Static and Rotating Neutron Stars in a General Relativistic Formulation of all Fundamental Interactions

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Abstract

We formulate the equations of equilibrium of static neutron stars taking into account strong, weak, electromagnetic, and gravitational interactions within the framework of general relativity. We develop the model fulfilling global and not local charge neutrality. The equilibrium conditions are given by our recently developed theoretical framework based on the Einstein-Maxwell-Thomas-Fermi equations along with the constancy of the general relativistic Fermi energies of particles, the "Klein potentials", throughout the configuration. From the microphysical point of view, the weak interactions are accounted for by requesting the β stability of the system, and the strong interactions by using the σ-ω-ρ nuclear model, where σ, ω and ρ are the mediator massive vector mesons. The equations are solved numerically in the case of zero temperatures and for selected parameterizations of the nuclear models. The solutions lead to a new structure of the star: a positively charged core at supranuclear densities surrounded by an electronic distribution of thickness $\sim \hbar/(m_e c) \sim 10^2\hbar/(m_\pi c)$ of opposite charge, as well as a neutral crust at lower densities. Inside the core there is a Coulomb potential well of depth $\sim m_\pi c^2/e$. The constancy of the Klein potentials in the transition from the core to the crust, impose the presence of an overcritical electric field $\sim (m_\pi/m_e)^2 E_c$, the critical field being $E_c = m_e^2 c^3/(e\hbar)$. The electron chemical potential and the density decrease, in the boundary interface, until values $\mu_e^{\rm crust} < \mu_e^{\rm core}$ and $\rho_{\rm crust} < \rho_{\rm core}$. For each central density, an entire family of core-crust interface boundaries and, correspondingly, an entire family of crusts with different mass and thickness, exist. The configuration with $\rho_{\rm crust} = \rho_{\rm drip} \sim 4.3 \times 10^{11}$ g/cm^3 separates neutron stars with and without inner crust. The equilibrium configurations of slowly rotating neutron stars are obtained by using the Hartle formalism in the case of the EMTF equations indicated above. We integrate these equations of equilibrium for different central densities ρ_c and circular angular velocities Ω and compute the mass M, polar R_p and equatorial $R_{\rm eq}$ radii, angular momentum J, eccentricity ϵ, moment of inertia I, as well as quadrupole moment Q of the configurations. Both the Keplerian mass-shedding limit and the axisymmetric secular instability are used to construct the new mass-radius relation. We compute the maximum and minimum masses and rotation frequencies of neutron stars. We compare and contrast our globally neutral solutions with the locally neutral obtained from the traditional Tolman-Oppenheimer-Volkoff treatment.

Thermodynamical Equilibrium, Charge Neutrality and Boundary Conditions

For a Fermionic self-gravitating system in GR, the thermodynamical equilibrium is ensured by the constancy both of the temperature (here $T = 0$) and the chemical potential (Klein Potential)

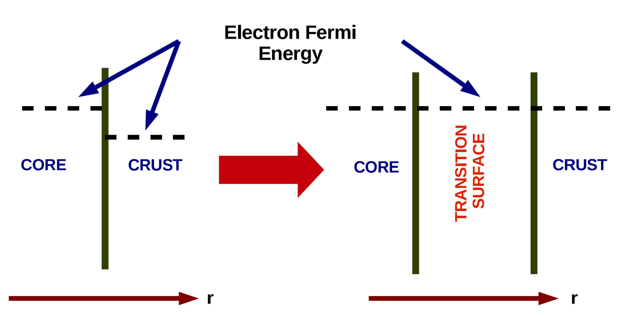
- gravitational field $\rightarrow \mu_{\rm K.P.} = \sqrt{g_{00}}\mu = const$ [1]
- gravitational and electric field + strong interactions $\rightarrow \mu^{(i)}_{\rm K.P.} = \sqrt{g_{00}}(\mu_i + q_i A_\alpha u^\alpha + S.\,I.) = const$ [2]

It has been shown[3] that

- a self-gravitating system of degenerate neutrons, protons and electrons in β-equilibrium fulfill Global Charge Neutrality and *NOT* Local Charge Neutrality, the latter violating the thermodynamical equilibrium conditions
- the mass-difference between electrons and protons induces the presence of an electric field $\rightarrow$ the core is charged
- to solve such a system, the Tolman-Oppheneimer-Volkoff (TOV) equations must be replaced by the general relativistic Thomas-Fermi equations, coupled with the Einstein-Maxwell ones (EMTF equations)

To join the thermodynamical equilibrium we have to replace the TOV-like solution with a new configuration, where core and crust are separated by a transition layer, allowing the constancy of the Klein Potentials:

New Boundary Conditions...



