum Hall liquid (QHL)-to IP transition various values of the temperature scaling exponent ranging from 0.4 to 0.7 have been reported [19–23].

Here I present some results of a new study of the temperature scaling exponents for the low Landau index integer quantum Hall effect (IQHE) plateau-to-plateau as well as for the $i = 1$ plateau-to-insulating phase transitions in 2DEG in $In_{0.53}Ga_{0.47}As/InP$ modulation-doped heterostructures. Preliminary results have already been published in [25], and a more detailed account is in preparation [26].

2. Samples and experiments

The samples used in this study were liquid phase epitaxially grown modulation-doped $In_{0.53}Ga_{0.47}As/InP$ heterostructures [27-30]. The 2DEG density and mobility were $(0.3{-}4) \times 10^{11}$ cm^{-2} and $(2{-}6) \times 10^{4}$ cm^2/Vs respectively. The strength of the disorder potential in our samples was assessed from the broadening of the Landau levels and from the ratio of the transport scatterig time to the quantum scattering time, τ_t/τ_q, deduced from the decay of the amplitude of the Shubnikov-de Haas oscillations in low magnetic fields [29, 31, 32]. The values of τ_t/τ_q increased with increasing 2DEG concentration. Typical values were ~ 1.5, 2 to 3, and 6 to 8 respectively for concentrations of $(4{-}5) \times 10^{10}$, $(1{-}2) \times 10^{11}$, and $(3{-}4) \times 10^{11}$ cm^{-2} respectively, reflecting the dominant nature of small-angle scattering, i.e. of alloy disorder scattering and of scattering on interface irregularities at low 2DEG concentrations [31, 32].

Magnetotransport measurements were carried out on photolithographically defined double cross Hall bars in the temperature range from 40 mK to 4.2 K in a superconducting magnet up to about 6 T, and also in a resistive magnet up to 20 T. Persistent photoconductivity was used to control the 2DEG density. Both conventional dc and low-frequency lock-in techniques were used. For the plateau-to-plateau transitions in the IQHE the scaling exponents were extracted from the temperature dependence of the linewidth of the ρ_{xx} peaks and from the maximum slope of ρ_{xy} between plateaus, i.e. $(\Delta B)^{-1} \sim T^{-\varkappa}$ and $(d\rho_{xy}/dB)_{\max} \sim T^{-\varkappa}$. For the QHL $(i = 1)$ plateau-to-IP transition the scaling exponent was determined using the appropriate scaling relationships.

3. Results

In Fig. 1 the Hall resistivity ρ_{xy} and its derivative with respect to the magnetic field $d\rho_{xy}/dB$ are shown for a sample with $n_s = 1.20 \times 10^{11}$ cm^{-2} and $\mu = 3 \times 10^4$ cm^2/Vs as a function of the magnetic field at several temperatures. The spin splitting of the first Landau level $(N = 1)$ is not resolved in this particular sample due to the relatively large broadening of the Landau levels as indicated also by the low mobility. The maxima of $d\rho_{xy}/dB$ clearly increase with decreasing temperature.

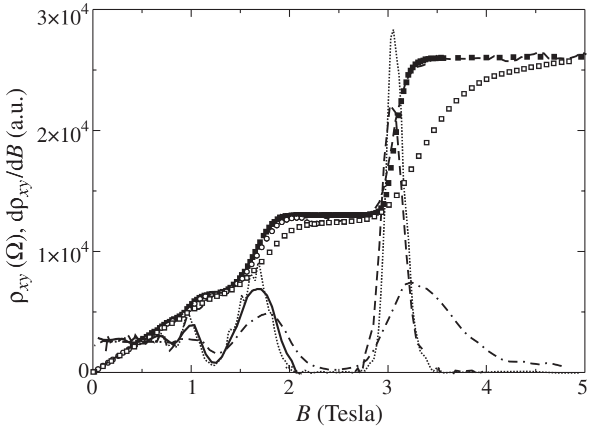

Figure 1. Hall resistivity ρ_{xy} and its derivative with respect to the magnetic field $d\rho_{xy}/dB$ at temperatures 0.75, 0.81, 1.53, and 4.2 K for a sample with $n_s = 1.20 \times 10^{11}$ cm^{-2} and $\mu = 3 \times 10^4$ cm^2/Vs.

