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## Lateral surface superlattices: minibands, transport, and chaos

## U. Rössler

Fakultät für Physik, Universität Regensburg, D-93040 Regensburg, Germany

The two-dimensional electron gas (2DEG), realized in semiconductor heterostructures and quantum wells, has become a model system for many different investigations. One field of research is based on applying modern semiconductor technologies, to impose a lateral periodic structure onto the 2DEG with periods a (now in the range of 100 nm) which are much smaller than the electron mean-free path  $l_{mfp}$  at low temperatures but comparable to the Fermi wavelength  $\lambda_F$  [1–3]. Thus, in a low-temperature transport experiment, the electrons can explore the periodic potential landscape without being scattered by impurities or phonons ( $l_{mfp} \ll a$ ). Considering at the same time that for the prevailing carrier densities  $\lambda_F = 2\pi/k_F$  is comparable to a, the electrons in a lateral superlattice represent a model of a solid, but with lattice constants very much larger than the crystalline ones of a few Å. The characteristic energy spectrum of electrons in a periodic potential is a band structure, which due to the scaling of the lattice constant results here in minibands, with widths and gaps in the meV range. The demonstration of miniband formation has been one of the challenges of this system.

Experimental investigations of these systems preferentially employ magnetotransport measurements with the magnetic field applied perpendicular to the plane of the 2DES. With the magnetic length  $l_B = \sqrt{\hbar/eB}$  a new scale is introduced which can be tuned from the outside to study different regimes. If  $l_B \ll a$ , the periodic modulation becomes irrelevant and the electrons behave (up to small modifications) as the free 2D electrons in high magnetic field where the magnetoconductivity shows Shubnikov–de-Haas (SdH) oscillations. However, for magnetic fields with  $l_B \simeq a$ , the conductivity is significantly determined by the periodic structure, showing for  $\lambda < a$  and strong potential modulation pronounced commensurability peaks [3], which are well described by classical [4] and quantum-mechanical [5, 6] transport simulations using the Kubo formula.

For even smaller magnetic fields in only moderately modulated systems and  $\lambda_F \simeq a$ , the electron motion can be described in the semiclassical picture which employs the formation of a band structure (with small gaps) and Fermi contours defining **k**-space orbits for electrons moving in the applied magnetic field. Within this picture, observed magnetotransport oscillations have been identified as finger-prints of a band structure in lateral surface superlattices [7].

In the classical regime, the 2DEG with periodic potential and perpendicular magnetic field represents a system with nonlinear dynamics. Depending on the relation between  $l_B$ , a, and the cyclotron radius  $R_c = l_B^2 k_F$ , the electrons move on (distorted) localized cyclotron orbits (for high magnetic field) but explore the hole space in chaotic orbits at lower fields. Commensurability peaks can be well understood as a result of this chaotic behavior by simulating magnetotransport with the classical Kubo formula [4]. In a quantum-mechanical picture the signatures of chaos are identified by level statistics [8–12].

One of the most striking properties of the electronic structure in a periodic potential in dependence on a magnetic field is the Hofstadter butterfly [13, 14]: depending on the magnetic flux threading a unit cell the electronic spectrum shows a fractal structure of bands and gaps which after long efforts has finally been verified in magnetotransport experiments [15].

Besides by electrostatic modulation investigations have been carried out also for lateral surface superlattices with magnetostatic modulation [16, 17], with a periodic structure created by superconducting islands [18], and for 2D hole systems [19].

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## Разбавленные ферромагнитные полупроводники. Теория и эксперимент

К. А. Кикоин

Университет Бен-Гурион, Беер-Шева, Израиль

Разбавленные магнитные полупроводники (РМП) — это полупроводниковые соединения группы  $A_3B_5$ , в которых часть атомов одной из подрешеток замещена примесями переходных или редкоземельных металлов. При достаточно высокой концентрации магнитных ионов в сплаве возникает ферромагнитный порядок. Если температура Кюри  $T_C$  достаточно высока, то появление такого материала чрезвычайно расширяет возможности современной микроэлектроники и спинтроники. Марганец оказался наиболее подходящей магнитной примесью благодаря своей высокой растворимости и диффузионной