EMTF System of Equations[4]

$$\left.\begin{aligned} e^{-\lambda(r)}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} &= -8\pi G T_0^0 \\ e^{-\lambda(r)}\left(\frac{1}{r^2} + \frac{1}{r}\frac{d\nu}{dr}\right) - \frac{1}{r^2} &= -8\pi G T_1^1 \end{aligned}\right\} \textit{TOV - like eqs.}$$

$$\left.\frac{d^2V}{dr^2} + \frac{dV}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] = -e^{\lambda} e J_0^{ch}\right\} \textit{Electric field}$$

$$\left.\begin{aligned} \frac{d^2\sigma}{dr^2} + \frac{d\sigma}{dr}\left[\frac{2}{r} + \frac{1}{2}\left(\frac{d\nu}{dr} - \frac{d\lambda}{dr}\right)\right] &= e^{\lambda}\left[\partial_\sigma U(\sigma) + g_s n_s\right] \\ \frac{d^2\omega}{dr^2} + \frac{d\omega}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] &= -e^{\lambda}\left[g_\omega J_0^\omega - m_\omega^2\omega\right] \\ \frac{d^2\rho}{dr^2} + \frac{d\rho}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] &= -e^{\lambda}\left[g_\rho J_0^\rho - m_\rho^2\rho\right] \end{aligned}\right\} \textit{Strong Ints.}$$

$$\left.\begin{aligned} E_e &= e^{\nu/2}\mu_e - eV = \text{constant} \\ E_p &= e^{\nu/2}\mu_p + \mathcal{V}_p = \text{constant} \\ E_n &= e^{\nu/2}\mu_n + \mathcal{V}_n = \text{constant} \end{aligned}\right\} \textit{Klein Potentials}$$

NS Global Structure

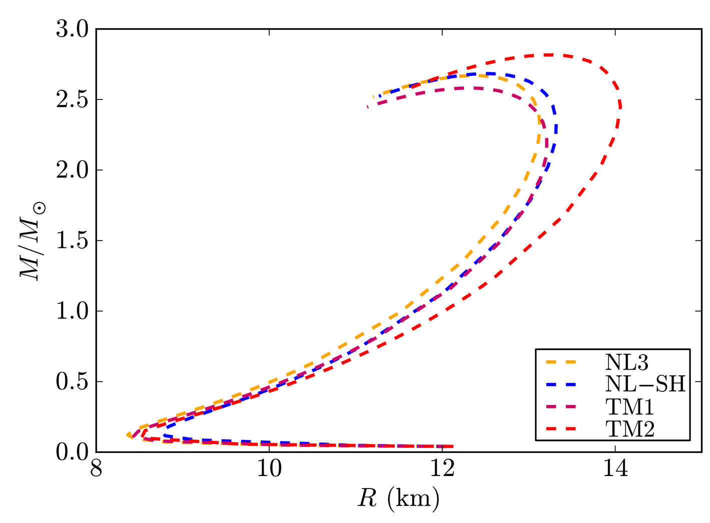


Figure: Total mass-radius relation for four nuclear models. In the crust we have used the Baym-Pethick-Sutherland (BPS) equation of state.