Fig. 2 shows $(d\rho_{xy}/dB)_{\max}$ for Landau levels $N = 0$ $(i = 1$ to 2 transition) and $N = 1$ $(i = 2$ to 4 transition), and the reciprocal halfwidth $(\Delta B)^{-1}$ of the ρ_{xx} peak $N = 0$ in function of the temperature. The temperature scaling behaviour is convincingly demonstrated. In this sample for the $i = 1$ to 2 transition the temperature dependence of the maximum slope of ρ_{xy} resulted in $\varkappa = 0.75 \pm 0.12$ while the width of the ρ_{xx} peak gave $\varkappa = 0.63 \pm 0.08$. According to [6] for spin degenerate levels the scaling exponent is half of that of the non-degenerate levels, i.e. $\varkappa/2$. In this sample the experimental value for the $N = 1$ level was found to

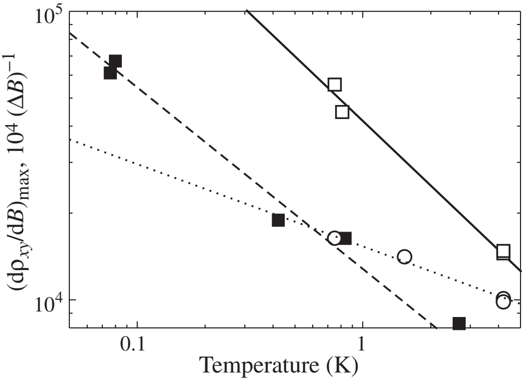

Figure 2. Maxima of $d\rho_{xy}/dB$ for $N = 0$ (squares) and 1 (circles), and reciprocal halfwidth of ρ_{xx} for $N = 0$ (filled squares) as function of the temperature.

be $\varkappa/2 = 0.29 \pm 0.03$, which is close to but somewhat less than the half of the value derived for the $N = 0$ level.

In the analysis of data taken in other similar samples transitions corresponding to $i = 1$ to 2 ($N = 0$ spin-down Landau level), to $i = 2$ to 4 ($N = 1$, no spin splitting) and $i = 2$ to 3 and 3 to 4 ($N = 1$ spin-splitted levels), and in a few cases also to $i = 4$ to 6 ($N = 2$, no spin splitting) were evaluated.

The values of the $\varkappa$ scaling exponent for the plateau-to-plateau transitions involving the $N = 0$ and the spin degenerate $N = 1$ Landau levels were found to lie consistently in the range of 0.6 to 0.8, however the estimated error was rather large, usually $\pm$ (0.1 to 0.2). In some samples the $i = 2$ to 3 and 3 to 4 transitions in the $N = 1$ level were partially resolved. In these cases the scaling exponent $\varkappa$ for the separate spin-up and spin-down levels were in the range from 0.45 to 0.55, i.e. smaller than the values obtained for the $N = 0$ level, but still greater than the presumed universal value [1, 4]. Finally the $i = 4$ to 6 transition in the spin degenerate $N = 2$ Landau level yielded values for $\varkappa/2$ in the range from 0.22 to 0.33.

Beyond the $N = 0$ Landau level with increasing magnetic field an insulating phase emerges [17, 19, 30], the onset of which is shifted toward higher fractional filling factors $\nu = en_s/hB$, i.e. lower magnetic field with increasing disorder in the 2DEG. For a sample with high disorder (and very low 2DEG concentration) Fig. 3 shows the QHL ($i = 1$)-to-IP transition. The transition from the quantum Hall liquid to the insulating phase occurring at $B = B_c$ is determined by the crossing points of the $\rho_{xx}(B)$ curves measured at different temperatures. At this point the direction of the temperature dependence of the resistivity changes sign. For

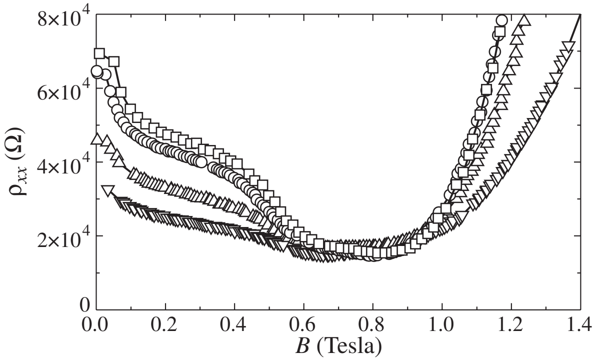

Figure 3. QHL ($i = 1$)-to-IP transition with increasing magnetic field in a sample with strong disorder ($n_s = 2.7 \times 10^{10}$ cm^{-2}). Down triangles — 800 mK, up triangles — 400 mK, circles — 200 mK, squares — 100 mK.