Transition Layer

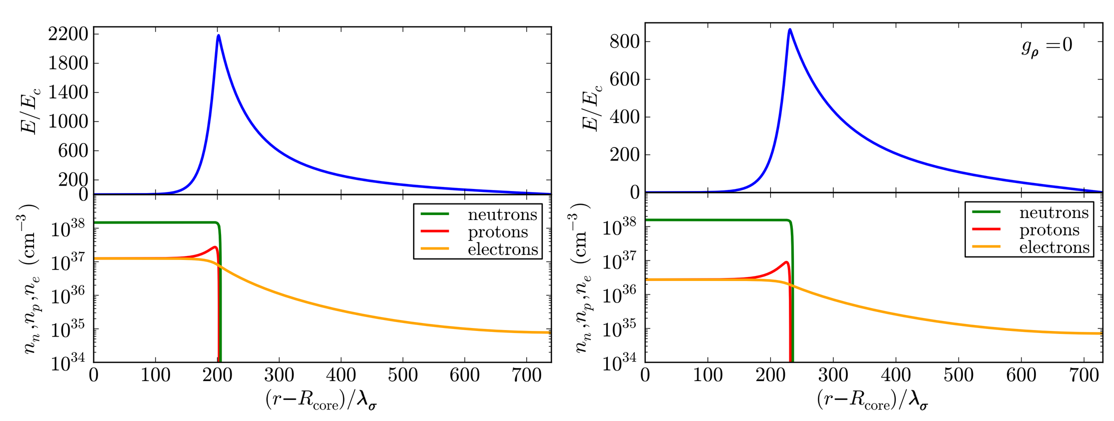


Figure: Left: upper panel: electric field in the transition layer in units of the critical field for vacuum polarization $E_c = m_e^2 c^3/(e\hbar) \sim 1.32 \times 10^{16}$V/cm; lower panel: particle density profiles in the boundary interface in units of cm^{-3}. We use the NL3 equation of state for the nuclear interactions. $\lambda_\sigma = \hbar/(m_\sigma c) \sim 0.4$ fm denotes the sigma-meson Compton wavelength. The density at the edge of the crust in this example is $\rho_{\rm crust} = \rho_{\rm drip}$ and $g_\rho \neq 0$. Right: the same for $g_\rho = 0$.

- Left region: mean-field-like region $\rightarrow$ all the fields vary slowly with length scale $\sim \lambda_e$ (bump due to Coulomb repulsion)
- Central region: strongly interacting region $\rightarrow \sim \lambda_\pi$ (n_n and n_p decreasing due to surface tension plus skin effect).
- Right region: Thomas-Fermi-like region $\rightarrow \sim \lambda_e$ (total screening of the core due to the electronic layer).

Hartle[5,6] Slow Rotation Approximation and Thermodynamical Equilibrium

- Solution obtained thorough a perturbative method, expanding the metric functions up to the second order in the angular velocity Ω.
- The structure of compact objects can be approximately described by M, J and Q.
- Slow rotation regime $\rightarrow$ perturbations owing to the rotation $<$ the known non-rotating geometry.
- Interior solution derived by solving numerically a system of ordinary differential equations for the perturbation functions.
- The exterior solution for the vacuum surrounding the star, can be written analytically in terms of M, J, and Q.

As in the static case, the Klein Potentials have the form

$$\frac{1}{u^t}\left[\mu_i + (q_i A_\alpha + g_\omega \omega_\alpha + g_\rho \tau_{3,i}\rho_\alpha)u^\alpha\right] = \text{constant}$$

but:

- Static Case: only the time components of the vector fields, A_0, ω_0, ρ_0 are present.
- Rotating Case: the fluid inside the star moves with a four-velocity of a rigid rotating body, $u^\alpha = (u^t, 0, 0, u^\phi)$, with

$$u^t = (g_{tt} + 2\Omega g_{t\phi} + \Omega^2 g_{\phi\phi})^{-1/2}, \qquad u^\phi = \Omega u^t$$

where ϕ is the azimuthal angular coordinate with respect to which the metric is symmetric, namely the metric is independent of ϕ (axial symmetry)[7].

Instabilities

- Secular Axisymmetric Instability $\rightarrow$ Turning Point Method

$$\left[\frac{\partial M(\rho_c, J)}{\partial \rho_c}\right]_{J=\text{constant}} = 0$$

- Keplerian Mass-Shedding Instability

$$\Omega_K^{J\neq 0}(r) = \sqrt{\frac{M}{r^3}}\left[1 - jF_1(r) + j^2F_2(r) + qF_3(r)\right]$$

where $j = J/M^2$ and $q = Q/M^3$ are the dimensionless angular momentum and quadrupole moment.

- Gravitational Binding Energy $\rightarrow W_{J\neq 0} < 0$

$$W_{J\neq 0} = W_{J=0} + \delta W\ ,\ \ \delta W = \frac{J^2}{R^3} - \int_0^R 4\pi r^2 B(r)dr$$

where $W_{J=0} = M_0 - M^0_{\rm rest}$ and $M^0_{\rm rest} = m_b A_{J=0}$, being $M^0_{\rm rest}$ is the rest-mass of the star, m_b is the rest-mass per baryon, and $A_{J=0}$ is the total number of baryons inside the star.

Secular Instability Boundary

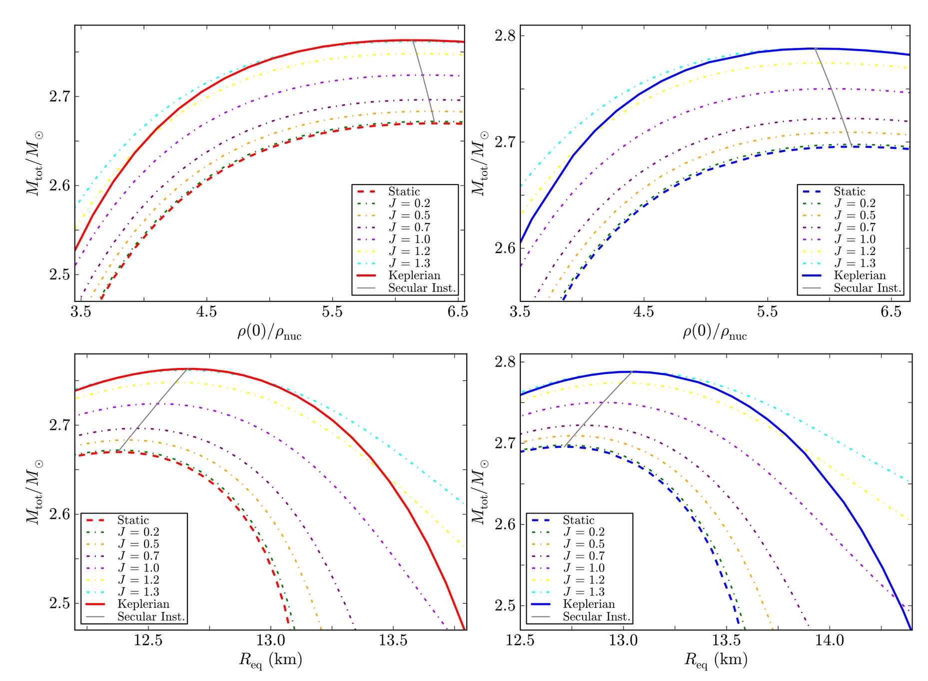
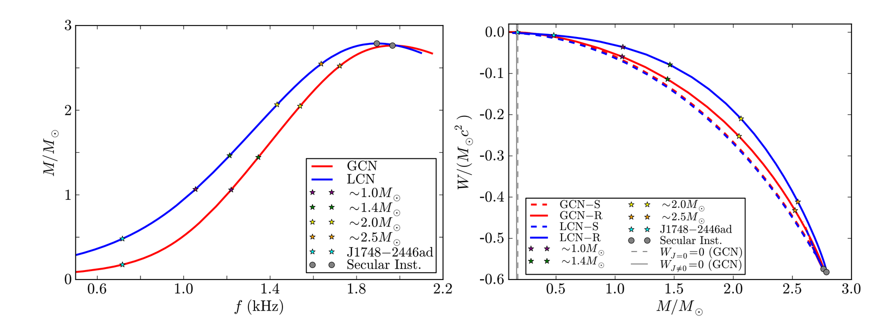
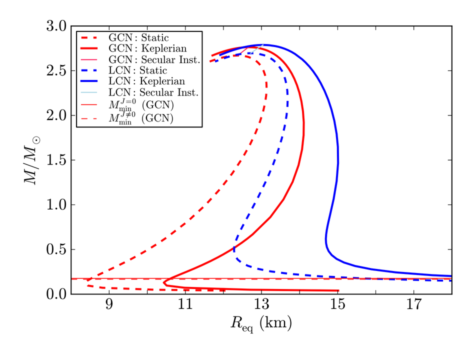


Figure: Left: Global Charge Neutrality. Right: Local Charge Neutrality.