$B < B_c$ the longitudinal resistivity decreases with decreasing temperature (metallic behaviour in the QHL), for $B > B_c$ the longitudinal resistivity increases with decreasing temperature (insulating behaviour in the IP). Supposing the validity of the scaling law, $\rho_{xx}(B) = f(\mid B - B_c \mid T^{-\varkappa})$, the scaling exponent is obtained from $(d\rho_{xx}/dB)|_{B=B_c} \sim T^{-\varkappa}$. For the correct value of $\varkappa$ the ρ_{xx} curves measured at different temperatures, when plotted in function of $\mid B - B_c \mid T^{-\varkappa}$ collapse into

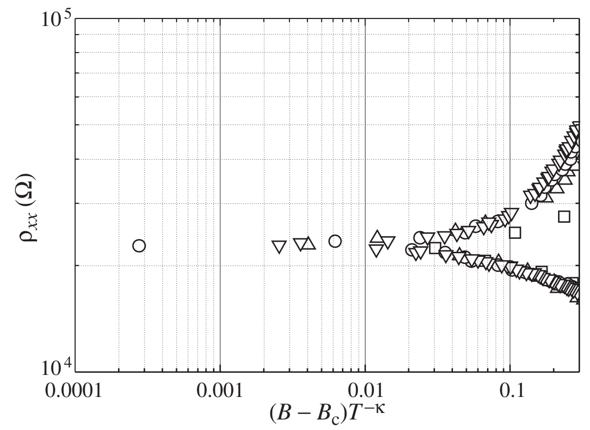

Figure 4. Longitudinal resistivity ρ_{xx} versus $\mid B - B_c \mid T^{-\varkappa}$ with the scaling exponent determined as $\varkappa = 0.77 \pm 0.08$ to collapse the data points at different temperatures to a single curve.

one single curve with two branches, one branch for $B < B_c$, the other one for $B > B_c$, as shown in Fig. 4, with $\varkappa = 0.77 \pm 0.08$ for this particular sample.

In other samples too the values of the $\varkappa$ exponent for the QHL $(i = 1)$-to-IP transition were also found in the range of 0.6 to 0.8, with an estimated error of about ± 0.1. The critical filling factor $\nu_c = en_s/hB_c$ for this transition was in the range from about 0.5 to 1, and the critical value of ρ_{xx} at the transition amounted to $\rho_c = (0.8 - 1.3)h/e^2$.

4. Discussion

The experimentally determined scaling exponent for the $i = 1$ to 2 plateau-to-plateau transition and for the $i = 1$ Hall plateau-to-insulating phase transition is the same within experimentally error. This indicates that both transitions belong to the same universality class, in accordance with the literature. However the value of the $\varkappa$ exponent found in our liquid phase epitaxially grown InGaAs/InP samples for these two transitions is significantly greater than the value of ~ 0.42 hitherto considered to be universal. However the value of $\varkappa = 0.6 - 0.8$ for the QHL $(i = 1)$-to-IP transition found here agrees with the values found recently for the same transition in vapour phase epitaxial InGaAs/InP heterostructures [21, 23] and also in Si/SiGe heterostructures [20].

The larger values of the scaling exponent for the plateau-to-plateau transition in the low index ($N = 0$, 1 and 2) Landau levels found in this work, especially their deviation from the presumed universal value are similar to that found in several other works [5, 8, 33]. As already mentioned above the scaling exponent can be expressed as $\varkappa = p/2\alpha$ [1, 4, 5, 13]. The numerically calculated value of the localization length exponent (which is predicted to be universal [1]) is $\alpha = 2.35 \pm 0.03$ in the lowest Landau level for a potential with short-range correlations [13, 34]. For zero magnetic field $p = 1$ in the "dirty metal limit" and $p = 2$ for "clean samples" in the Fermi liquid theory [33]. The value of $\varkappa = 0.42$, considered universal in the literature, is in accordance with $p = 2$. On the other hand the experimental values of $\varkappa$ found in this work would indicate larger values of the inelastic scattering rate p in the range from 2.8 to 3.7, at least for the InGaAs/InP system with relatively large disorder studied here. Our conclusions concerning the larger values of the p exponent are in accordance among others with the results obtained in the AlGaAs/GaAs heterostructures [5, 33], where on the basis of similar p values it was stated that the actual value of p can be substantially larger in high magnetic fields compared to the theoretical zero-magnetic-field results.

5. Conclusions

We have studied the scaling behaviour of the quantum Hall plateau ($i = 1$)-to-insulating phase transition and of the transitions between the low index quantum Hall plateaus in 2DEG in InGaAs/InP heterostructures.

The scaling exponent for the transition from the first quantum Hall plateau to the insulating phase (supposed Hall insulator) was found equal to that for the quantum Hall plateau-to-plateau transition in the same material system, i.e. $\varkappa \approx 0.6-0.8$. This value of the scaling exponent is significantly greater than the value of $\varkappa \approx 0.42$, considered up to now universal in the literature. The experimental results have been interpreted by considering the enhanced values of the inelastic scattering rate exponent.

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