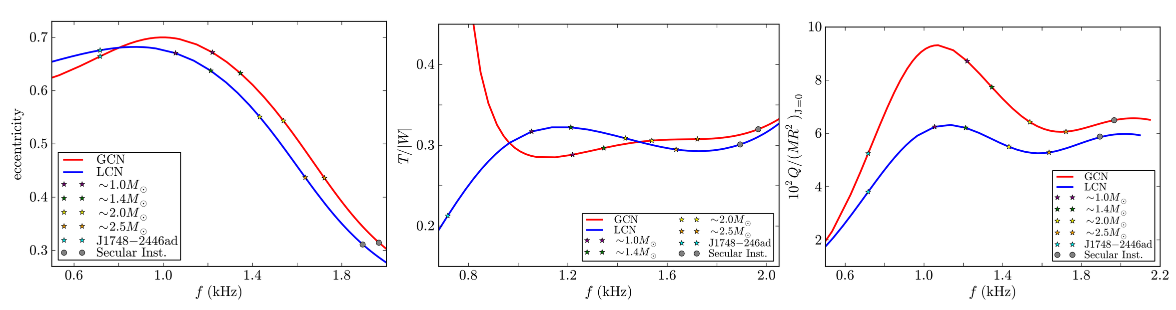
Max and Min Mass and Rotation Frequency



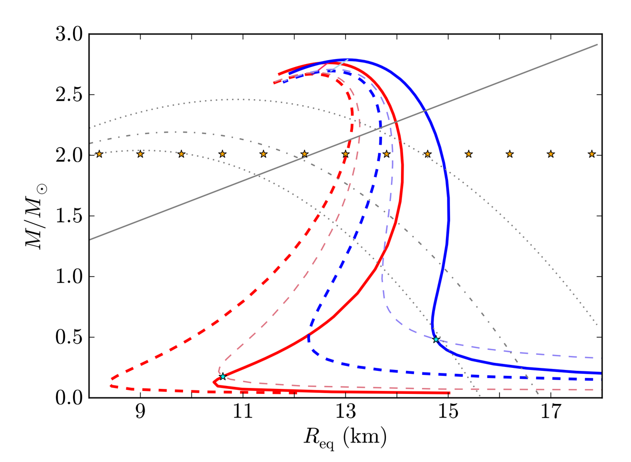


	Global Neutrality	Local Neutrality
$M^{J=0}_{\rm max}$ ($M_\odot$)	2.67	2.70
$M^{J=0}_{\rm min}$($M_\odot$)	0.18	–
$R^{J=0}_{\rm max}$ (km)	12.38	12.71
$M^K_{\rm max}$ ($M_\odot$)	2.76	2.79
$M^K_{\rm min}$($M_\odot$)	0.17	–
$R^K_{\rm max}$ (km)	12.66	13.06
$\delta M_{\rm max}$	3.37%	3.33%
$\delta R^{\rm max}_{\rm eq}$	2.26%	2.75%
$f^K_{\rm max}$ (kHz)	1.97	1.89
$P^K_{\rm min}$ (ms)	0.51	0.53
$f^K_{\rm min}$ (kHz)	0.70	–

ϵ , $T/|W|$, Q



NS Structure[8] and Obs. Constraints



- Global Charge Neutrality. Local Charge Neutrality. Dashed lines:Static configurations. Solid lines: Keplerian sequences.
- solid line $\rightarrow$ upper limit of the surface gravity of XTE J1814-338.
- dotted-dashed curve $\rightarrow$ lower limit to the radius of RX J1856-3754.
- dotted curves $\rightarrow$ 90% confidence level contours of constant R_∞ of the neutron star in the low-mass X-ray binary X7.
- any mass-radius relation must have a maximum mass larger than the mass of PSR J1614-2230, $M = 2.01 \pm 0.04 M_\odot$.

References

1 O. Klein, Rev. Mod. Phys., **21**, 531 (1949)
2 J. A. Rueda, R. Ruffini and S.-S. Xue, Nucl. Phys. A, **872**, 286 (2011)
3 M. Rotondo, Jorge A. Rueda, R. Ruffini and S.-S. Xue, Phys. Lett. B, **701**, 667 (2011)
4 R. Belvedere, D. Pugliese., J. A. Rueda, R. Ruffini and S.-S. Xue, Nucl. Phys. A, **883**, 1 (2012)
5 J. B. Hartle, ApJ, **150**, 1005 (1967)
6 J. B. Hartle and K. S. Thorne, ApJ, **153**, 807 (1968)
7 J. B. Hartle and D. H. Sharp, ApJ, **147**, 317 (1967)
8 R. Belvedere, K. Boshkayev, J. A. Rueda and R. Ruffini, Nucl. Phys. A, **921**, 33 (2014)

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